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# Intensity of gravitational wave emitted by an oscillating Keplerian binary

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## HIGHLIGHTS

- Calculate intensity of gravitational wave for oscillating orbit.
- Calculate intensity of gravitational wave for oscillating circular orbit.
- Calculate intensity of gravitational wave for oscillating elliptic orbit.

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## ABSTRACT

This paper attempts to formulate a way for calculating the intensity of gravitational wave from two point masses in Keplerian circular and elliptic orbits. The intensity is calculated with the assumption that the orbital plane of the binary undergoes small oscillation about the equilibrium x-y plane. This problem is simplification of a physically possible orbit where one of the point masses is spinning whereby the spin-orbit force drives the orbital plane to wobble in a complicated manner. It is shown that the total energy of gravitational wave emitted by the binary in this case is dominated by the parameters which take into account the oscillation of the plane. The results presented are in fact a generalization of the classic results of Landau and Lifshitz.

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## 1. Introduction

Gravitational waves (GWs) have always attracted theoreticians and experimentalists in the field of cosmology and gravitation as a new tool to understand the universe. Scientists have been working vigorously ever since their prediction by the General Theory of Relativity to meet the challenge of detecting the ultra-weak quivers generated by these waves, and thereby unlocking the wealth of information contained in them about our universe and its evolution. As a result, the study of gravitational waves has become the focus of many physicists and lately there have been several works in both to develop theories as well as to improve the technology for detecting them. Earth-based laser-interferometric detectors of gravitational wave are now collecting data, and LIGO has just completed the longest scientific run (Abbott et al. 2016) to date and confirmed their existence. In such an exciting and important time of gravitational wave research, every bit of information on gravitational wave, be it theoretical, computational or experimental, is valuable. Binary star systems, consisting of compact objects such as black holes and neutron stars, are relatively strong sources of gravitational wave. Calculation of energy emitted due to gravitational wave by point masses in Keplerian elliptical orbit was performed by Peters and Mathews (1963); Landau and Lifshitz (1975). In current literature, the objectives of gravitational wave research are more focused on detection of GW (Grote 2008; Berti et al. 2008; Mendell and Wette 2008; Shoemaker et al. 2008) and on evolution of GW sources (Cutler, Kennefick and Poisson 1994; Ryan 1995; Glampedakis and Kennefick 2002; Gergely and Keresztes 2003). In present paper we consider the case that the orbital plane does not remain invariant on a plane. This is quite possible because in a binary where one of the bodies is spinning, the spin orbit-force drives the orbital plane to precession (Vecchio 2004) or to oscillate in a complex manner (Mashhoon and Singh 2006). Such precession or oscillation modulates the GW signal and the total energy emitted also changes. Here we consider a simplified problem from the scenario reported by Mashhoon and Singh (2006). Let two point masses in a Keplerian binary revolve round the center-of-mass in circular orbit and at the same time, the plane of the orbit is undergoing small oscillation about the

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equilibrium x-y plane. We consider that the amplitude of angular oscillation about the x-y plane is very small compared to the radius of the orbit. We then calculate the energy emitted separately in the two polarization modes of gravitational wave and the total energy emitted i.e. intensity in all directions. We found that the amount of emitted energy depends on the nature of oscillation of the plane - in particular, the angular frequency of oscillation about the x-y plane. This is an important finding and we feel that many researchers would like to know the way to this result. The paper is organized as follows: In Section 2, we briefly summarize the important formulae of gravitational wave. Section 3 is about the review of the problem and represents the subsequent calculation of gravitational radiation from oscillating circular orbits and from oscillating elliptic orbits respectively of a Keplerian binary. Finally, Section 4 contains the conclusion.

## 2. Gravitational wave

Gravitational radiation emission from various astrophysical sources has been the focus of many researches (Zimmermann and Szedenits Jr 1999; Beltrami and Chau 1985; Dionysiou 1986; Shibata 1993; Moreno-Garrido, Buitrago and Mediavilla 1994; Moreno-Garrido, Buitrago and Mediavilla 1995; Blanchet 1996). Let us consider a source of gravitational radiation characterized by a mass quadrupole moment tensor $D_{\alpha\beta}$ with the six elements $D_{xx}$, $D_{yy}$, $D_{zz}$, $D_{xy}$, $D_{yz}$, $D_{zx}$, with respect to a set of fixed inertial axes $(x,y,z)$. We define $D_{\alpha\beta}$ as by Landau and Lifshitz (1975), that is,

$$D_{\alpha\beta} = \int \rho\left(3x^{\alpha}x^{\beta} - \delta^{\alpha\beta}r^2\right)dV \tag{1}$$

where $\rho$ is the mass density, and $r^2 = x^2 + y^2 + z^2$, $dV = dxdydz$. The waves can be taken to be plane in view of the typically large distance between the source and the observer. The two independent polarization states of the gravitational wave can be represented by the three-dimensional symmetric, unit polarization tensor $e_{\alpha\beta}$ satisfying the relations

$$e_{\alpha\alpha} = 0,\ e_{\alpha\beta}\, n_{\beta} = 0,\ e_{\alpha\beta}e_{\alpha\beta} = 1, \tag{2}$$

where $\hat{n}$ is a unit vector in the direction of propagation of the wave. Let us label the two polarizations by (Peters and Mathews 1963)

$$e_{+} = \frac{1}{\sqrt{2}}\left(\hat{\theta}\hat{\theta} - \hat{\varphi}\hat{\varphi}\right),\ e_{\times} = \frac{1}{\sqrt{2}}\left(\hat{\theta}\hat{\varphi} + \hat{\varphi}\hat{\theta}\right), \tag{3}$$

where $\theta$ and $\varphi$ are conventional polar coordinates. In this basis, the waveform can be written as (Kochanek et al. 1990)

$$rh = (\ddot{D}_{\theta\theta} - \ddot{D}\varphi\varphi)e_{+} + 2\ddot{D}_{\theta}\varphi e_{\times}, \tag{4}$$

where $h$ is the metric perturbation or the GW waveform, and $D_{\theta\theta}$, $D_{\theta\varphi}$, $D_{\varphi\varphi}$ are the physical components of $D_{ij}$ (the Cartesian components of quadrupole tensor) projected along the directions of the spherical unit vectors $\hat{\theta}$ and $\hat{\varphi}$. There exists canonical procedure for obtaining these components, but we simply quote the results from Kochanek et al. (1990):

$$\begin{aligned} D_{\theta\theta} &= \left(D_{xx}cos^2\varphi + D_{yy}sin^2\varphi + D_{xy}sin2\varphi\right)cos^2\theta \\ &\quad + D_{zz}sin^2\theta - (D_{xz}cos\varphi + D_{yz}sin\varphi)sin2\theta, \\ D_{\varphi\varphi} &= D_{xx}sin^2\varphi + D_{yy}cos^2\varphi - D_{xy}sin2\varphi, \\ D_{\theta\varphi} &= -\frac{1}{2}(D_{xx} - D_{yy})cos\theta sin2\varphi + D_{xy}cos\theta cos2\varphi \\ &\quad + (D_{xz}sin\varphi - D_{yz}cos\varphi)sin\theta. \end{aligned} \tag{5}$$

The expressions for the intensity of radiation of a given polarization into solid angle $d\Omega$ are (Landau and Lifshitz 1975)

$$dI = \frac{G}{72\pi c^5}\left(\frac{d^3D_{\alpha\beta}}{dt^3}e_{\alpha\beta}\right)^2 d\Omega \tag{6}$$

where $G$ is the Newton's gravitational constant and c is the speed of light in free space. Using Eqs. (3) and (4), we can write for the intensity of GW in $(\times)$ polarization as

$$\frac{dI_1}{d\Omega} = \frac{G}{72\pi c^5}\left(2\frac{d^3D_{\theta\varphi}}{dt^3}\frac{1}{\sqrt{2}}\right)^2 = \frac{G}{36\pi c^5}\left(\frac{d^3D_{\theta\varphi}}{dt^3}\right)^2, \tag{7}$$

and that in $(+)$ polarization as

$$\begin{aligned} \frac{dI_2}{d\Omega} &= \frac{G}{72\pi c^5}\left[\left(\frac{d^3D_{\theta\theta}}{dt^3} - \frac{d^3D_{\varphi\varphi}}{dt^3}\right)\frac{1}{\sqrt{2}}\right]^2 \\ &= \frac{G}{144\pi c^5}\left(\frac{d^3D_{\theta\theta}}{dt^3} - \frac{d^3D_{\varphi\varphi}}{dt^3}\right)^2 \end{aligned} \tag{8}$$

Next we apply these formulae to find out the intensity of gravitational wave emitted by a Keplerian binary whose orbital plane is oscillating about the equilibrium x-y plane.

## 3. Intensity of gravitational wave from a binary with oscillating orbital plane

In many astrophysical binary star systems, the orbit of the stars undergoes precession and oscillation due to many perturbing forces, such as, spin-orbit, spin-spin interactions. Specifically, the spin-orbit force drives the orbital plane to oscillate about the equilibrium plane in a quite complicated manner. One typical case is analyzed by Mashhoon and Singh (2006).

We consider a simplified situation defined by an almost fixed orbital plane confined to the x-y plane, but the orbital plane undergoes very small angular oscillation about the equilibrium x-y plane. This situation simulates some of the characteristics of orbital motion of a Keplerian binary with one particle having small spin. Now, we define the orbit by the following orbital variables:

$$r = constant,\ \theta = \frac{\pi}{2} - bsin\frac{\omega}{n}t,\ \varphi = \omega t \tag{9}$$

where $\omega$ is the Newtonian angular frequency of the orbit in the x-y plane, $r = |\vec{r_1} - \vec{r_2}|$; $\vec{r_1}$, $\vec{r_2}$ being the positions of the particles of mass $m_1$ and $m_2$, respectively, and $b$ is a very small parameter $(b \ll 1)$ characterizing the angular oscillation about the x-y plane. Now to simplify, let us consider the frequency of oscillation of the orbital plane is same as the frequency of orbital motion (Mashhoon and Singh 2006). That is the time taken for one complete orbital motion is the same as the time taken for a complete oscillation of the plane. So $\omega$ is same for both. The complicated wobble motion can be represented by a single $\omega$. But when the frequencies are not same, the resulted frequency of the complicated motion can be presented by $\frac{\omega}{n}$ where n is a parameter that relates the two frequencies. That is when $n = 1$, the two frequencies are same.

Now, we approximate the Cartesian components of the vector $\vec{r}$ as:

$$\begin{aligned} x &\cong rcos\omega t \\ y &\cong rsin\omega t \\ z &\cong rbsin\frac{\omega}{n}t \end{aligned} \tag{10}$$

Then, the quadrupole moments are:

$$\begin{aligned} D_{xx} &= \mu r^2\left(3cos^2\omega t - 1\right), \qquad D_{yy} = \mu r^2\left(3sin^2\omega t - 1\right), \\ D_{xy} &= \frac{3}{2}\mu r^2 sin2\omega t, \qquad D_{zz} = \mu r^2\left(3b^2sin^2\frac{\omega}{n}t - 1\right), \\ D_{xz} &= 3\mu br^2\left(cos\omega t sin\frac{\omega}{n}t\right),\ D_{yz} = 3\mu br^2\left(sin\omega t sin\frac{\omega}{n}t\right) \end{aligned} \tag{11}$$

where $\mu = \frac{m_1m_2}{m_1+m_2}$ is the reduced mass of the binary. Now, since the system is rapidly rotating about the z-axis, average over the angle $\varphi$ is appropriate. Next, we take an average over the orbital period. Using Eqs. (5) and (7) and after doing a somewhat lengthy calculation we obtain

$$\left\langle \frac{dI_1}{d\Omega} \right\rangle = \frac{G\mu^2 r^4 \omega^6}{8\pi c^5} \times \left[ 16cos^2\theta + b^2 \left\{ \begin{array}{l} \left(\frac{3+n^2}{n^2}\right)^2 \left(\frac{1}{2} - \frac{n}{8\pi} sin\frac{4\pi}{n}\right) \\ + \left(\frac{1+3n^2}{n^3}\right)^2 \left(\frac{1}{2} + \frac{n}{8\pi} sin\frac{4\pi}{n}\right) \end{array} \right\} sin^2\theta \right] \tag{12}$$

This equation gives the average intensity of gravitational radiation per unit solid angle which is dependent on the oscillation parameters $b$ and $n$. Moreover, the result correctly reduces to the classic result of Landau and Lifshitz (1975) for circular orbit with invariant orbital plane, i.e. when we put $b = 0$ and $n = 1$. Similarly, for (+) polarization

$$\begin{aligned} \left\langle \frac{dI_2}{d\Omega} \right\rangle = \frac{G\mu^2 r^4 \omega^6}{8\pi c^5} \times \Bigg( & cos^4\theta \left[ 4 + 8\frac{b^4}{n^6}\left(\frac{1}{2} - \frac{n}{16\pi} sin\frac{8\pi}{n}\right) \right. \\ & \left. - b^2 \left\{ \begin{array}{l} \left(\frac{3+n^2}{n^2}\right)^2 \left(\frac{1}{2} - \frac{n}{8\pi} sin\frac{4\pi}{n}\right) \\ + \left(\frac{1+3n^2}{n^3}\right)^2 \left(\frac{1}{2} + \frac{n}{8\pi} sin\frac{4\pi}{n}\right) \end{array} \right\} \right] \\ & + cos^2\theta \left[ 8 - 16\frac{b^4}{n^6}\left(\frac{1}{2} - \frac{n}{16\pi} sin\frac{8\pi}{n}\right) \right. \\ & \left. + b^2 \left\{ \begin{array}{l} \left(\frac{3+n^2}{n^2}\right)^2 \left(\frac{1}{2} - \frac{n}{8\pi} sin\frac{4\pi}{n}\right) \\ + \left(\frac{1+3n^2}{n^3}\right)^2 \left(\frac{1}{2} + \frac{n}{8\pi} sin\frac{4\pi}{n}\right) \end{array} \right\} \right] \\ & + 4 + 8\frac{b^4}{n^6}\left(\frac{1}{2} - \frac{n}{16\pi} sin\frac{8\pi}{n}\right) \Bigg) \end{aligned} \tag{13}$$

This result also reduces to the classic result of Landau and Lifshitz (1975) in the $b = 0$ and $n = 1$ limit. The total intensity of GW radiation in the present context is found by summing Eqs. (12) and (13). Let us consider

$$P = \left(\frac{3+n^2}{n^2}\right)^2 \left(\frac{1}{2} - \frac{n}{8\pi} sin\frac{4\pi}{n}\right) + \left(\frac{1 + 3n^2}{n^3}\right)^2 \times \left(\frac{1}{2} + \frac{n}{8\pi} sin\frac{4\pi}{n}\right), Q = \left(\frac{1}{2} - \frac{n}{16\pi} sin\frac{8\pi}{n}\right)$$

Using this, we obtain the total intensity as

$$\left\langle \frac{dI}{d\Omega} \right\rangle = \frac{G\mu^2 r^4 \omega^6}{8\pi c^5} \left[ 4 + b^2 P + 8\frac{b^4}{n^6} Q + \left(24 - 16\frac{b^4}{n^6} Q\right) cos^2\theta + \left(4 + 8\frac{b^4}{n^6} Q - b^2 P\right) cos^4\theta \right] \tag{14}$$

Integrating this expression over all directions, we get the energy radiated in gravitational wave in all directions per unit time or intensity as

$$-\frac{dE}{dt} = I = \frac{2G\mu^2 r^4 \omega^6}{5c^5} \left[ 16 + b^2 P + \frac{16}{3}\frac{b^4}{n^6} Q \right] \tag{15}$$

which reduces to the classic result of Landau and Lifshitz (1975), for the case of circular binary orbit fixed in the x-y plane, using $b = 0$ and $n = 1$, to

$$I = \frac{32G\mu^2 r^4 \omega^6}{5c^5} \tag{16}$$

We see that the total energy radiated in gravitational wave by a Keplerian binary system which is undergoing small oscillation of the orbital plane is dependent on the values of $b$ and $P$, $Q$ that is in fact on the value of $n$. Since $b$ and $n$ cannot have negative value, the amount of radiated energy is more than that emitted by a binary system with invariant orbital plane. Hence, we found an important result, namely, the intensity of GW emitted by an astrophysical binary that is undergoing small orbital plane oscillation about the equilibrium x-y plane, given by Eq. (15).

We now extend the calculation of the GW intensity to the case of elliptic binary orbit. In reality, astrophysical binary orbits are elliptic. The orbit may also undergo oscillation. Therefore, a calculation of the GW intensity from such binaries will be worthwhile to be carried out. With this motivation, we proceed with the assumption that the orbital plane undergoes small oscillation similar to what we have considered in the previous chapter. For this, we consider the same parameters '$n$' and '$b$' as we did in the previous chapter. Also, here we use the same notations of the former chapter. We define the orbital elements as that of a Keplerian elliptic orbit. In particular, we assume the following orbital parameters:

$$\begin{aligned} \theta &= \frac{\pi}{2} - bsin\varphi, \; r = \frac{a(1-e^2)}{1+ecos\varphi}, \\ \frac{d\varphi}{dt} &= \frac{\left[G(m_1+m_2)a(1-e^2)\right]^{\frac{1}{2}}}{nr^2} \end{aligned} \tag{17}$$

where $a$ and $e$ are the semi-major axis and eccentricity respectively of an elliptic binary orbit. The quadrupole moments are

$$\begin{aligned} D_{xx} &= \mu r^2(3cos^2\varphi - 1), & D_{yy} &= \mu r^2(3sin^2\varphi - 1) \\ D_{zz} &= \mu r^2(3b^2 sin^2\varphi - 1), & D_{xy} &= \tfrac{3}{2}\mu r^2 sin2\varphi \\ D_{xz} &= \tfrac{3}{2}\mu b r^2 sin2\varphi, & D_{yz} &= 3\mu b r^2 sin^2\varphi \end{aligned} \tag{18}$$

We obtain the intensity of (×) polarization component of gravitational wave from elliptical and oscillating binary, after a long calculation as

$$\left\langle \frac{dI_1}{d\Omega} \right\rangle = \frac{G^4 m_1^2 m_2^2 (m_1+m_2)}{n^5 \pi c^5 a^5 (1-e^2)^{7/2}} \left[ \begin{array}{l} \left(2 + \frac{97}{16}e^2 + \frac{49}{64}e^4\right)\cos^2\theta \\ + b^2\left(2 + \frac{111}{16}e^2 + \frac{29}{32}e^4\right)\sin^2\theta \end{array} \right] \tag{19}$$

This equation gives the average intensity of gravitational wave per unit solid angle which depends on the oscillation parameters $b$, $n$ and eccentricity $e$. If $e = 0$, i.e., for circular orbit, this equation reduces to Eq. (12). Note that $\omega$ in Eq. (12) is now given by $\frac{d\varphi}{dt}$ of Eq. (17) with $e = 0$. The exactness of Eqs. (12) and (19) for $e = 0$ is clearly evident. For the (+) polarization the calculations are more involved and lengthy. One would find the following result, (we have neglected terms of order higher than $b^2$):

$$\left\langle \frac{dI_2}{d\Omega} \right\rangle = \frac{G^4 m_1^2 m_2^2 (m_1+m_2)}{n^5 \pi c^5 a^5 (1-e^2)^{7/2}} \times \left[ \begin{array}{l} \left(\frac{1}{2} + \frac{99}{64}e^2 + \frac{51}{256}e^4 - 2b^2 - \frac{131}{16}e^2b^2 - \frac{63}{64}e^4b^2\right)cos^4\theta \\ + \left(1 + \frac{95}{32}e^2 + \frac{47}{128}e^4 + 2b^2 + \frac{63}{8}e^2b^2 + \frac{17}{16}e^4b^2\right)cos^2\theta \\ + \frac{1}{2} + \frac{99}{64}e^2 + \frac{51}{256}e^4 - \frac{15}{48}e^2b^2 - \frac{5}{64}e^4b^2 \end{array} \right] \tag{20}$$

For $e = 0$, this equation correctly gives formula (13) for circular orbit (note that in (20) no $b^4$ terms are included, however, in (13) there are). The results we have obtained here coincide with the results of Peters and Mathews (1963) when we put $b = 0$ and $n = 1$ in the formulae (19) and (20).

Next, we calculate the total energy radiated in GW in all directions per unit time or intensity as

$$-\frac{dE}{dt} = I = \frac{32G^4 m_1^2 m_2^2 (m_1+m_2)}{5n^5 c^5 a^5 (1-e^2)^{7/2}} \times \left[ 1 + \frac{73}{24}e^2 + \frac{37}{96}e^4 + b^2\left(1 + \frac{53}{16}e^2 + \frac{41}{72}e^4\right) \right] \tag{21}$$

This is the generalization of the result of Landau and Lifshitz (1975) from a Keplerian binary in elliptic orbit because we take into account the oscillation of the plane. The presence of small oscillation is evident from theoretical investigation performed by Mashhoon and Singh (2006). The amount of radiated energy can be calculated by putting the values of corresponding parameters. From the equation we can see that the amount of radiated energy varies greatly with $n$ since $n$ has power of 5. Though the exact values of $b$ and $n$ are not known yet, we hope in near future the values can be obtained by observing the astrophysical binaries more acutely.

## 4. Conclusion

The energy radiated in gravitational wave by a Keplerian astrophysical binary system has been discussed earlier in the context of a binary in circular or elliptical orbit with the plane of the orbit invariant (Peters and Mathews 1963; Landau and Lifshitz 1975). But it is seen in literature (Mashhoon and Singh 2006) that due to spin-orbit interaction, the orbital plane in a physical binary may undergo oscillation. Here we consider the whole scenario, that is the orbital plane of the binary undergoes small oscillation about the equilibrium x-y plane. Our problem is the canonical problem of GW radiation from a circular or elliptic binary, but with additional parameters $b$ and $n$ where $b$ is the amplitude of angular oscillation of the orbital plane and $n$ compares the angular frequency of oscillation to the actual orbital frequency [see, Eqs. (9) and (10)]. Our final results are depicted in Eqs. (15) and (21), which give the intensity or total energy radiated in all directions in GW per unit time averaged over the orbital period. The formulae we obtain, in Eqs. (15) and (21), are an extension of the classic results of Landau and Lifshitz (1975) for the case of GW from circular or elliptic binary with fixed orbital plane. The oscillation we considered is simply dependent on the parameters $b$ and $n$. We consider our simple calculation presented here as a valuable addition to gravitational wave phenomenology that can arouse interest in practical GW researchers.

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## References

Abbott, B.P., et al., 2016. Observation of gravitational waves from a binary black hole merger. Phys. Rev. Lett. 116, 061102.
Beltrami, H., Chau, W.Y., 1985. Analytic result for the properties of gravitational waves emitted by a large class of model sources. Astrophys. Sp. Sci. 111, 335.
Berti, E., Cardoso, V., Gonzalez, J.A., Sperhake, U., Brugmann, B., 2008. Multipolar analysis of spinning binaries, IOP publishing. Class. Quantum Grav. 25, 114035.
Blanchet, L., 1996. Energy losses by gravitational radiation in inspiraling compact binaries to 5/2 post-Newtonian order. Phys. Rev. D 54, 1417.
Cutler, C., Kennefick, D., Poisson, E., 1994. Gravitational radiation reaction for bound motion around a Schwarzschild black hole. Phys. Rev. D 50, 3816.
Dionysiou, D.D., 1986. The Post-Newtonian relative orbit of the two-body system. Gravitational Radiation. Astrophys. Sp. Sci. 125, 115.
Gergely, L., Keresztes, Z., 2003. Gravitational radiation reaction in compact binary systems: contribution of the quadrupole-monopole interaction. Phys. Rev. D 67, 024020.
Glampedakis, K., Kennefick, D., 2002. Zoom and whirl: Eccentric equatorial orbits around spinning black holes and their evolution under gravitational radiation reaction. Phys. Rev. D 66, 044002.
Grote, H., 2008. The status of GEO 600, IOP publishing. Class. Quantum Grav. 25, 114043.
Kochanek, C.S., Shapiro, S.L., Teukolsky, S.A., Chernoff, D.F., 1990. Gravitational radiation from colliding clusters: Newtonian simulations in three dimensions. Astrophys. J. 358, 81.
Landau, L.D., Lifshitz, E.M., 1975. The Classical Theory of Fields, 4th Edition Pergamon Press, Oxford; New York.
Mashhoon, B., Singh, D., 2006. Dynamics of extended spinning masses in a gravitational field. Phys. Rev. D 74, 124006.
Mendell, G., Wette, K., 2008. Using generalized Power Flux methods to estimate the parameters of periodic gravitational waves, IOP publishing. Class. Quantum Grav. 25, 114044.
Moreno-Garrido, C., Buitrago, J., Mediavilla, E., 1994. Spectral analysis of the gravitational radiation emitted by binary systems in moderately eccentric orbits: application to coalescing binaries. Mon. Not. R. Astron. Soc. 266, 16.
Moreno-Garrido, C., Buitrago, J., Mediavilla, E., 1995. Gravitational radiation from point masses in elliptical orbits: spectral analysis and orbital parameters. Mon. Not. R. Astron. Soc. 274, 115.
Peters, P.C., Mathews, J., 1963. Gravitational radiation from point masses in a Keplerian orbit. Phys. Rev 131, 435.
Ryan, F.D., 1995. Gravitational waves from the inspiral of a compact object into a massive, axisymmetric body with arbitrary multipole moments. Phys. Rev. D 52, 3154.
Shibata, M., 1993. Gravitational waves induced by a particle orbiting around a rotating black hole: spin- orbit interaction effect. Phys. Rev. D 48, 663.
Shoemaker, D., Vaishnav, B., Hinder, I., Herrmann, F., 2008. Numerical relativity meets data analysis: spinning binary black hole case, IOP PUBLISHING. Class. Quantum Grav. 25, 114047.
Vecchio, A., 2004. LISA observations of rapidly spinning massive black hole binary systems. Phys. Rev. D 70, 042001.
Zimmermann, M., Szedenits Jr, E., 1999. Gravitational waves from rotating and precessing rigid bodies: simple models and applications to pulsar. Phys. Rev. D 20, 351.