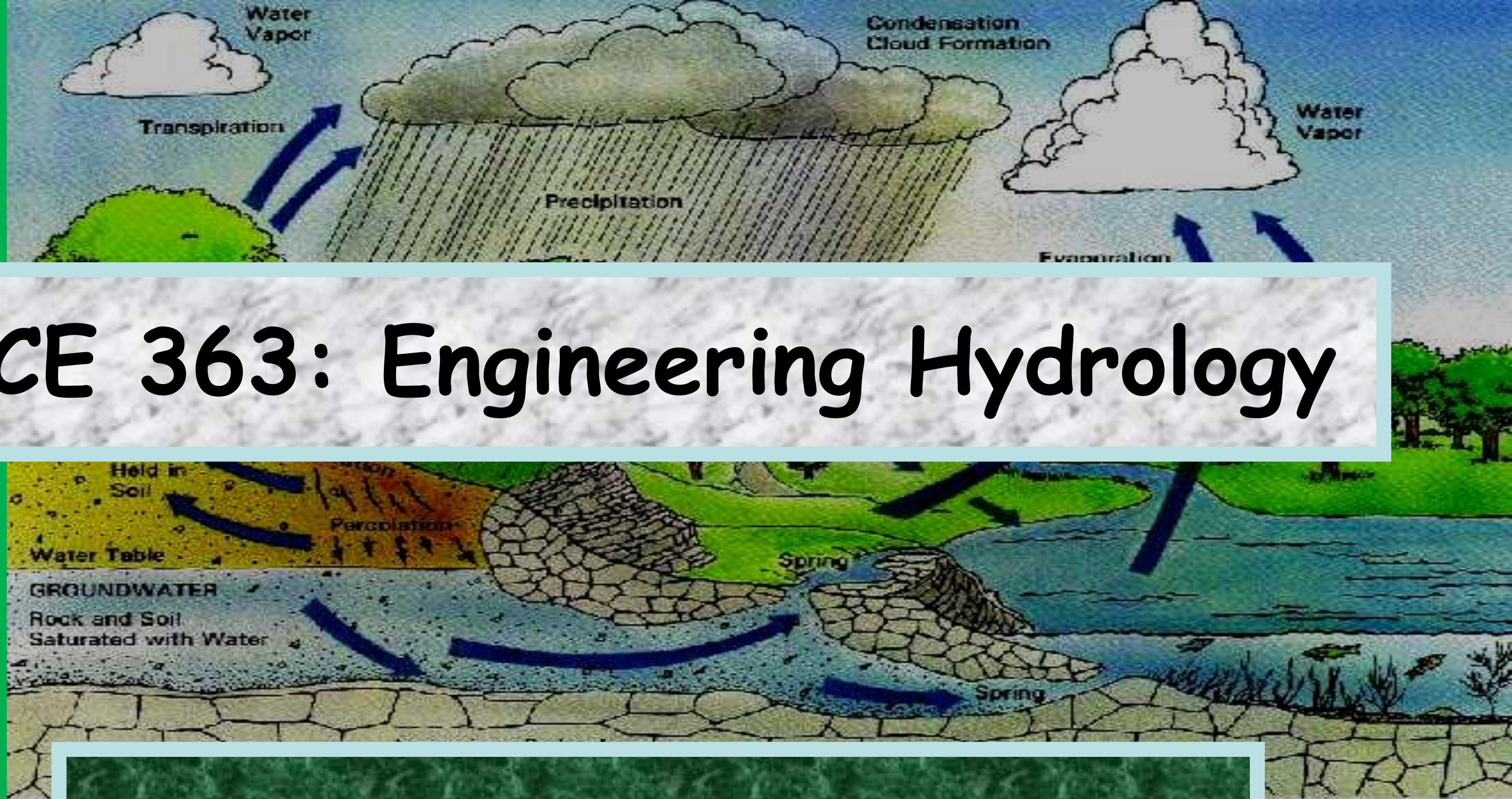
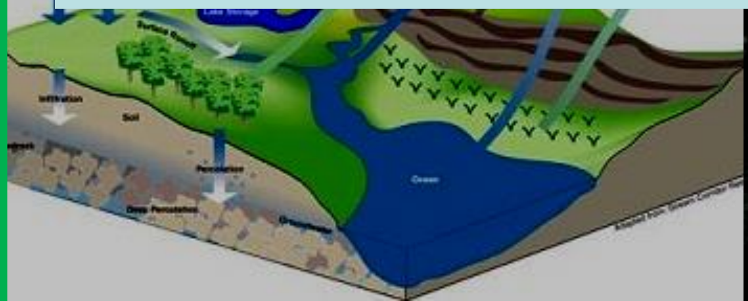


CE 363: Engineering Hydrology

Dr. M. R. Kabir

Professor and Head, Department of Civil Engineering
University of Asia Pacific (UAP), Dhaka



Objective of this course

To understand the following:

- Physical process and quantification of different sub-system of the hydrologic cycle.
- Weather hydrology.
- Rainfall-Runoff relation in a catchment.
- Flood routing for flood forecasting and protection
- Statistical Analysis of Hydrological data

Syllabus

Hydrologic Cycle, Weather and Hydrology,
Precipitation, Evaporation
Evapotranspiration, and Infiltration,
Stream Flow, Rainfall-Runoff Relations,
Hydrographs, Unit Hydrographs,
Hydrologic Routing,
Statistical Methods in Hydrology.

Different Chapters

Chapter	Name of Chapters	No. of Lectures
1	Hydrologic Cycle	3
2	Weather and Meteorology	2
3	Precipitation	3
4	Evaporation and Evapotranspiration	4
5	Infiltration and Other losses	2
	Midterm Examination	
6	Rainfall and Runoff Relationship	2
7	Hydrograph	5
8	Flood Routing	2
9	Statistical Methods of Hydrology	2
10	Stream Flow Measurement	2
	Final Examination	

Lecture Plan

Lecture 1	Introduction, Discuss about class grading.
Lecture 2	Definition of Hydrology, History, Hydrologic related works.
Lecture 3	Hydrologic Cycle, Weather Hydrology, Atmospheric Circulation.
Lecture 4	Front, Storms, Water Vapor, Climate of Bangladesh, Residence Time.
	Class Test – 1
Lecture 5	Water vapor in a static atmospheric column, Precipitable Water.
Lecture 6 -11	Precipitation
	Class Test – 2
Lecture 12 – 14	Evaporation
Lecture 15 – 16	Evapotranspiration

Lecture	17	Initial Loss, Infiltration, Φ – Index
		Class Test – 3
		MID TERM EXAMINATION
Lecture	18	Runoff
Lecture	19 – 22	Hydrograph
Lecture	23 – 26	Unit Hydrograph
		Class Test – 4
Lecture	27	Synthetic Unit Hydrograph
Lecture	28 – 30	Flood Routing
Lecture	31 – 33	Floods
		Class Test – 5
Lecture	34 – 35	Stream flow Measurement
Lecture	36	Review Class
		FINAL EXAMINATION

Reference Books

Engineering Hydrology	Linsley
Applied Hydrology	VT Chow
Engineering Hydrology	**K Subramanya**
Introduction to Hydrology	Warren Viessman
Hydrology and Water Quality Control	Martin Wanielista

Assessment

- Class Assessment 30%

  (Attendance : 10, Min. 4 class test : 20)

- Mid Term Examination 20%

- Final Examination 50%

Thank You

CE 363 : Engineering Hydrology

Fall 2015

Class started on 11. 10. 2015

Course Teacher

Dr. M. R. Kabir

Lecture Plan

Lecture	1	Introduction, Discuss about class grading.
Lecture	2	Definition of Hydrology, History, Hydrologic related works.
Lecture	3	Hydrologic Cycle, Weather Hydrology, Atmospheric Circulation.
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Lecture	6 -11	Precipitation
		Class Test – 2
Lecture	12 – 14	Evaporation
Lecture	15 – 16	Evapotranspiration

Lecture	17	Initial Loss, Infiltration, Φ – Index
		Class Test – 3
		MID TERM EXAMINATION
Lecture	18	Runoff
Lecture	19 – 22	Hydrograph
Lecture	23 – 26	Unit Hydrograph
		Class Test – 4
Lecture	27	Synthetic Unit Hydrograph
Lecture	28 – 30	Flood Routing
Lecture	31 – 33	Floods
		Class Test – 5
Lecture	34 – 35	Stream flow Measurement
Lecture	36	Review Class
		FINAL EXAMINATION

Reference Books

1. Engineering Hydrology

   ---- Linsley

2. Applied Hydrology

   ---- VT Chow

3. Engineering Hydrology

   ---- Subramanya

4. Introduction to Hydrology

   ---- Viessman

5. Hydrology ---- Martin Wanielista

Assessment

- Class Assessment 30%

  (attendance : 10, Min. 4 class test : 20)

- Mid Term Examination 20%

- Final Examination 50%

Hydrology

Hydrology is a multidisciplinary subject that deals with the occurrence, circulation and distribution of waters of the earth.

The study and practice of hydrology aids in explaining and quantifying the occurrence of water on, under and over the earth's surface.

The boundaries between hydrology and other earth sciences such as meteorology, geology, ecology and oceanology are not distinct.

History

- Aristotle : Conversion of moist air into water deep inside the mountains as the source of springs and streams.
- Rome : (97AD) Stream flow measurement based on cross – sectional area of flow.
- Leonards da Vinci : Proper relationship between area velocity and flow rate.
- Perrault (17 century) : Recorded rainfall and surface flow . Relation between rainfall and surface flow was published in 1694.
- Halley (1656 – 1742) : Used small pan to estimate evaporation from the Mediterranean sea.
- 18th Century : Bernoulli's theorem, the pitot tube and Chezy formula.
- 19th Century : signicant advances in G.W hydrology
  - ✓ Darcy's law
  - ✓ Dupuit Thiem well formula

Modern History

- Shereman’s (1932) : Unit hydrograph
- Horton’s (1933) : Infiltration theory
- Theis’s (1935) : Non equilibrium equation in well hydraulics
- Gumbel (1941) : Use of extreme – value distribution for frequency.
  analysis of hydrologic data.

Hydrologic related works

- Flood mitigation
- Roadway design
- Irrigation system
- Navigation
- Water supply
- pollution control
- Hydropower development
- Ecological protection

Hydrologic Cycle

The hydrologic cycle is a continuous process by which water is transported from the oceans to the atmosphere to the land and back to the sea.

The subsystems are :

- Precipitation
- Evaporation
- Evapotranspiration
- Infiltration
- Overland flow
- Stream flow
- Groundwater flow

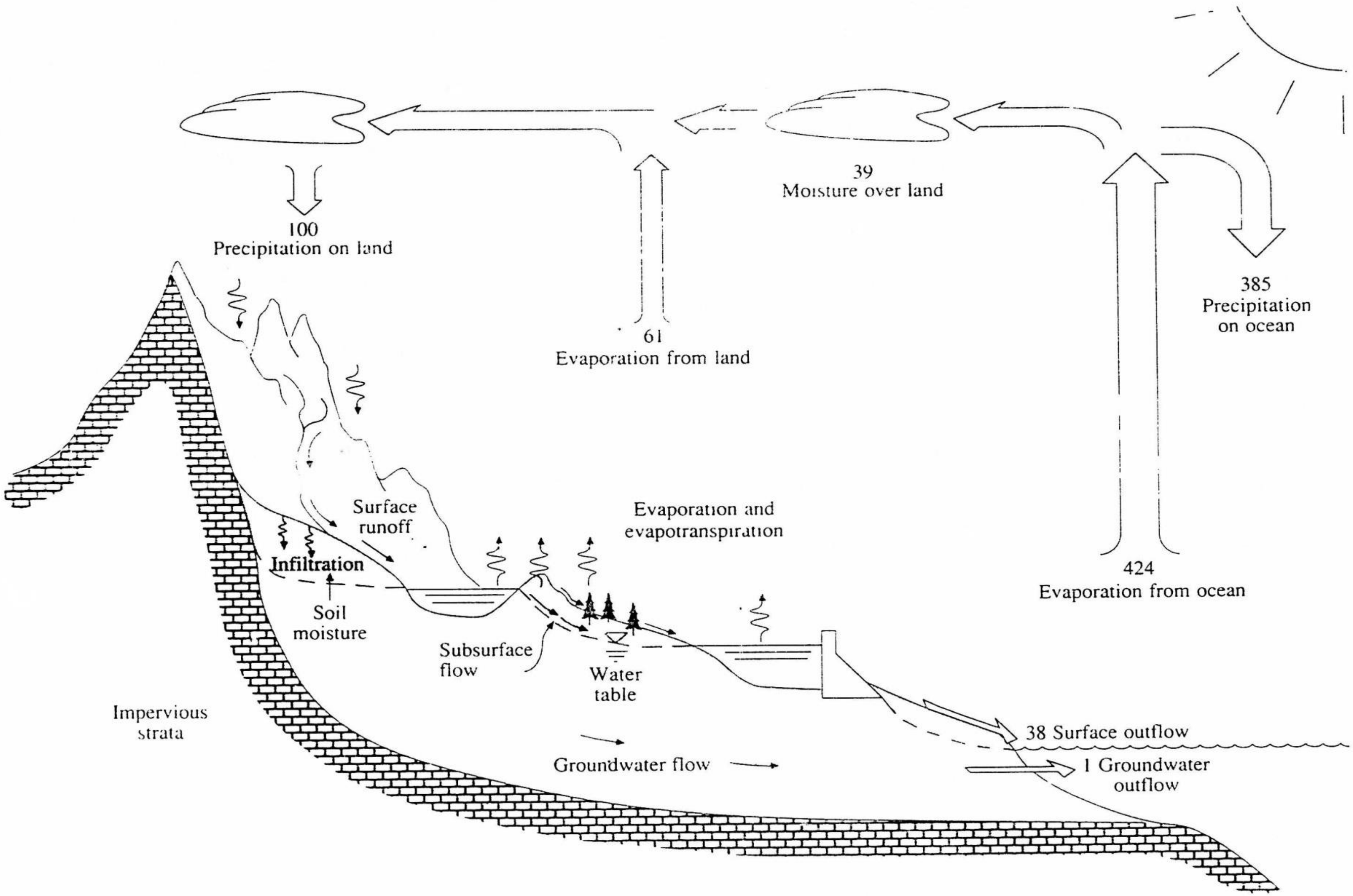
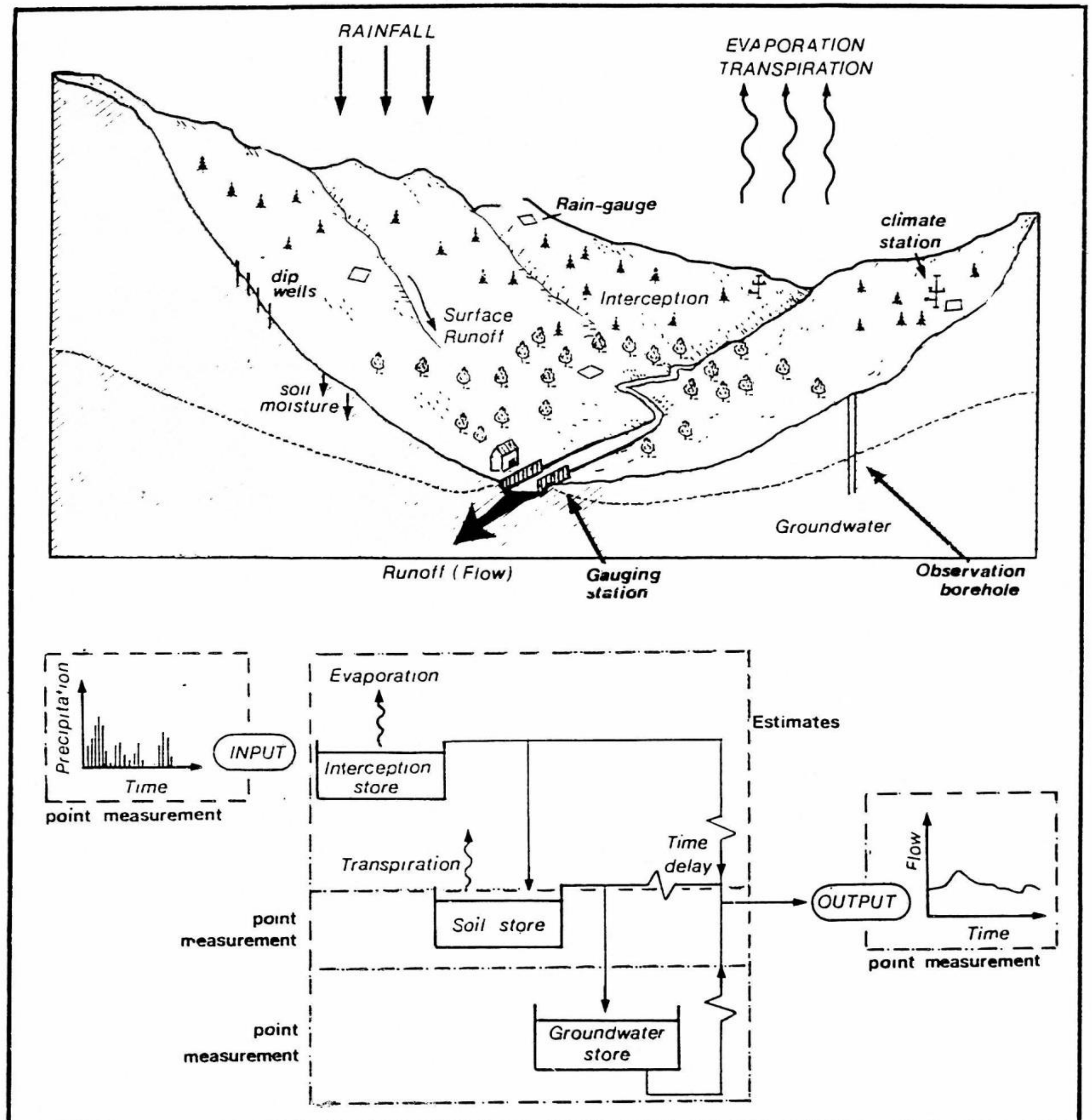
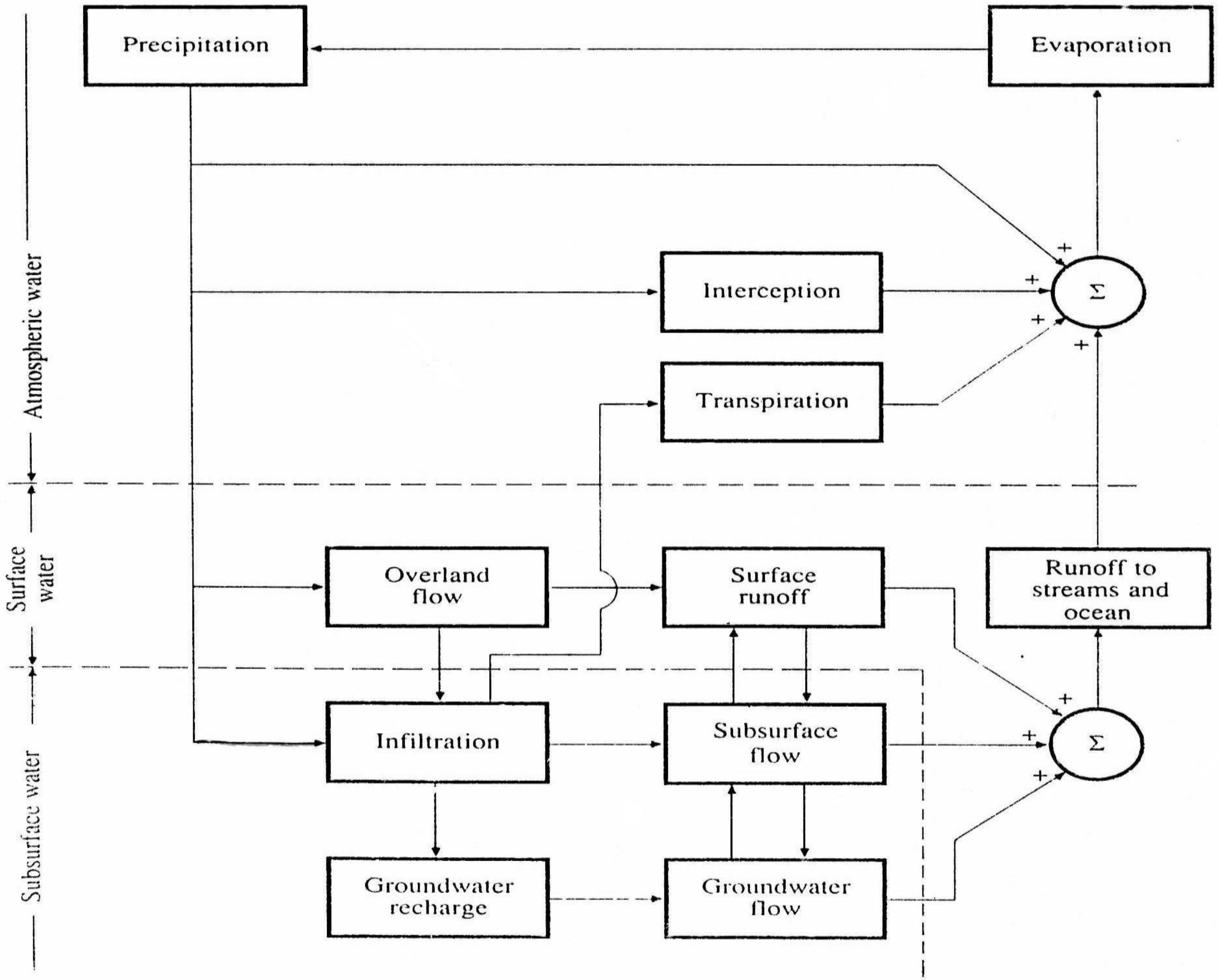

39
Moisture over land
100
Precipitation on land
61
Evaporation from land
385
Precipitation on ocean
Surface runoff
Evaporation and evapotranspiration
Infiltration
Soil moisture
424
Evaporation from ocean
Subsurface flow
Water table
Impervious strata
38 Surface outflow
Groundwater flow
1 Groundwater outflow

Fig. 2.2. The catchment hydrological cycle displayed as a landscape view and as a series of 'tank' stores which route an input precipitation through to a river flow (shown as time series). An emphasis is put on measurements in the cycle.





Precipitation
Evaporation
Interception
Transpiration
Σ
Overland flow
Surface runoff
Runoff to streams and ocean
Infiltration
Subsurface flow
Σ
Groundwater recharge
Groundwater flow
Atmospheric water
Surface water
Subsurface water

Weather hydrology (Meteorology)

The Atmosphere

1. dry air
    - Nitrogen (N_2) (78.09%)
    - Oxygen (O) (20.95%|)
    - Argon (Ar) (0.93%)
    - Carbon – dioxide (Co_2) (0.03%)
2. water vapor
3. impurities

Hadley Circulation

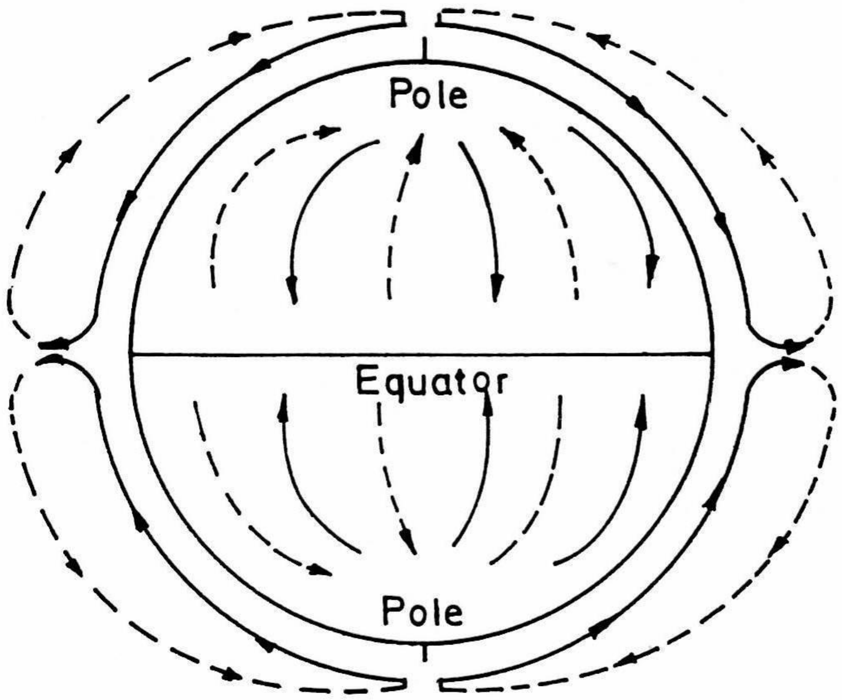
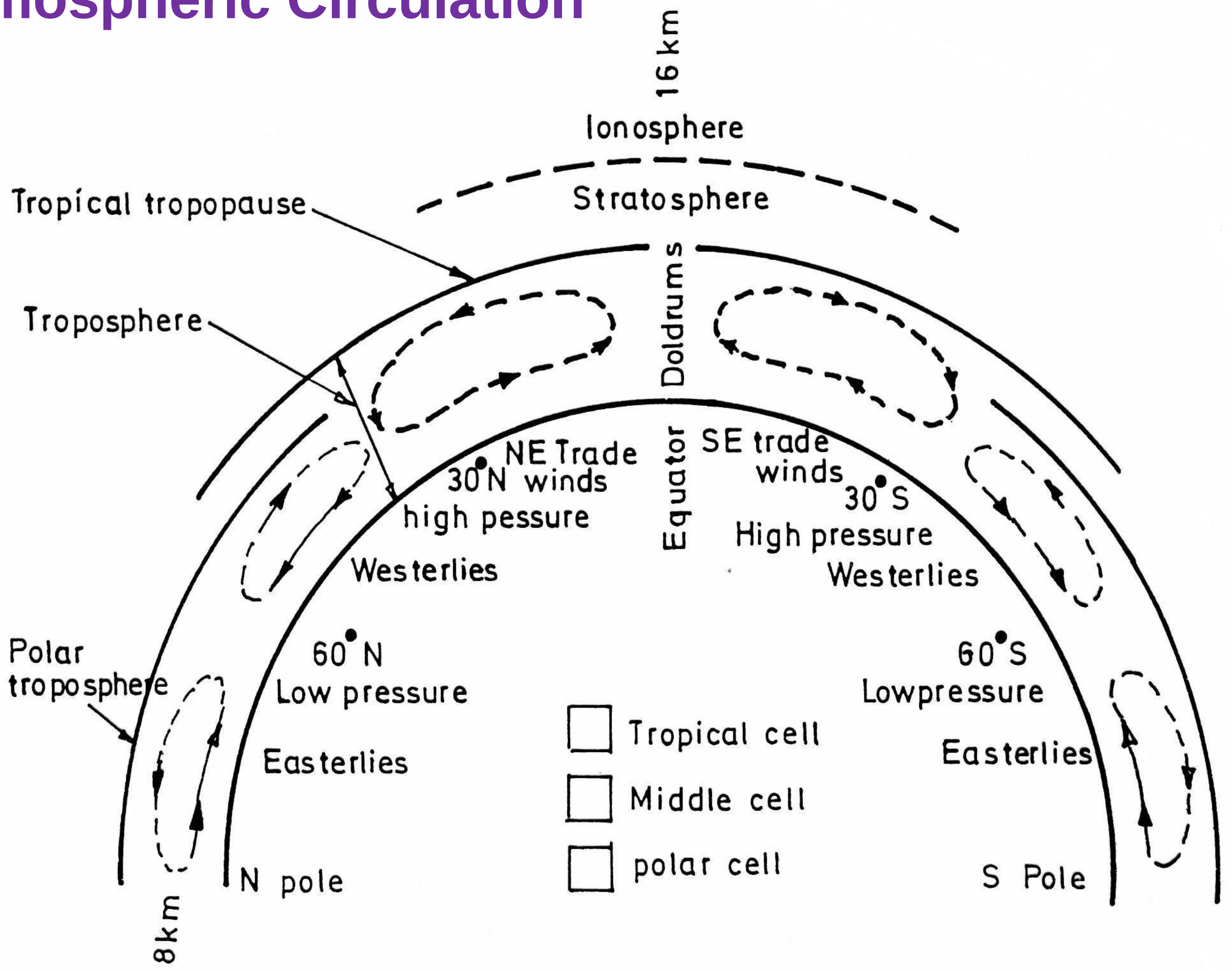


Figure Hadley Circulation

Atmospheric Circulation

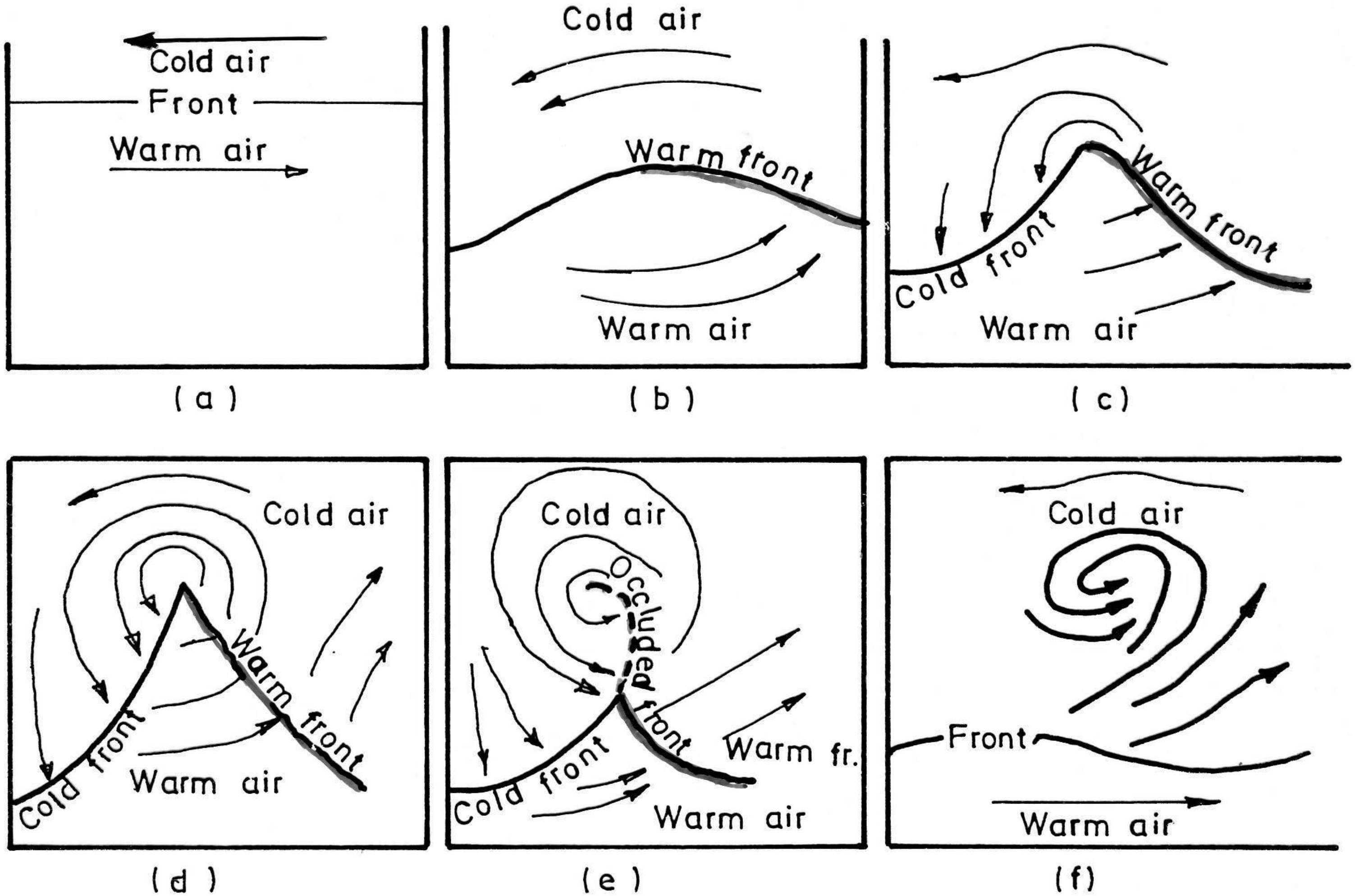


Front

The border between air masses of different temperature, pressure and moisture content is called a front.

There are two types of front

- cold front
- warm front

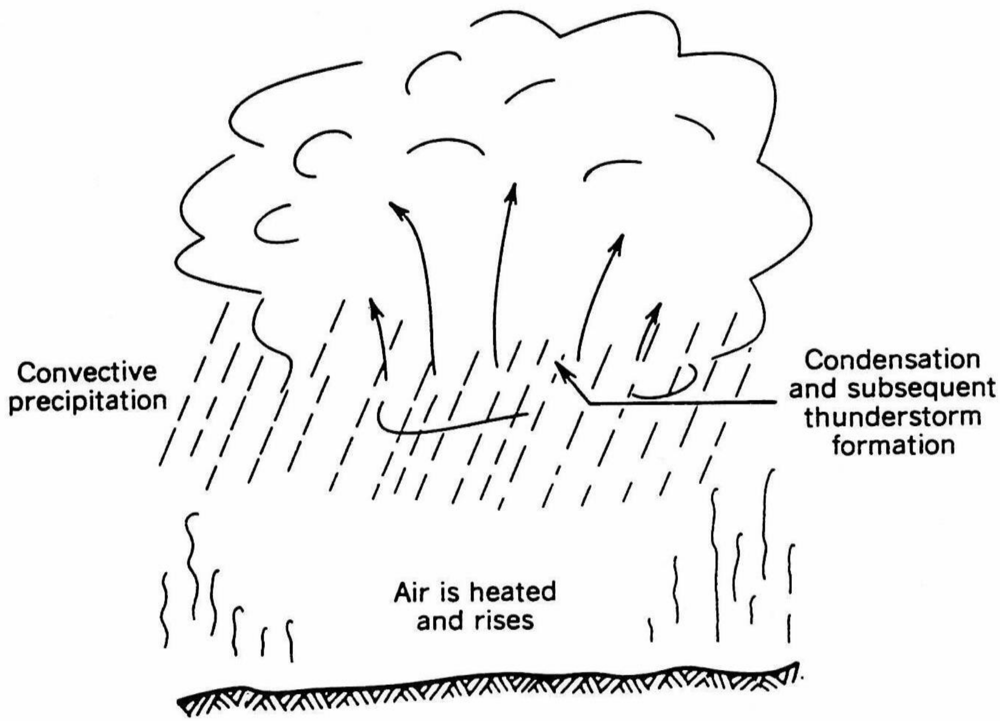

Cold air
Front
Warm air
(a)
Cold air
Warm front
Warm air
(b)
Warm front
Cold front
Warm air
(c)
Cold air
Warm front
Cold front
Warm air
(d)
Cold air
Occluded front
Warm fr.
Cold front
Warm air
(e)
Cold air
Front
Warm air
(f)

Storms

There are four types major of storms

- Convective storms
- Orographic storms
- Cyclonic storms
- Tropical storms

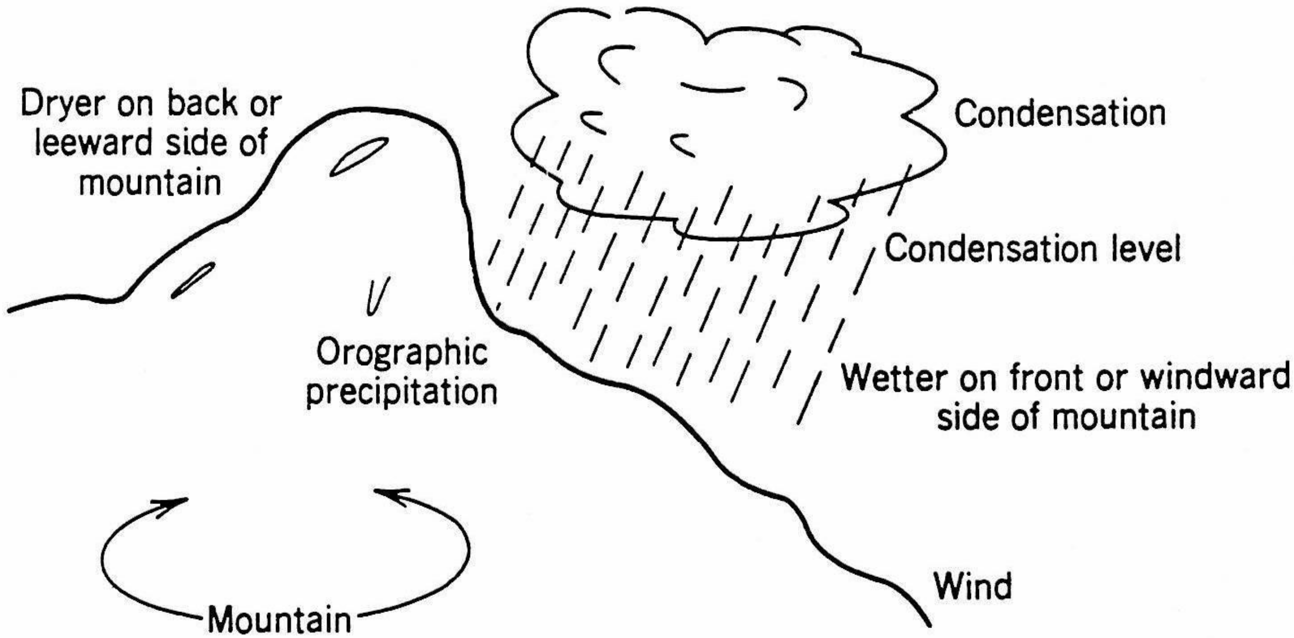
- **Convective storms**



Convective storm.

FIGURE Convective storm.

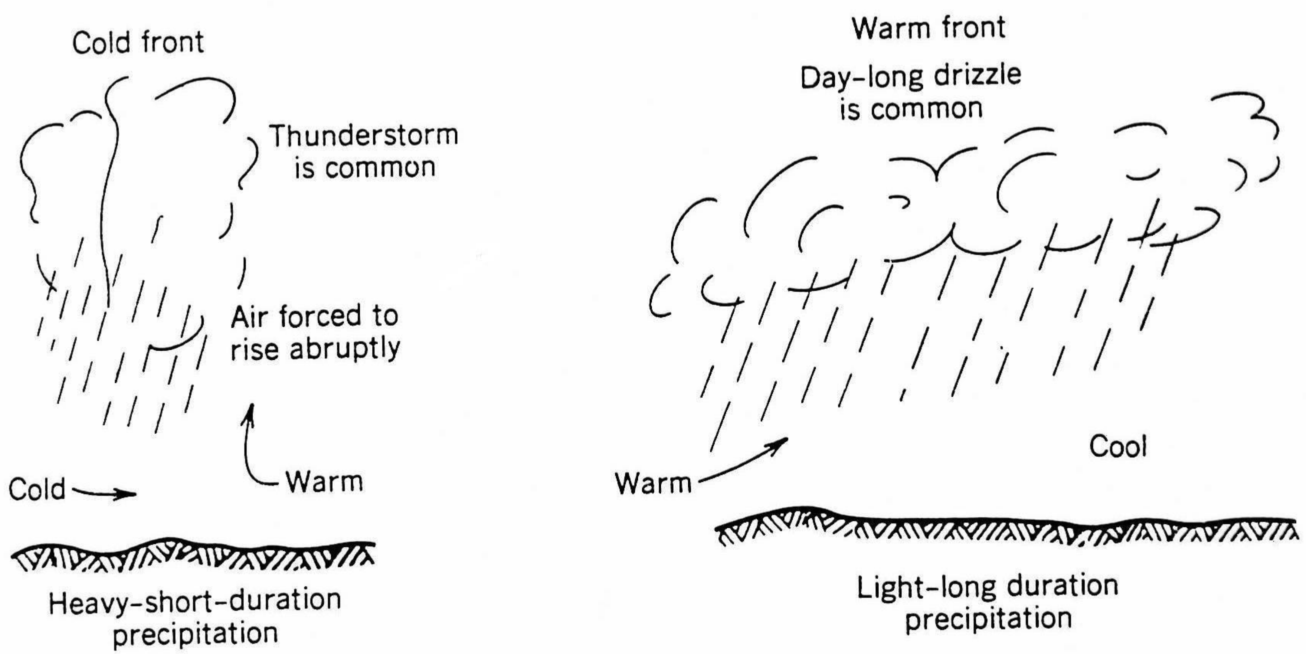
- **Orographic storms**



Orographic storm.

FIGURE Orographic storm.

- **Cyclonic storms**



Cyclonic storms in mid-latitude.

FIGURE Cyclonic storms in mid-latitude.

Water vapor

The amount of water vapor can be expressed as the pressure that vapor would exert in the absence of other gases, and is known as vapor pressure

Lapse Rate

The variation of temperature with altitude is known as lapse rate (9.80^{o} C/Km)

Relative humidity

$$f = 100\frac{e}{e_s}$$

e	=	actual vapor pressure (m bar)
e_s	=	saturated vapor pressure (m bar)

Saturated vapor pressure at the ground

$$e = 611 \exp\left(\frac{17.27T}{237.3+T}\right)$$

where,

T	=	Temperature in ^{0}C
e	=	Saturated vapor presence in kpa

Specific humidity at the ground surface

$$f_v = 0.622\frac{e}{p} \;;\; p = 101.3\,kpa$$

Wind

Movement of air. Measurements of wind are speed and direction. Wind speed is important because it can be related to water losses and precipitation events. Instrument for measuring wind speed are called anemometers.

Temperature

Temperature influences the form of precipitation and the rates of evaporation, transpiration and snowmelt.

Climate of Bangladesh

Temperature

- Maximum temperature : last week of March and of April (30.4^{o} C – 36^{o} C)
- Due to monsoon rain temp. : June to October (around 31^{o} C – 34^{o} C)
- Lowest temp. : November to December (8 – 9^{o} C) (minimum 2^{o} C)
- temperature ranges : February to March (27 -31^{o} C)

Rainfall

Three main sources of rainfall

- the western depression of winter
- The early summer thunderstorms known as Nor'westers
- The summer rains known as monsoons.

Fog Mist dew

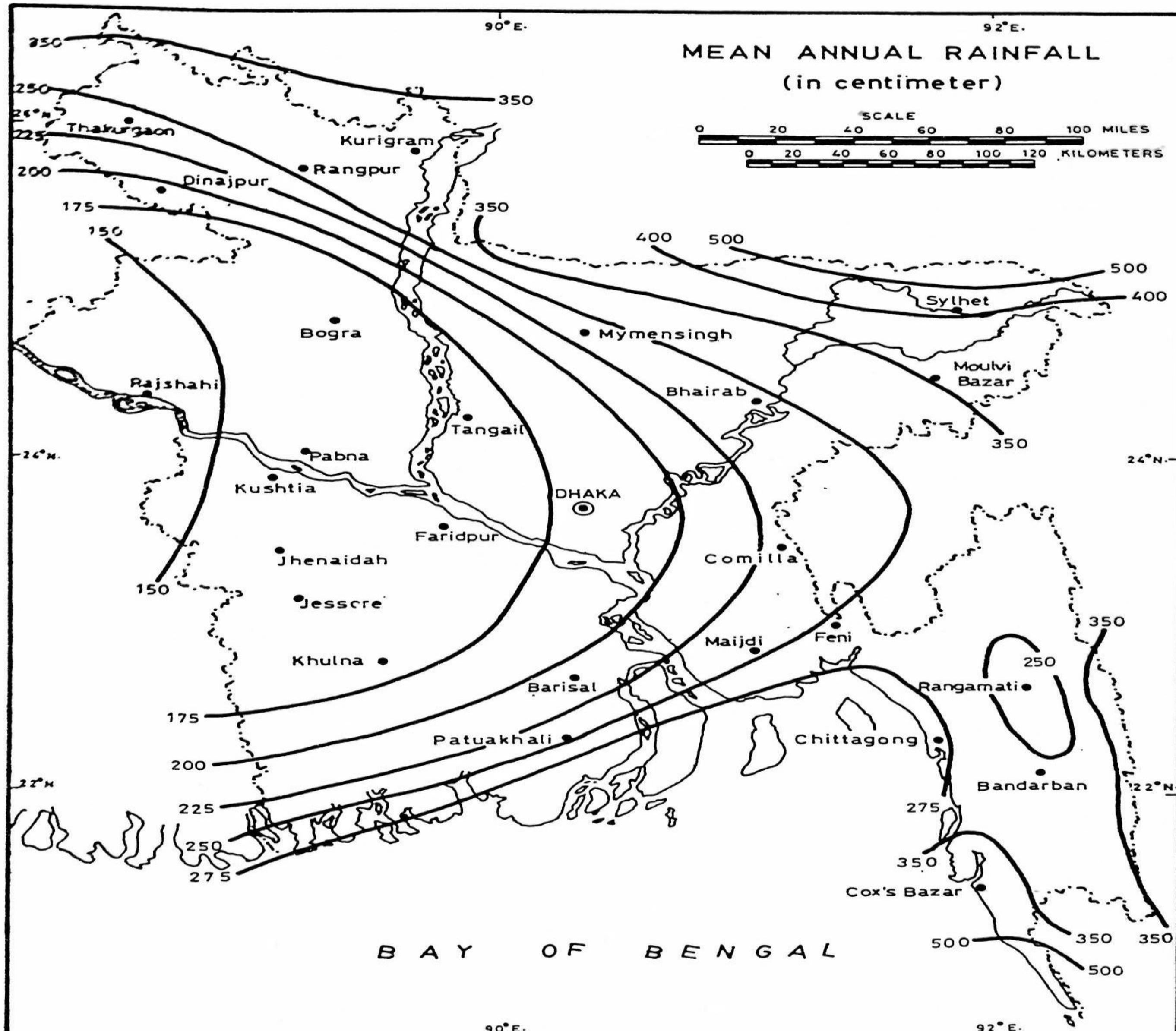
- November to march

Humidity

- humidity is high throughout the year
- march and April are the least humid months

Winds

- November to February : Wind direction is mostly from north
- March to May : Southerly, south – westerly
- June to September : South, south – easterly, easterly


MEAN ANNUAL RAINFALL
(in centimeter)
SCALE
0 20 40 60 80 100 MILES
0 20 40 60 80 100 120 KILOMETERS
90° E.
92° E.
26° N
24° N.
22° N
Thakurgaon
Dinajpur
Rangpur
Kurigram
Bogra
Rajshahi
Pabna
Kushtia
Tangail
Mymensingh
Sylhet
Moulvi Bazar
Bhairab
DHAKA
Faridpur
Jhenaidah
Jessore
Khulna
Barisal
Patuakhali
Comilla
Maijdi
Feni
Rangamati
Chittagong
Bandarban
Cox's Bazar
BAY OF BENGAL
150
175
200
225
250
275
350
400
500

The Residence Time

The residence time T_r is the average duration for a water molecule to pass through 2 subsystem of the hydrologic cycle. It is calculated by dividing the volume of water S in storage by the flow rate Q.

$$T_r = S/Q$$

TABLE 1.1.2
Global annual water balance

		Ocean	Land
Area (km^2)		361,300,000	148,800,000
Precipitation	(km^3/yr)	458,000	119,000
	(mm/yr)	1270	800
	(in/yr)	50	31
Evaporation	(km^3/yr)	505,000	72,000
	(mm/yr)	1400	484
	(in/yr)	55	19
Runoff to ocean			
Rivers	(km^3/yr)	–	44,700
Groundwater	(km^3/yr)	–	2200
Total runoff	(km^3/yr)	–	47,000
	(mm/yr)	–	316
	(in/yr)	–	12

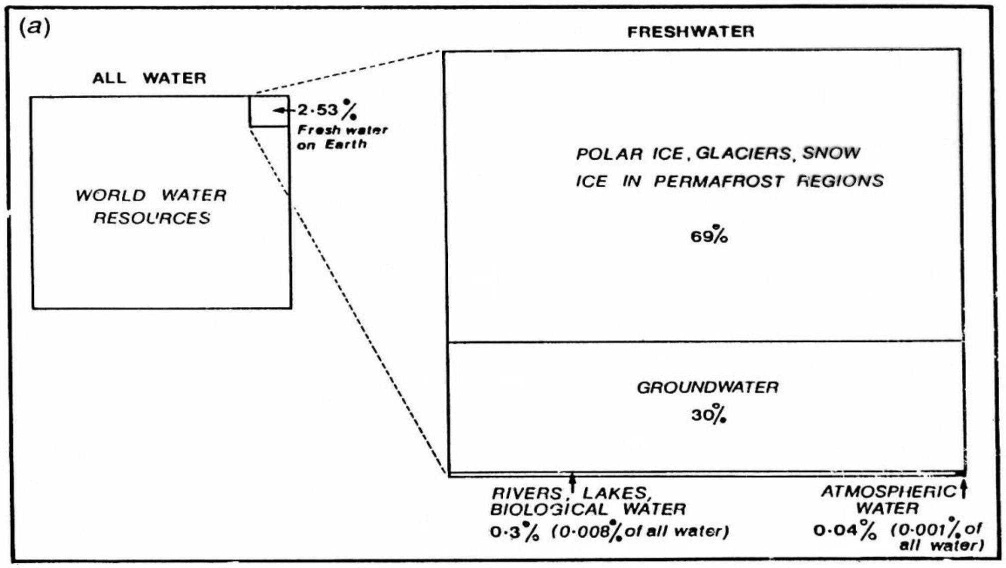
Table from World Water Balance and Water Resources of the Earth, Copyright, UNESCO, 1978

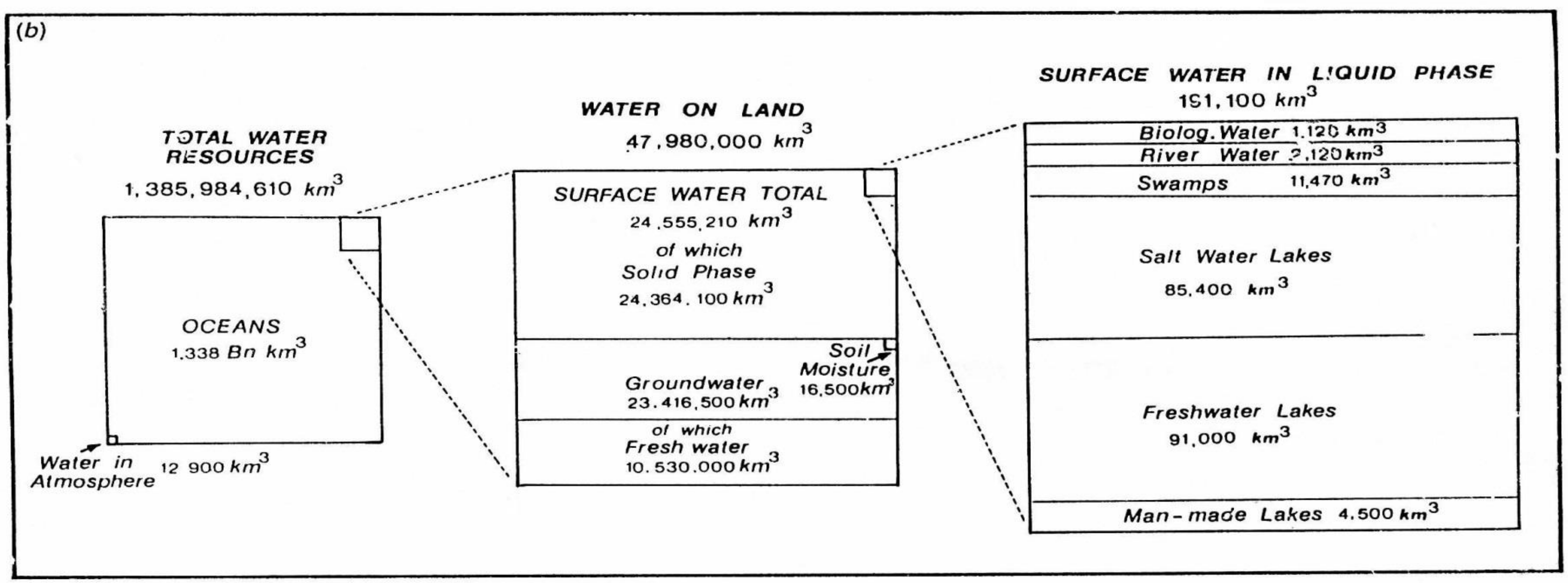
TABLE 1.1 ESTIMATED WORLD WATER QUANTITIES

Item	Area (M km^2)	Volume (M km^3)	Percent total water	Percent fresh water
1. Oceans	361.3	1338.0	96.5	–
2. Groundwater				
(a) fresh	134.8	10.530	0.76	30.1
(b) saline	134.8	12.870	0.93	–
3. Soil moisture	82.0	0.0165	0.0012	0.05
4. Polar ice	16.0	24.0235	1.7	68.6
5. Other ice and snow	0.3	0.3406	0.025	1.0
6. Lakes				
(a) fresh	1.2	0.0910	0.007	0.26
(b) saline	0.8	0.0854	0.006	–
7. Marshes	2.7	0.01147	0.0008	0.03
8. Rivers	148.8	0.00212	0.0002	0.006
9. Biological water	510.0	0.00112	0.0001	0.003
10. Atmospheric water	510.0	0.01290	0.001	0.04
Total : (a) All kinds of water	510.0	1386.0	100.0	
(b) Fresh water	148.8	35.0	2.5	100.0

Table from WORLD WATER BALANCE AND WATER RESOURCES OF THE EARTH, © UNESCO, 1975. Reproduced by the permission of UNESCO

Fig. 1.1. (a) The world's freshwater resources as a proportion of planetary water. (b) Geographical detail of water on the planet's land mass. (After Keller, 1984.)





Example :

Estimate the residence time of global atmospheric moisture.

Solution :

The volume of atmospheric moisture = 12,900 Km^3

The flow rate of moisture from the atmosphere as precipitation = ocean + land

= 458,000 + 119,000

= 577,000 Km^3 / yr

$T_r = S/Q$

= 12,900 / 577,000

= 0.022 yr

= 8.2 days

The very short residence time for moisture in the atmosphere is one reason why weather cannot be forecast accurately more than a few days ahead

Example :

Assuming that all the water in the oceans is involved in the hydrologic cycle, calculate the average residence time of ocean water

Water vapor in a Static Atmospheric Column

Two laws govern the properties of water vapor in a static column, the ideal gas law

$$p = \rho_a R_a T \tag{3.2.12}$$

and the *hydrostatic pressure law*

$$\frac{dp}{dz} = -\rho_a g \tag{3.2.13}$$

The variation of air temperature with altitude is described by

$$\frac{dT}{dz} = -\alpha \tag{3.2.14}$$

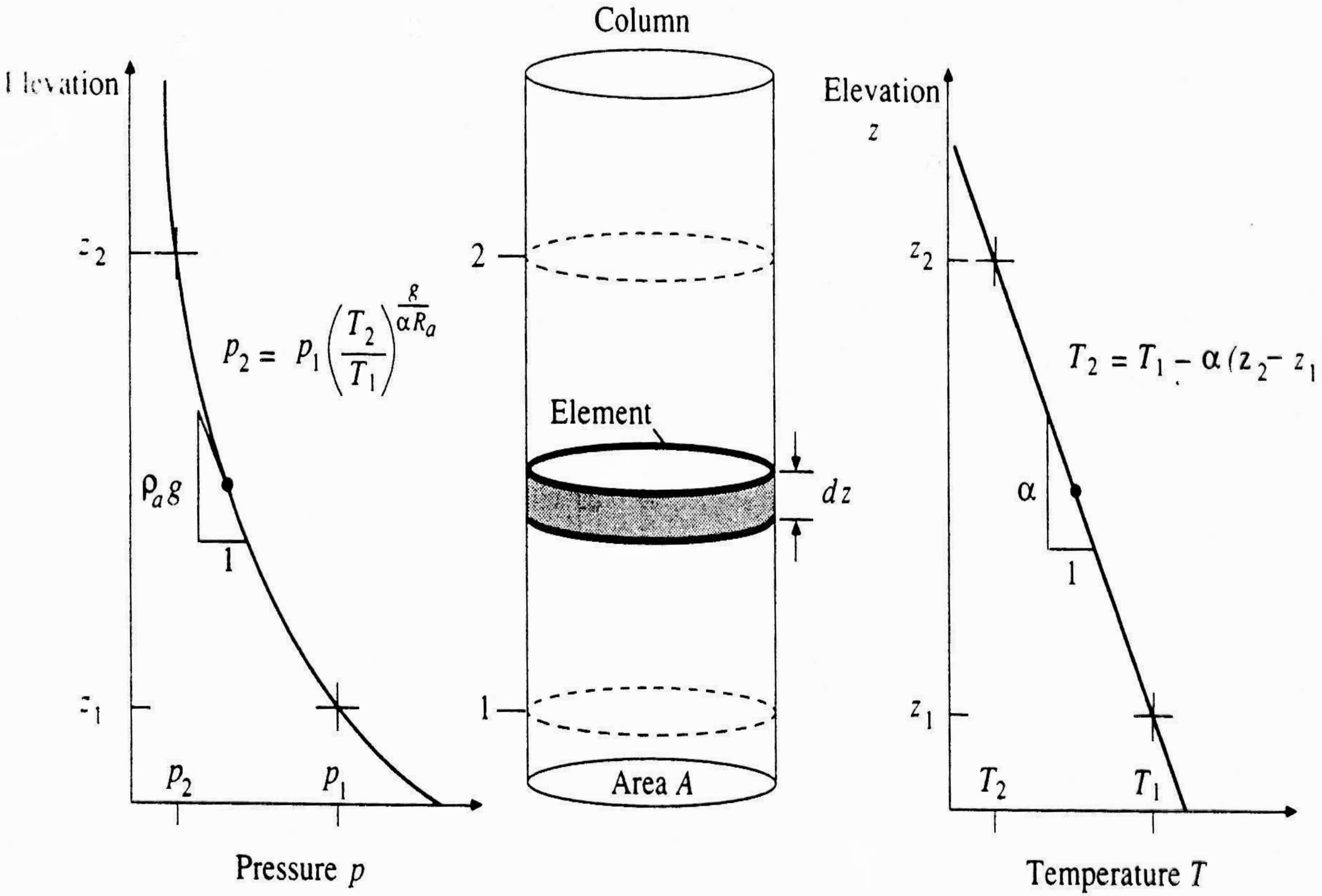
where α is the lapse rate. As shown in Fig. 3.2.2, a linear temperature variation combined with the two physical laws yields a nonlinear variation of pressure with altitude. Density and specific humidity also vary nonlinearly with altitude. From (3.2.12), $\rho_a = p/R_a T$, and substituting this into (3.2.13) yields

$$\frac{dp}{dz} = \frac{-pg}{R_a T}$$

or

$$\frac{dp}{p} = \left(\frac{-g}{R_a T}\right) dz$$

Substituting $dz = -dT/\alpha$ from (3.2.14):



Elevation
z_2
z_1
$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{g}{\alpha R_a}}$
$\rho_a g$
1
p_2
p_1
Pressure p
Column
2
Element
dz
1
Area A
Elevation
z
z_2
$T_2 = T_1 - \alpha(z_2 - z_1)$
α
1
z_1
T_2
T_1
Temperature T

FIGURE 3.2.2
Pressure and temperature variation in an atmospheric column.

$$\frac{dp}{p} = \left(\frac{g}{\alpha R_a}\right)\frac{dT}{T}$$

and integrating both sides between two levels 1 and 2 in the atmosphere gives

$$\ln\left(\frac{p_2}{p_1}\right) = \left(\frac{g}{\alpha R_a}\right)\ln\left(\frac{T_2}{T_1}\right)$$

or

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{g/\alpha R_a} \tag{3.2.15}$$

From (3.2.14) the temperature variation between altitudes z_1 and z_2 is

$$T_2 = T_1 - \alpha(z_2 - z_1) \tag{3.2.16}$$

Precipitable Water

The amount of moisture in an atmospheric column is called its precipitable water.

$$m_p = \int_{z_1}^{z_2} q_v \rho_a A \, dz$$

The integral (3.2.17) is calculated using intervals of height Δz, each witl an incremental mass of precipitable water

$$\Delta m_p = \bar{q}_v \bar{\rho}_a A \Delta z \qquad (3.2.18$$

where $\bar{q}_v$ and $\bar{\rho}_a$ are the average values of specific humidity and air density ove the interval. The mass increments are summed over the column to give the tota precipitable water.

Example 3.2.2. Calculate the precipitable water in a saturated air column 10 km high above 1 m^2 of ground surface. The surface pressure is 101.3 kPa, the surface air temperature is 30°C, and the lapse rate is 6.5°C/km.

Solution. The results of the calculation are summarized in Table 3.2.2. The increment in elevation is taken as Δz = 2 km = 2000 m. For the first increment, at $z_1 = 0$ m, T_1 = 30°C = (30 + 273) K = 303 K; at $z_2 = 2000$ m, by Eq. (3.2.16) using α = 6.5°C/km = 0.0065°C/m,

$$
\begin{aligned}
T_2 &= T_1 - \alpha(z_2 - z_1) \\
&= 30 - 0.0065(2000 - 0) \\
&= 17^\circ\mathrm{C}
\end{aligned}
$$

TABLE 3.2.2
Calculation of precipitable water in a saturated air column (Example 3.2.2)

Column	1 Elevation z (km)	2 Temperature (°C)	3 Temperature (°K)	4 Air pressure p (kPa)	5 Density ρ_a (kg/m^3)	6 Vapor pressur e (kPa)
	0	30	303	101.3	1.16	4.24
	2	17	290	80.4	0.97	1.94
	4	4	277	63.2	0.79	0.81
	6	−9	264	49.1	0.65	0.31
	8	−22	251	37.6	0.52	0.10
	10	−35	238	28.5	0.42	0.03

Column	7 Specific humidity q_v (kg/kg)	8 Average over increment $\bar{q}_v$ (kg/kg)	9 Average over increment $\bar{\rho}_a$ (kg/m^3)	10 Incremental mass Δm (kg)	11 % of total mass
	0.0261				
	0.0150	0.0205	1.07	43.7	57
	0.0080	0.0115	0.88	20.2	26
	0.0039	0.0060	0.72	8.6	11
	0.0017	0.0028	0.59	3.3	4
	0.0007	0.0012	0.47	1.1	2
				77.0	

$$= (17 + 273)\ \text{K}$$
$$= 290\ \text{K}$$

as shown in column 3 of the table. The gas constant R_a can be taken as 287 J/kg·K in this example because its variation with specific humidity is small [see Eq. (3.2.8)]. The air pressure at 2000 m is then given by (3.2.15) with $g/\alpha R_a = 9.81/(0.0065 \times 287) = 5.26$, as

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{g/\alpha R_a}$$
$$= 101.3 \left(\frac{290}{303}\right)^{5.26}$$
$$= 80.4\ \text{kPa}$$

as shown in column 4.

The air density at the ground is calculated from (3.2.12):

$$\rho_a = \frac{p}{R_a T}$$
$$= \frac{101.3 \times 10^3}{(287 \times 303)}$$
$$= 1.16\ \text{kg/m}^3$$

and a similar calculation yields the air density of 0.97 kg/m^3 at 2000 m. The average density over the 2 km increment is therefore $\bar{\rho}_a = (1.16 + 0.97)/2 = 1.07$ kg/m^3 (see columns 5 and 9).

The saturated vapor pressure at the ground is determined using (3.2.9):

$$e = 611 \exp\left(\frac{17.27T}{237.3 + T}\right)$$

$$= 611 \exp\left(\frac{17.27 \times 30}{237.3 + 30}\right)$$

$$= 4244 \text{ Pa}$$

$$= 4.24 \text{ kPa}$$

The corresponding value at 2000 m where $T = 17°\text{C}$, is $e = 1.94$ kPa (column 6). The specific humidity at the ground surface is calculated by Eq. (3.2.6):

$$q_v = 0.622\frac{e}{p}$$

$$= 0.622 \times \frac{4.24}{101.3}$$

$$= 0.026 \text{ kg/kg}$$

At 2000 m $q_v = 0.015$ kg/kg. The average value of specific humidity over the 2-km increment is therefore $\bar{q}_v = (0.026 + 0.015)/2 = 0.0205$ kg/kg (column 8). Substituting into (3.2.18), the mass of precipitable water in the first 2-km increment is

$$\Delta m_p = \bar{q}_v \bar{\rho}_a A \, \Delta z$$

$$= 0.0205 \times 1.07 \times 1 \times 2000$$

$$= 43.7 \text{ kg}$$

By adding the incremental masses, the total mass of precipitable water in the column is found to be m_p = 77 kg (column 10). The equivalent depth of liquid water is $m_p / \rho_w A$ = 77/(1000 × 1) = 0.077 m = 77 mm.

The numbers in column 11 of Table 3.2.2 for percent of total mass in each increment show that more than half of the precipitable water is located in the first 2 km above the land surface in this example. There is only a very small amount of precipitable water above 10 km elevation. The depth of precipitable water in this column is sufficient to produce a small storm, but a large storm would require inflow of moisture from surrounding areas to sustain the precipitation.

Precipitation

Precipitation denotes all forms of water that reach the earth from the atmosphere. The usual forms are rainfall, snowfall, hail, frost and dew.

To form precipitation

There are four conditions that must be present for the production of precipitation

(i)Atmosphere must have moisture

(ii)Must be sufficient nuclei present to aid condensation

(iii)Weather conditions must be good for condensation of water vapor to take place

(iv) product of condensation must reach the earth

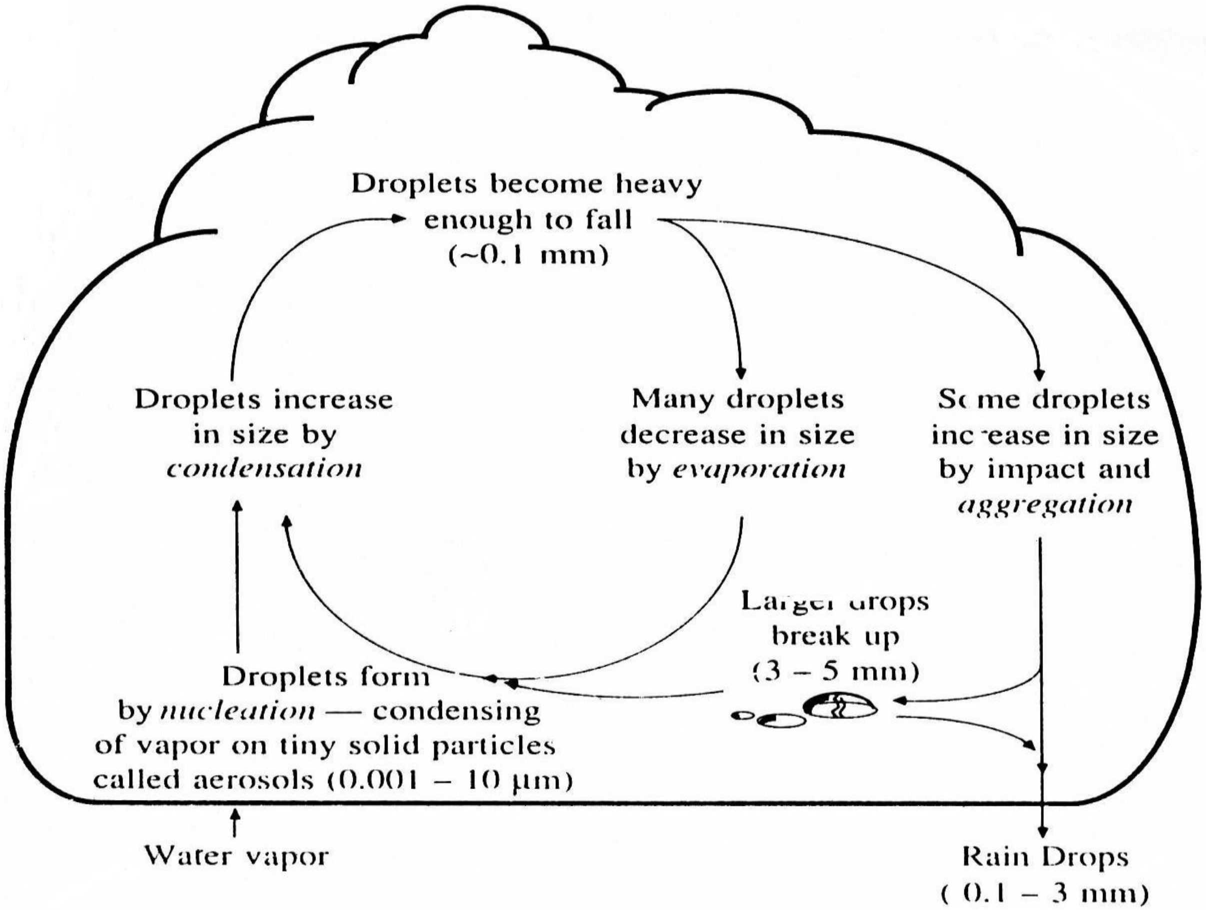


FIGURE 3.3.1
Water droplets in clouds are formed by nucleation of vapor on aerosols, then go through many condensation-evaporation cycles as they circulate in the cloud, until they aggregate into large enough drops to fall through the cloud base.

Different forms of precipitation

Rainfall

Sizes of raindrop 0.5 mm to 6 mm

Type	Intensity
Light rain Moderate rain Heavy rain	trace to 2.5 mm/h 2.5 mm/h to 7.5 mm/h >7.5 mm/h

Snow

Density varies from 0.06 to 0.15 gm/ cm^3 (average density 0.1 gm/ cm^3).

Drizzle

Water droplets of size less than 0.5 mm and intensity less than 1 mm/h.

Glaze

When rain or drizzle comes in contact with cold ground at around 0^o c, the water drops freeze to form an ice coasting called glaze.

Sleet

When rain falls through air at sub – freezing temperature, the frozen raindrop called sleet.

Hail

It is a showery precipitation in the form of irregular pellets or lumps of ice of size more than 8 mm.

Important considerations for setting a raingauge

- In a flat surface
- It must be placed near the ground
- Placed in an open space (area of 5.5 m x 5.5 m).
- No obstruction within 30 m.

Types of raingauge

- Non – recording Gauge
- Recording Gauge

Symon's non-recording gauge

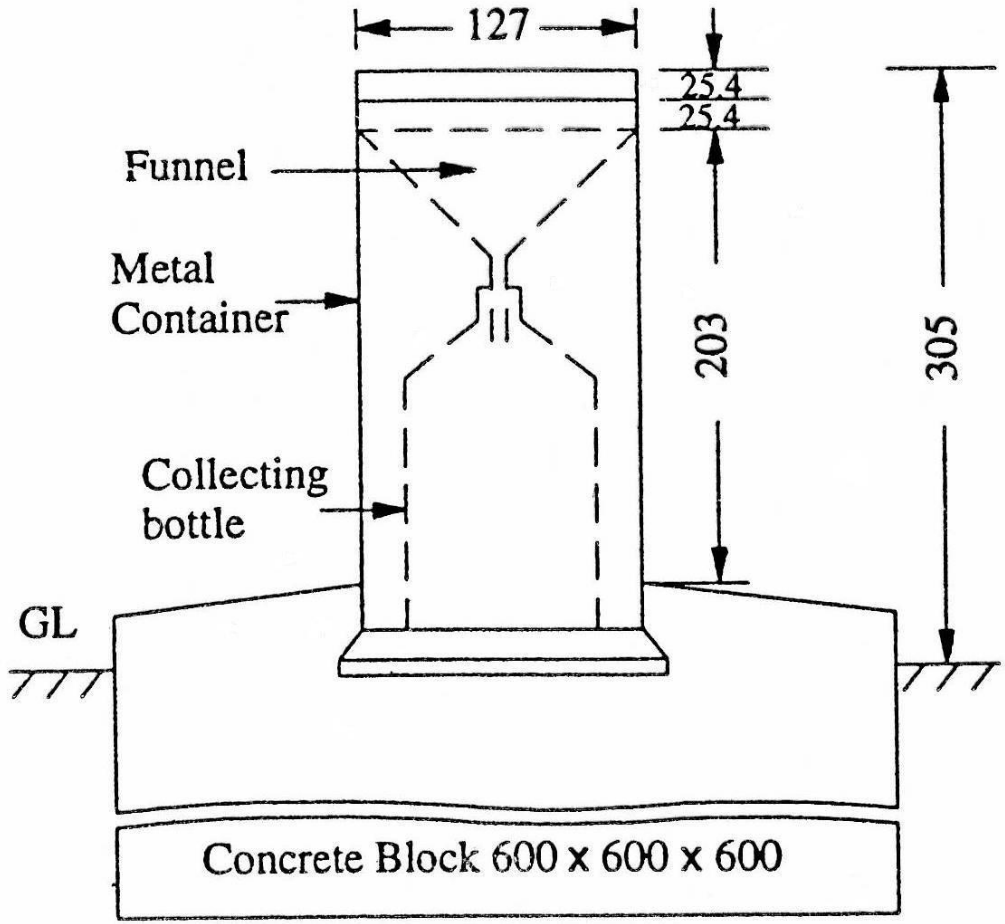


Fig. Nonrecording raingauge (Symons' gauge)

Recording Gauges

Advantage

- We can identify storm event
- We find intensity of rainfall
- We find storm duration

Tipping-Bucket Type

- 30.5 cm size raingauge
- Two compartment bucket size 0.1 mm/0.25 mm

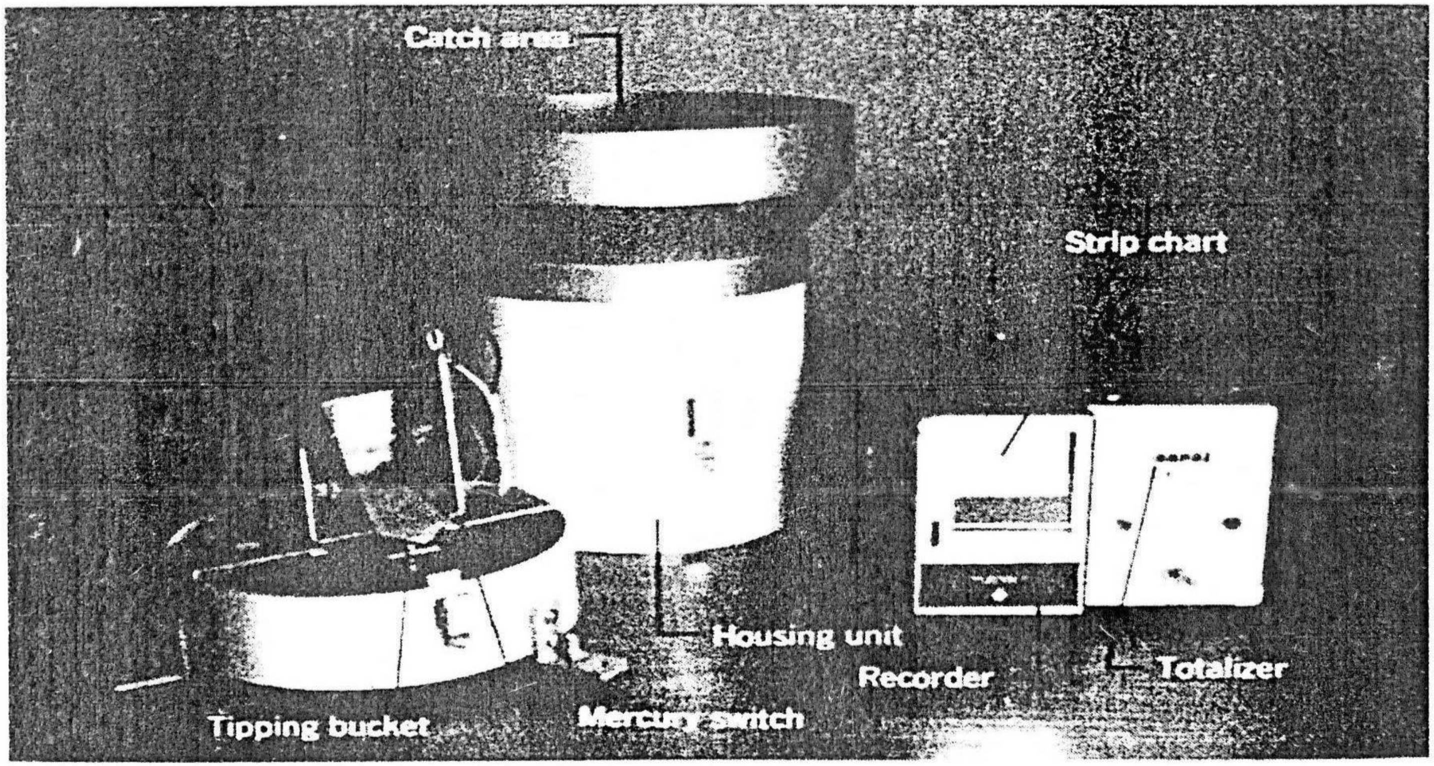


FIGURE Tipping bucket housing unit mercury switch recorder strip chart totalizer.

Weighing – Becket Type

Natural Syphon Type

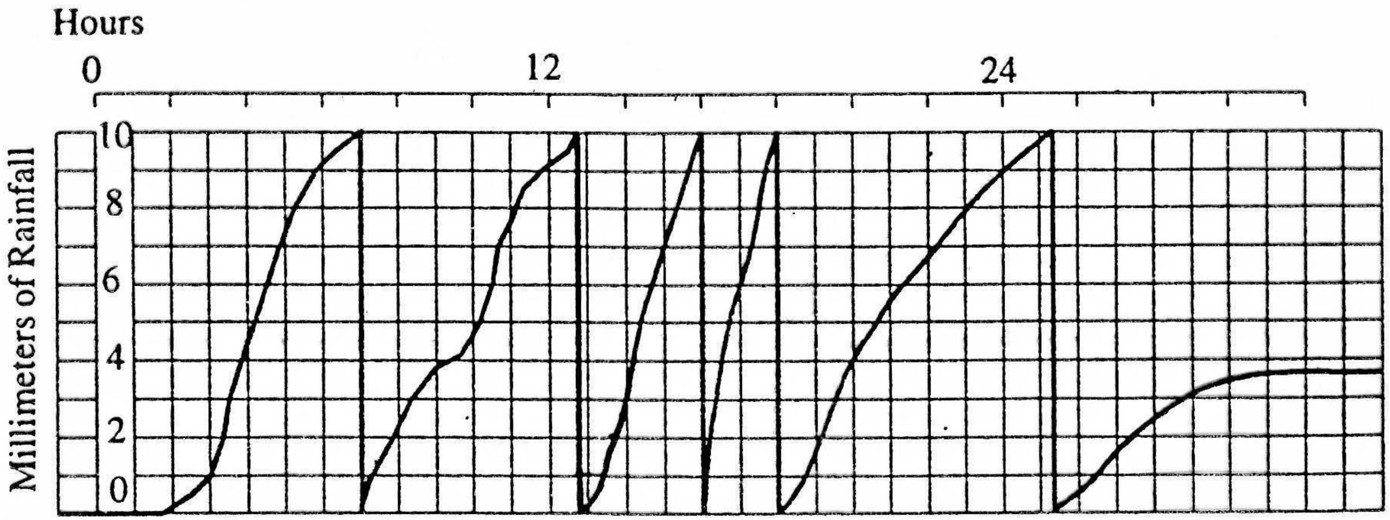


Fig. Recording from a natural syphon–type gauge (schematic)

Raingauge Network

World Meteorological Organization (WMO) recommends the following densities.

1. Flat regions : ideal – 1 station for 600 – 900 Km^2
   acceptable – 1 station for 900 - 3000 Km^2

2. Mountains regions : ideal – 1 station for 100 – 250 Km^2
   acceptable - 1 station for 250 – 1000 Km^2

3. Arid and Polar Zones : 1- station for 1500 – 10,000 Km^2

Adequacy of Raingauge Stations

$$N = \left(\frac{C_v}{\varepsilon} \right)^2$$

where,

N = optimal number of stations,

ϵ = allowable degree of error in the estimate of the mean rainfall, and

C_v = coefficient of variation of the rainfall values at the existing m stations (in per cent).

If there are ***m*** stations in the catchment each recording rainfall values $P_1, P_2, \ldots, P_p \ldots\ldots P_m$ in a known time, the coefficient of variation C_v is calculated as:

$$C_v = \frac{100 \times \sigma_{m-1}}{\overline{P}}$$

where, standard deviation =

$$\sigma_{m-1} = \sqrt{\frac{\sum_{1}^{m}\left(p_i - \overline{p}\right)^2}{m-1}}$$

P_i = precipitation magnitude in the i^{th} station

$$\overline{P} = \frac{1}{m}\left(\sum_{1}^{m} P_i\right) = \text{mean precipitation}$$

In calculating optimal number of stations N it is usual to take ϵ = 10%. It is seen that if the value of ϵ is small, the number of raingauge stations will be more.

Example A catchment has six raingauge stations. In a year, the annual rainfalls recorded by the gauges are as follows:

Station	A	B	C	D	E	F
Rainfall	82.6	102.9	180.3	110.3	98.8	136.7

For a 10% error in the estimation of the mean rainfall, calculate the optimum number of stations in the catchment.

Solution

For this data,

m = 6

$\overline{P} = 118.6$

$\sigma_{m-1} = 35.04$

$\epsilon = 10$

$$C_v = \frac{100 \times 35.04}{118.6} = 29.54$$

$$N = \left(\frac{29.54}{10}\right)^2 = 8.7$$

Provide 9 stations

Estimation of Missing Data

Given the annual precipitation values, P_1, P_2, P_3, ... P_m at neighboring M stations 1,2,3, ..., M respectively, it is required to find the missing annual precipitation P_x at a station X not included in the above M stations. Further, the normal annual precipitations N_1, N_2, ..., N_i ... at each of the above (M+1) stations including station X are known.

If the normal annual precipitations at various stations are within about 10% of the normal annual precipitation at station X, then a simple arithmetic average procedure is followed to estimate P_x. Thus

$$P_x = \frac{1}{M}\left[P_1 + P_2 + \ldots + P_m\right]$$

If the normal precipitations vary considerably, then Px is estimated by weighing the precipitation at the various stations by the ratios of normal annual precipitations. This method, known as the normal ratio method, gives P_x as

$$P_x = \frac{N_x}{M}\left[\frac{P_1}{N_1} + \frac{P_2}{N_2} + \ldots + \frac{P_m}{N_m}\right]$$

Test for Consistency of Record

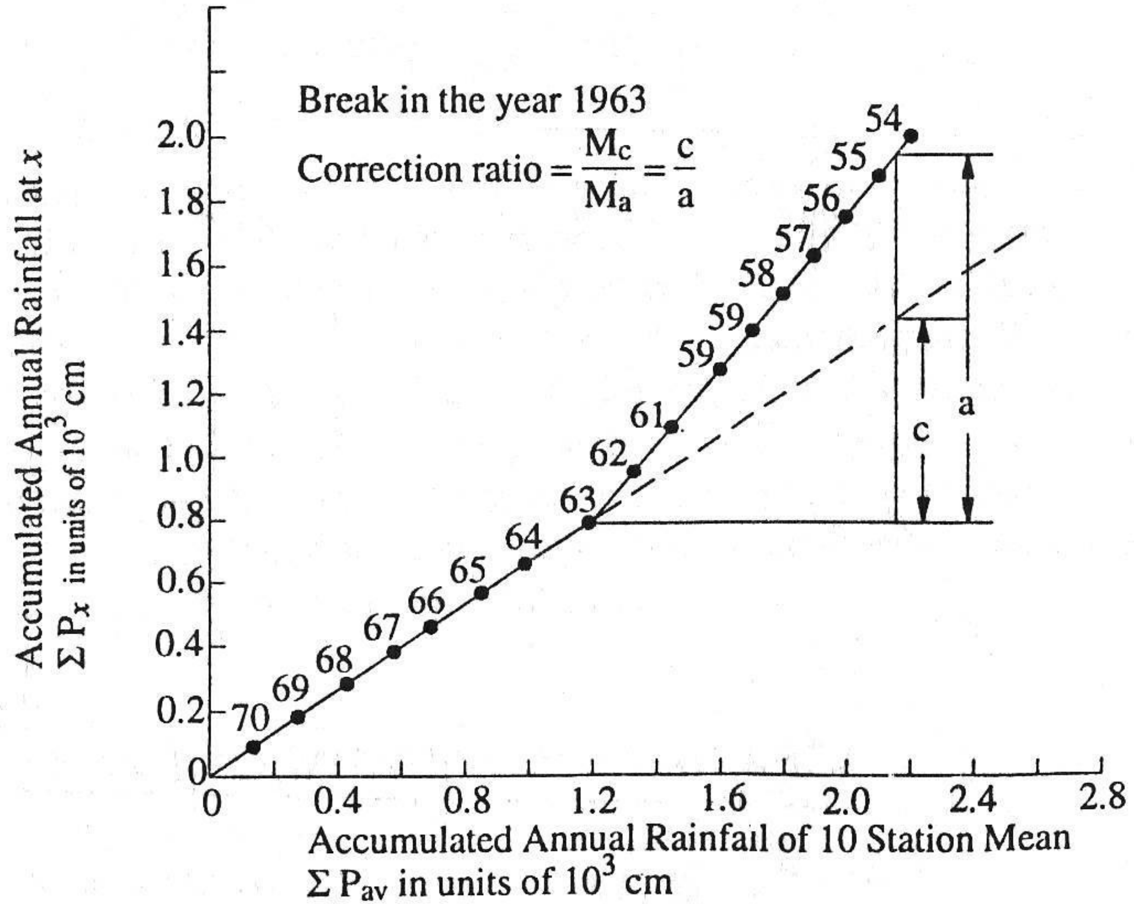


Fig. 2.7 Double–mass curve

$$P_{cx} = P_x \frac{M_c}{M_a}$$

Where P_{cx} = Corrected precipitation at any time period t_1 at station X

P_x = original recorded precipitation at time period t_1 at station x

M_c = corrected slope of the double – mass curve

M_a = original slope of the mass curve

Arithmetic Average Method

$$\overline{P} = \sum_{i=1}^{n} P_i / n$$

$\overline{P}$ = average precipitation depth (mm or in.).

P_i = precipitation depth at gage (within the topographic basin), (mm or in.)

n = total number of gauging stations within the topographic basin

Thiessen Polygon Method

$$W_i = A_p/A$$

Where,

W_i = weighted area, dimensionless
A_p = area of the polygon within the topographic basin (km^2)
A = total area (km^2)

The average precipitation using the Thiessen method is

$$\overline{P} = \sum_{i=1}^{n} W_i P_i$$

where,

$\overline{P}$ = average precipitation (mm)
P_i = gage precipitation for polygon i
n = total number of polygons

Isohyetal Method

$$\overline{P} = \sum_{i=1}^{n} W_i P_i$$

Where,

$\overline{P}$	=	isohyetal average precipitation (mm)
P_i	=	isohyetal cell average precipitation (mm)
W_i	=	Ai/A; Ai – area of cell (km^2)
A	=	total area (km^2)
n	=	total number of cells

Example :

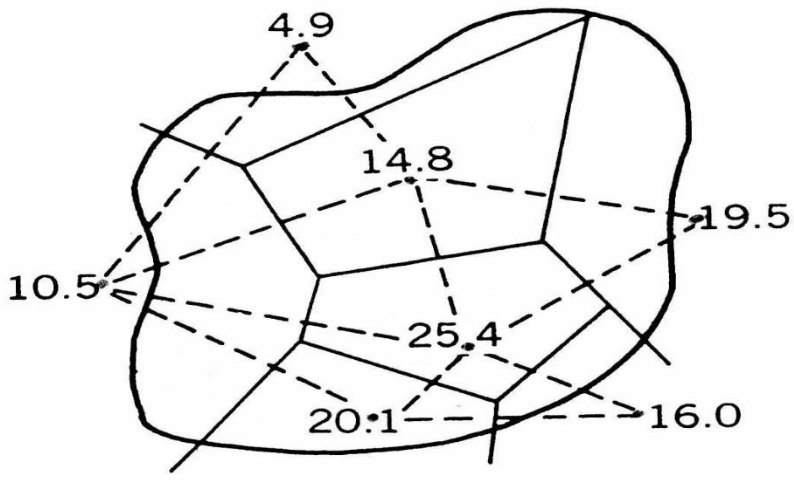
For the following watershed, estimate using three methods the average precipitation. The watershed in figure.

Solution

a. Arithmetic average (add those within the watershed)

$$\overline{P} = \sum_{i=1}^{n} P_i / n$$

$$\overline{P} = \frac{14.8 + 25.4 + 20.1}{3} = 20.1\ mm$$

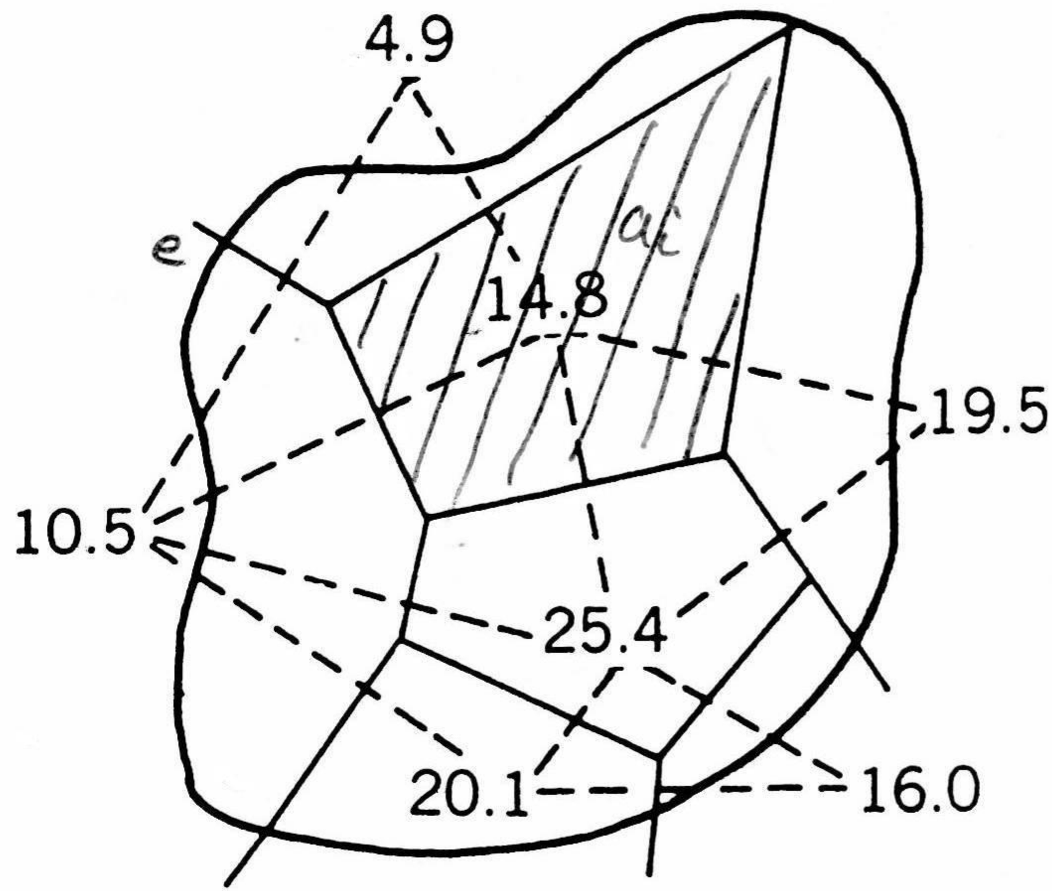


Thiessen Method

b. Thiessen method

$$\overline{P} = \sum_{i=1}^{n} W_i P_i$$

OBSERVED PRECIPITATION (P_i)	WEIGHTED AREA (W_i) $= \frac{a_i}{A}$	W_iP_i (mm)
14.8	0.29	4.3
19.5	0.16	3.1
4.9	0.07	0.3
10.5	0.19	2.0
25.4	0.15	3.8
20.1	0.12	2.4
16.0	0.02	0.3
	$\overline{P}$	= 16.2 mm



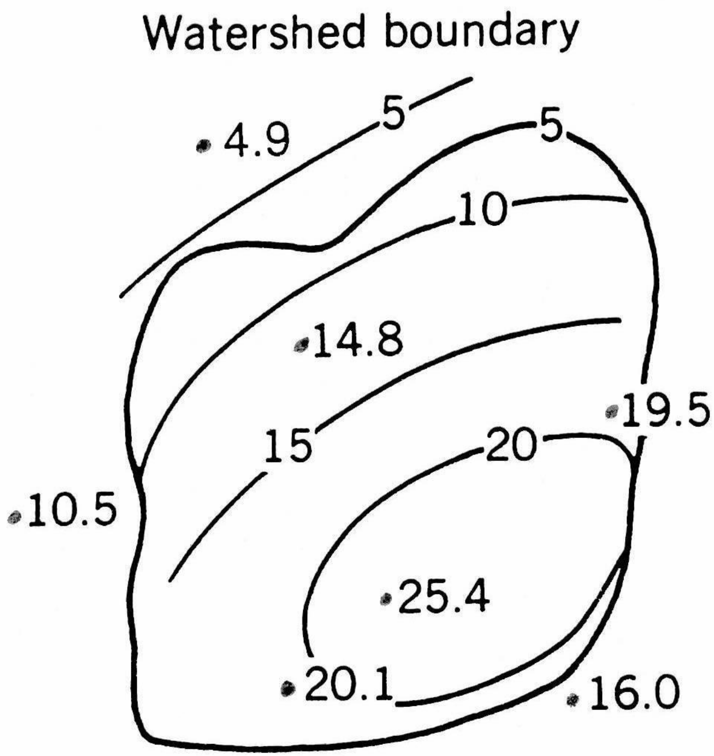
Thiessen
Method

c. **Isohyetal method**

$$\overline{P} = \sum_i W_i P_i$$

ISOHYET (mm)	WEIGHTED AREA (W_i)	AVERAGE PRECIPITATION (P_i)	W_iP_i (mm)
>20	0.30	22.7*	6.8
10	0.58	15.0	8.7
5	0.12	7.5	0.9
			$\overline{P}$ = 16.4 mm

*(25.4 + 20)/2 = 22.7

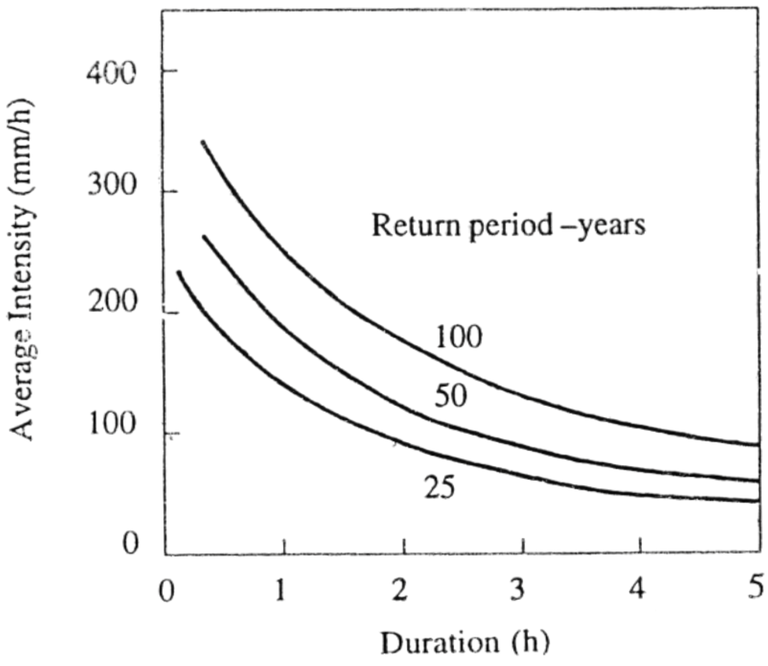


Isohyetal
Method

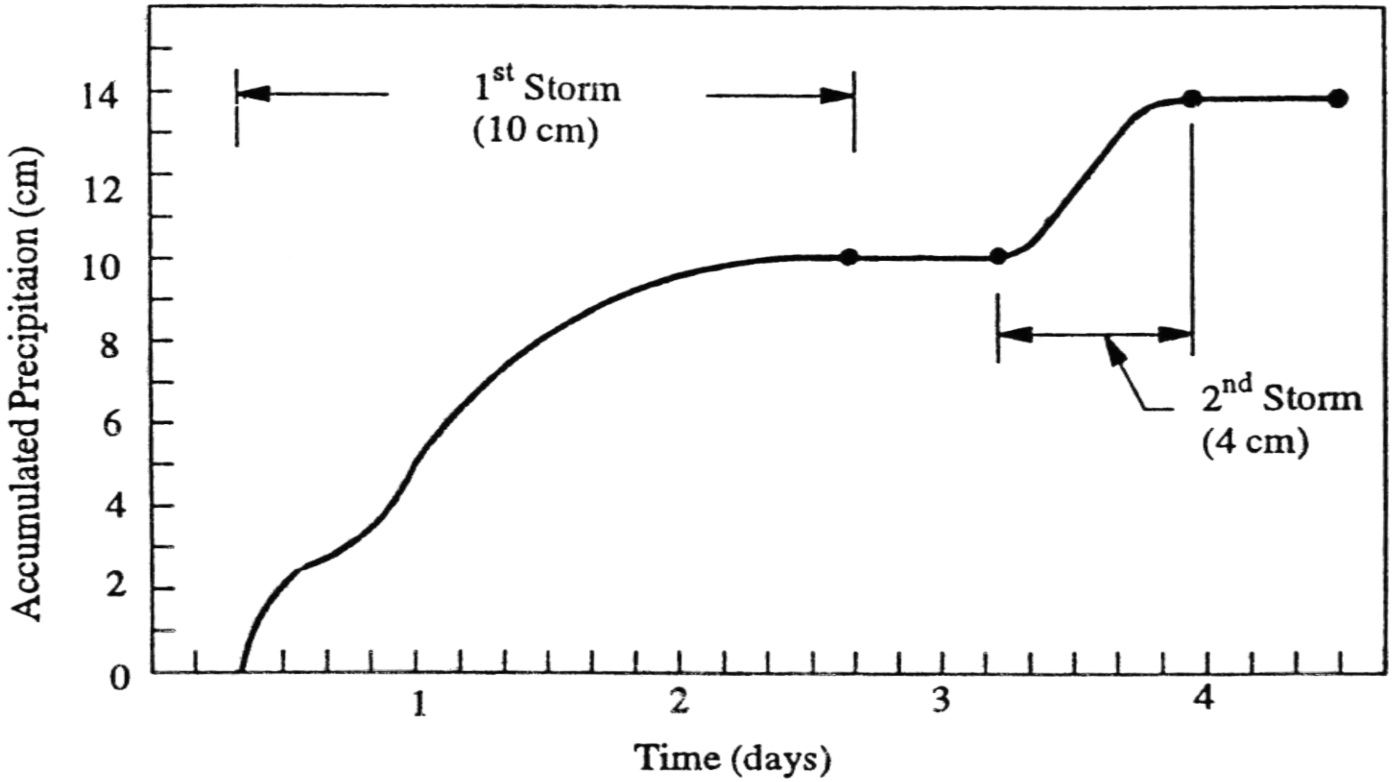
Intensity – Duration – Frequency relationship

$$i = \frac{KT^x}{(D+a)^n}$$

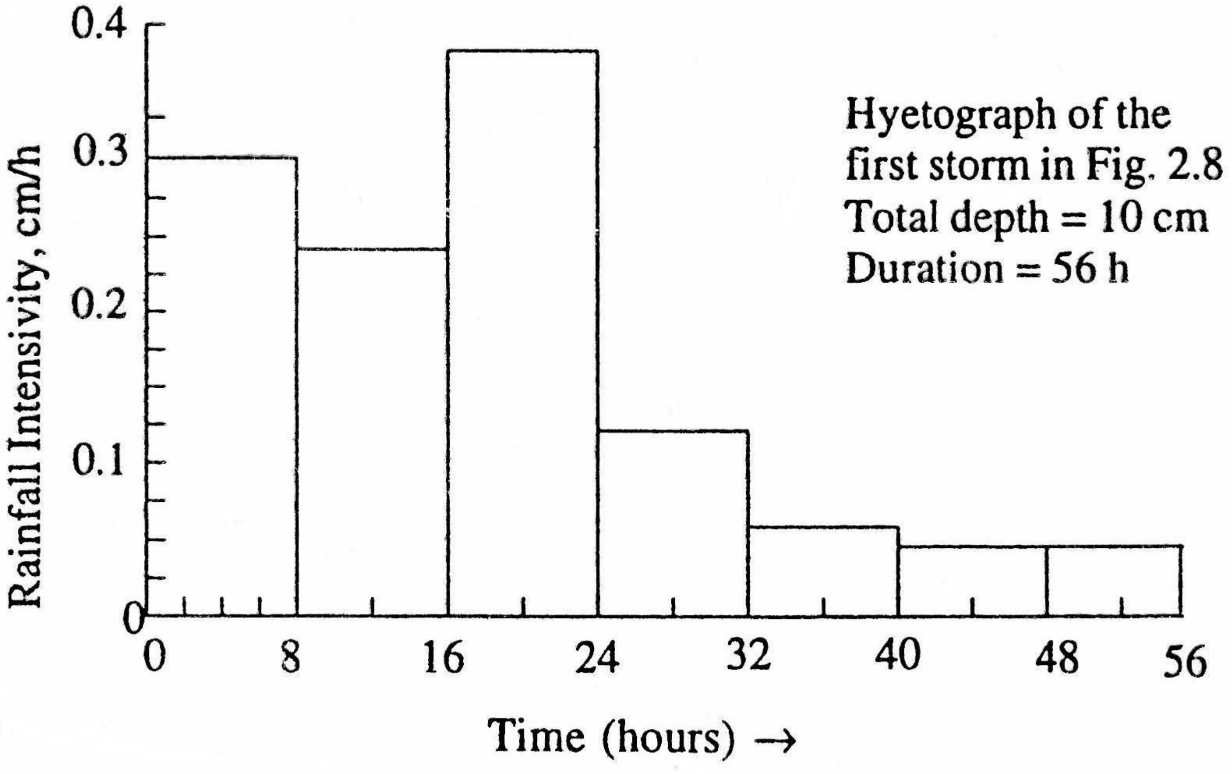
where K, x, a and n are constants for a given catchment.



Mass Curve of Rainfall



Hyetograph


0.4
0.3
0.2
0.1
0
Rainfall Intensivity, cm/h
0
8
16
24
32
40
48
56
Time (hours) →
Hyetograph of the
first storm in Fig. 2.8
Total depth = 10 cm
Duration = 56 h

Evaporation

Evaporation is the process in which a liquid changes to the gaseous state at the free surface, below the boiling point through the transfer of heat energy.

The rate of evaporation is depended on the following :

(i)Vapour Pressure

$$E_L = C\,(e_w - e_a)$$

Where E_L = rate of evaporation (mm/day)

C = a constant

e_w = the saturation vapour pressure at the water temperature in mm of mercury

e_a = the actual vapour pressure in the air in mm of mercury

This equation is known as Dalton’s law of evaporation after John Dalton (1802) Who first recognized this law. Evaporation continuous till $e_w = e_a$. If $e_w > e_a$ Condensation takes place

(ii) Temperature

The rate of evaporation increases with an increase in the water temperature.

(iii) Wind

The rate of evaporation increases with the wind speed up to a critical speed beyond which any further increase in the wind speed has no influence on the evaporation rate.

(iv) Atmospheric Pressure

A decrease in the barometric pressure , as in high altitudes, increases evaporation.

(v) Soluble Salts

When a solute is dissolved in water, the vapour pressure of the solution is less than that of pure water and hence causes reduction in the rate of evaporation. For example, under identical condition evaporation from sea water is about 2 – 3% less than the fresh water.

(vi) Heat Storage in Water Bodies

Deep water bodies have more heat storage than shallow ones.

Evaporimeters

The amount of water evaporated from a water Surface is estimated by the Following methods :

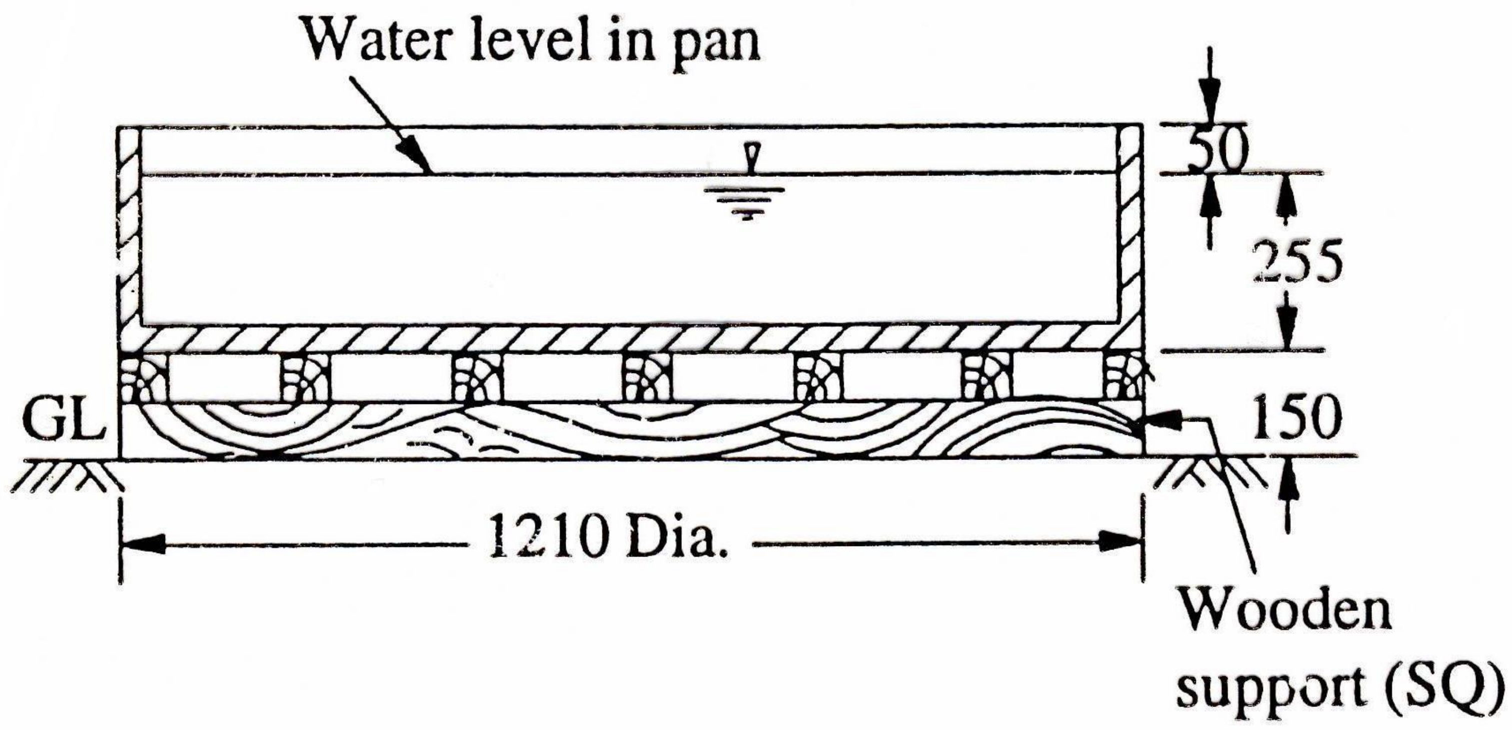
(i) using evaporimeter

(ii) empirical evaporation equations and

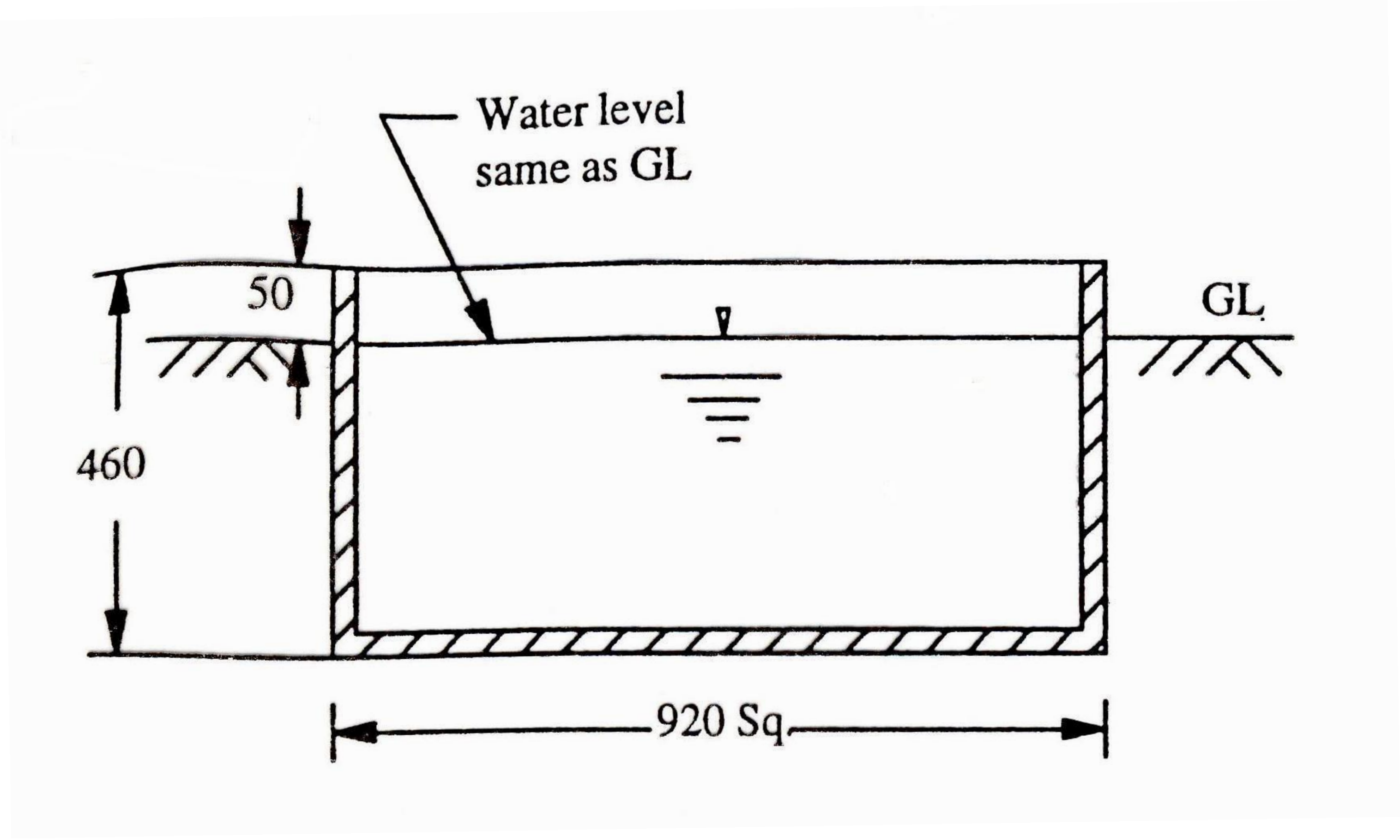
(iii) analytical methods

Types of Evaporimeters

Class A Evaporation Pan



Colorado Sunken Pan



US Geological Survey Floating Pan

Square pan 900 mm side and 450 mm depth supported by drum floats in the middle of a raft (4.25 m x 4.87 m) is set a float in a lake. The water level in the pan is kept at the same level as the lake leaving a rim of 75 mm.

Pan Coefficient Cp

Evaporation pan are not exact models of large reservoirs and have the following principle drawbacks :

1. They differ in the heat storing capacity and heat transfer from the sides and bottom. The sunken pan and floating pan aim to reduce this deficiency. As a result of this factor the evaporation from a pan depends to a certain extent on its size.
2. The height of the rim in an evaporation pan affects the wind action over the surface.
3. The heat transfer characteristics of the pan material is different from that of the reservoir.

Thus a coefficient is introduced as

Lake evaporation = C_p x pan evaporation

In which C_p = pan coefficient. The values of C_p in use for different pans are given in the following Table

VALUES OF PAN COEFFICIENT C_P

Sl. No.	Types of pan	Average value	Range
1.	Class A Land Pan	0.70	0.60-0.80
2.	Colorado Sunken Pan	0.78	0.75-0.86
3.	USGS Floating Pan	0.80	0.70-0.82

Evaporation Stations

The WMO recommends the minimum network of evaporimeter stations as Below.

1. Arid zones – one station for every 30,000 Km^2
2. Humid temperate climates – one station for every 50,000 Km^2, and
3. Cold regions – one station for every 100,000 Km^2.

Empirical evaporation Equations

Meyer's Formula (1915)

$$E_L = K_M (e_w - e_a) (1 + u_9/16)$$

In which, u_9 = monthly mean wind velocity in km/h at bout 9 m above ground and

K_M = coefficient accounting for various other factors with a value of 0.36 for large deep and 0.50 for small shallow waters.

Rohwer's Formula (1931)

$$E_L = 0.771(1.465 - 0.000732\ P_a)\ (0.44 + 0.0733\ u_o)(e_w - e_a)$$

P_a = mean barometric reading in mm of mercury

U_o = mean wind velocity in km/h at ground level, which can be taken to be the velocity at 0.6 m height above ground.

The wind velocity can be assumed to follow the 1/7 power law

$$U_h = C\ h^{1/7}$$

Where, U_h = wind velocity at a height h above the ground and

C = constant.

This equation can be used to determine the velocity at any desired level.

Example : A reservoir with a surface area of 250hectares had the following average values of parameters during a week : water temperature = 20^o C, relative humidity = 40% wind velocity at 1.0 m above ground = 16km/h. Estimate the average daily evaporation from the lake and volume of water Evaporated from the lake during that one week.

Solution :

e_w = 17.54 mm of Hg

e_a = 0.40 x 17.54 = 7.02 mm of Hg

u_9 = wind velocity at a height of 9.0 m above ground

u_1 = 16 km/h $\qquad$ u_9 = ?

$u_h = C\,(h)^{1/7}$

$u_h = C\,(1)^{1/7} = 16$ km/h

$u_9/u_1 = C\,((9)^{1/7}) / C\,((1)^{1/7})$

$$u_9 = u_1\ (9)^{1/7}$$

$$= 16\ (9)^{1/7}$$

$$= 21.9 \text{ km/h}$$

By Meyer's formula

$$E = 0.36\ (17.54 - 7.02)\ (1 + 21.9/16)$$

$$= 8.97 \text{ mm/day}$$

Evaporated volume in 7 days

$$= 7 \times 8.97/1000 \times 250 \times 10000$$

$$= 157{,}000 \text{ m}^3$$

Analytical Methods of Evaporation Estimation

The analytical methods for the determination of Lake evaporation can be broadly classified into three categories as :

1. Water – budget method.
2. Energy – balance method, and
3. Mass – transfer method

Water – Budget Method

$$P + V_{is} + V_{ig} + = V_{os} + V_{og} + E_L + \Delta S + T_L$$

Where,

P = daily evaporation

V_{is} = daily surface inflow into the lake

V_{ig} = daily groundwater inflow

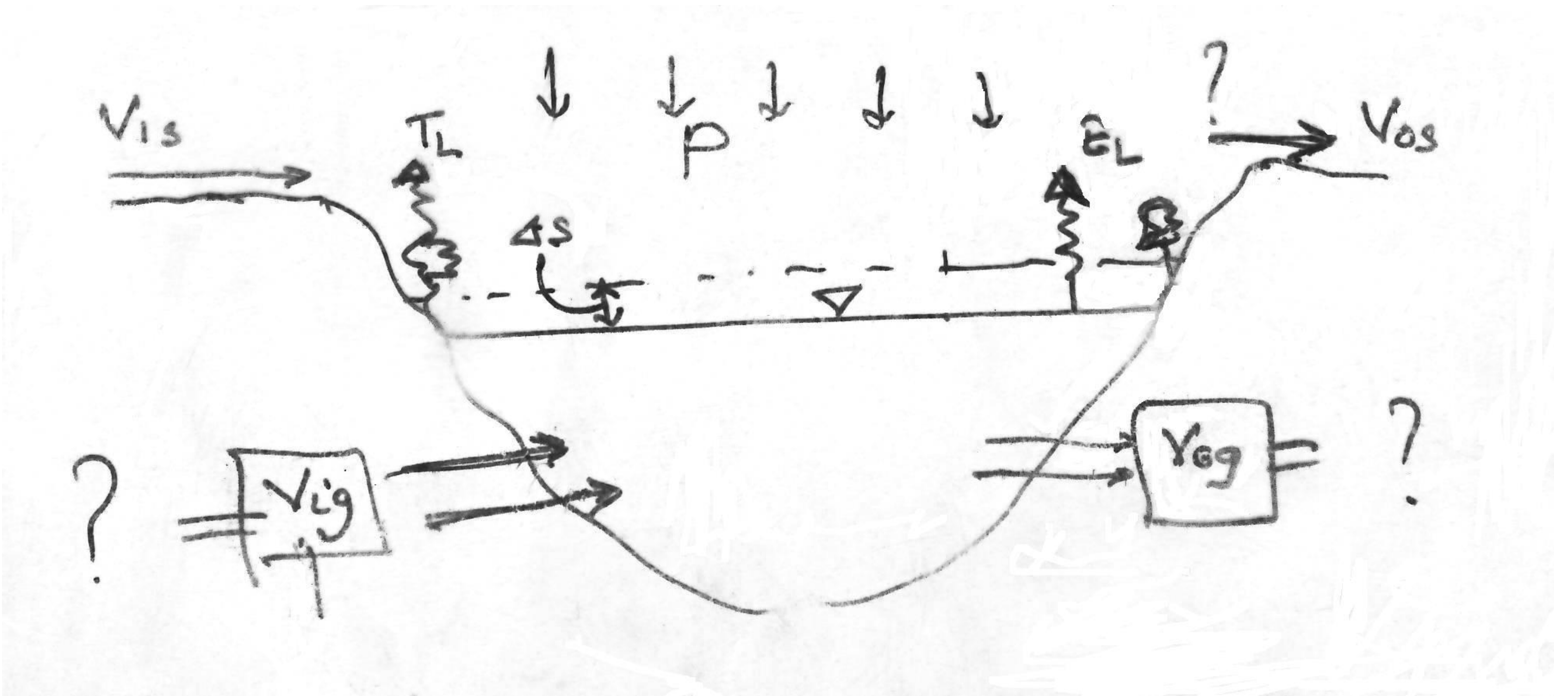
V_{os} = daily surface outflow from the lake

V_{og} = daily surface outflow

EL = daily lake evaporation

Δ S = increase in lake storage in a day

TL = daily transpiration loss



Energy – Budget Method

$$H_n = H_a + H_e + H_g + H_s + H_i$$

Where, H_n = net heat energy received by the water surface

$= H_c (1- r) - H_b$

$H_c (1 – r)$ = incoming solar radiation into a surface of reflection coefficient (albedo) r

H_b = back radiation (long wave) from water body

H_a = sensible heat transfer from water surface to air

H_e = heat energy used up in evaporation

$= \rho L E_L$ where ρ = density of water ,

L = latent heat of evaporation and

E_L = evaporation in mm

H_g = heat flux into the ground

H_s = heat stored in water body

H_i = net heat conducted out of the system by water flow (advected energy)

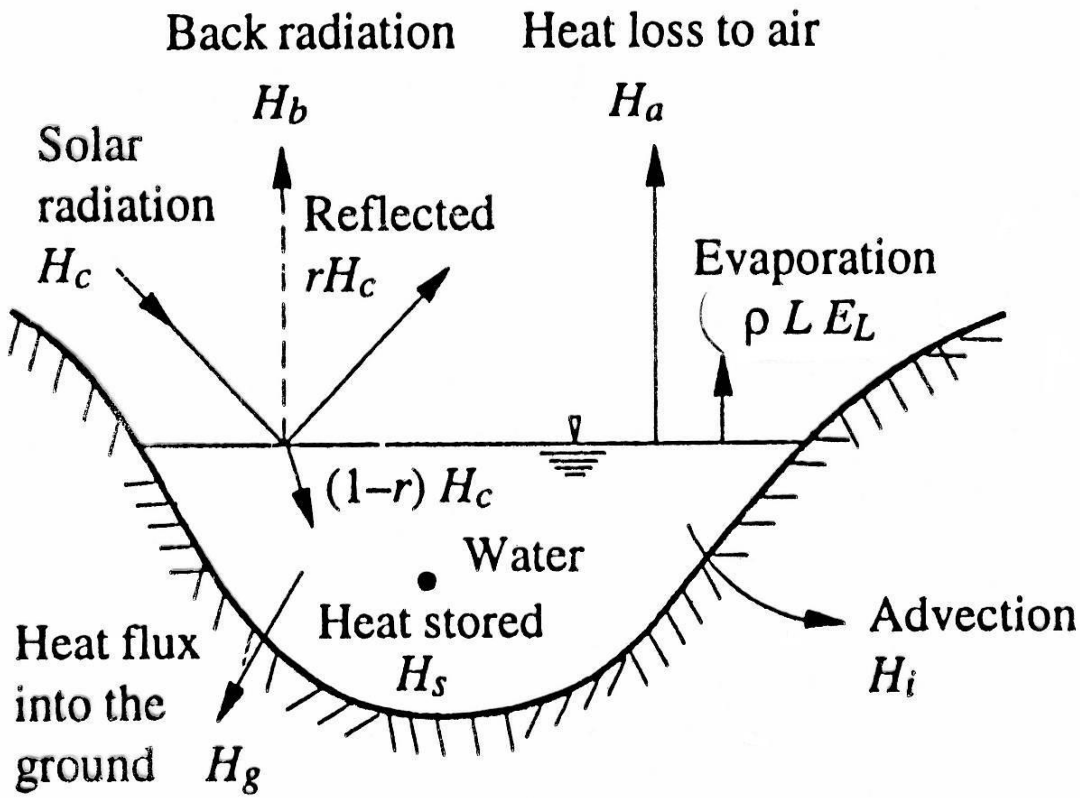


Fig. Energy balance in a water body

Evapotranspiration

Transpiration is the process by which water leaves the body of a living plant and reaches the atmosphere as water vapor. The water is taken up by the plant – root system and escapes through the leaves. The important factors affecting transpiration are: atmospheric vapor pressure, temperature, wind, light intensity and characteristics of the plant, such as the root and leaf systems.

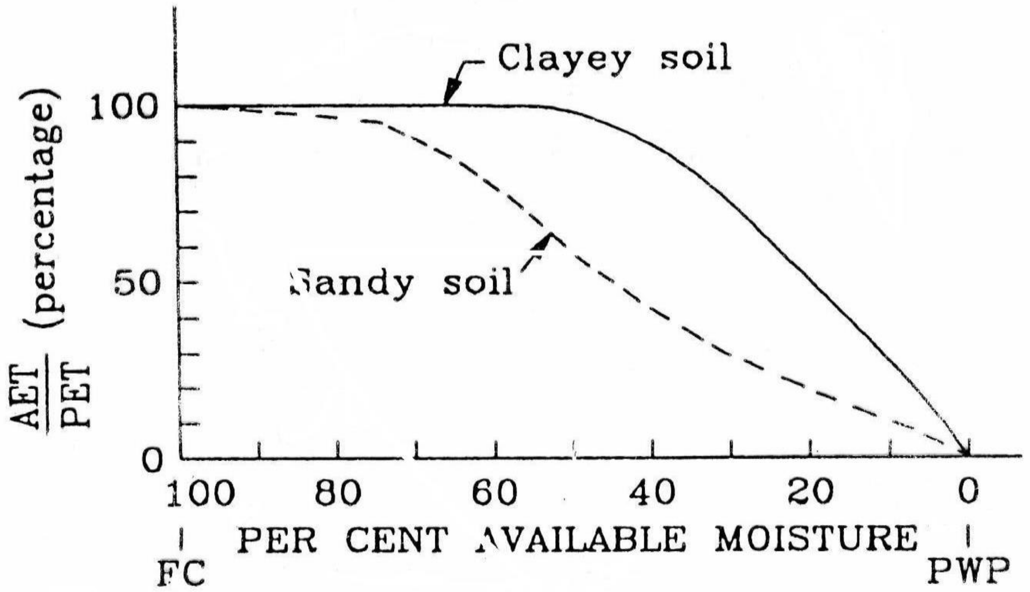


Fig. Variation of AET

Potential Evapotranspiration (PET)

If sufficient moisture is always available to completely meet the needs of vegetation fully covering the area, the resulting evapotranspiration is called potential evapotranspiration (PET).

Actual Evapotranspiration (AET)

The real evapotranspiration occurring in a specific situation is called actual evapotranspiration (AET).

Field Capacity (FC)

Field capacity is the maximum quantity of water that the soil can retained against gravity at field beyond which it simply drains away.

Permanent Wilting point (PWP)

permanent wilting point is the moisture content of a soil at which the moisture is no longer available in sufficient quantity to sustain the plants.

If the water supply to the plant is adequate, soil moisture will be at the field capacity and AET will be equal to PET. If the water supply is less than PET, the soil dries out and the ratio AET/PET with available moisture depends upon the type of soil and rate of drying. Generally, for clayey soils, AET/PET ≈ 1.0 for nearly 50% drop in the permanent wilting point, the AET reduces to zero as shown in figure.

For a catchment in a given period of time, the hydrologic budget can be written as

$$P - R_s - G_o - E_{act} = \Delta S$$

$$E_{act} = P - R_s - G_o \pm \Delta S$$

Where P = precipitation, R_s = surface runoff, G_o = subsurface outflow, E_{act} = actual evapotranspiration (AET) and ΔS = change in the moisture storage.

Measurement of Evapotranspiration

The measurement of evapotranspiration for a given vegetation type can be carried out in two ways:

(i) by using Lysimeters or

(ii) by the use of Field plots

Evapotranspiration = precipitation + irrigation input – increase in soil storage – ground water loss

Evapotranspiration Equations

Penman's Equation

$$\text{PET} = \frac{AH_n + E_a\gamma}{A+\gamma}$$

Where PET = daily potential evapotranspiration in mm per day

A = slope of the saturation vapour pressure vs temperature curve at the mean air temperature, in mm of mercury per °C

H_n = net radiation in mm of evaporable water per day

E_a = parameter including wind velocity and saturation deficit

γ = psychrometric constant = 0.49 mm of mercury/ °C

The net radiation estimated by the following equation:

$$H_n = H_a(1-r)\left(a + b\frac{n}{N}\right) - \sigma T^4{}_a\left(0.56 - 0.092\sqrt{e_a}\right)\left(0.10 + 0.90\frac{n}{N}\right)$$

Where

H_n = incident solar radiation outside the atmosphere on a horizontal surface, expressed in mm of evaporable water per day.

a =a constant depending upon the latitude Φ and is given by $a = 0.29 \cos\Phi$

b = a constant with an average value of 0.52

n = actual duration of a bright sunshine in hours

N = maximum possible hours of bright sunshine

r = reflection co-efficient (albedo)

Usual ranges of values of r are given below:

Surface	range of r values
Close ground crops	0.15 - 0.25
Bare lands	0.05 - 0.45
Water surface	0.05
Snow	0.45 - 0.95

σ = Stefan-Boltzman constant = 2.01×10^{-9} mm/day

T_a = mean air temperature in degrees Kelvin = 273+ °C

e_a = actual mean vapour pressure in the air in mm of mercury

The parameter E_a is estimated as

$$E_a = 0.35\left(1+\frac{u_2}{160}\right)\left(e_w - e_a\right)$$

in which

u_2 =mean wind speed at 2 m above ground in km/day

e_w = saturation vapour pressure at mean air temperature in mm of mercury

e_a = actual vapour pressure, defined earlier

For the computation of PET, data on n, e_a, u_2 mean air temperature and nature of surface (i.e. value of r) are needed. These can be obtained from actual observations or through available meteorological data of the region.

TABLE SATURATION VAPOUR PRESSURE OF WATER

Temperature (°C)	Saturation vapour pressure e_w (mm of Hg)	A (mm/°C)
0	4.58	0.30
5.0	6.54	0.45
7.5	7.78	0.54
10.0	9.21	0.60
12.5	10.87	0.71
15.0	12.79	0.80
17.5	15.00	0.95
20.0	17.54	1.05
22.5	20.44	1.24
25.0	23.76	1.40
27.5	27.54	1.61
30.0	31.82	1.85
32.5	36.68	2.07
35.0	42.81	2.35
37.5	48.36	2.62
40.0	55.32	2.95
45.0	71.20	3.66

$$e_w = 4.584 \exp\left(\frac{17.27\,t}{237.3 + t}\right) \text{ mm of Hg, where } t = \text{temperature in °C}$$

TABLE MEAN MONTHLY SOLAR RADIATION AT TOP OF ATMOSPHERE, H_a IN mm OF EVAPORABLE WATER/DAY

North latitude	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0°	14.5	15.0	15.2	14.7	13.9	13.4	13.5	14.2	14.9	15.0	14.6	14.3
10°	12.8	13.9	14.8	15.2	15.0	14.8	14.8	15.0	14.9	14.1	13.1	12.4
20°	10.8	12.3	13.9	15.2	15.7	15.8	15.7	15.3	14.4	12.9	11.2	10.3
30°	8.5	10.5	12.7	14.8	16.0	16.5	16.2	15.3	13.5	11.3	9.1	7.9
40°	6.0	8.3	11.0	13.9	15.9	16.7	16.3	14.8	12.2	9.3	6.7	5.4
50°	3.6	5.9	9.1	12.7	15.4	16.7	16.1	13.9	10.5	7.1	4.3	3.0

TABLE MEAN MONTHLY VALUES OF POSSIBLE SUNSHINE HOURS, N

North latitude	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0°	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1
10°	11.6	11.8	12.1	12.4	12.6	12.7	12.6	12.4	12.9	11.9	11.7	11.5
20°	11.1	11.5	12.0	12.6	13.1	13.3	13.2	12.8	12.3	11.7	11.2	10.9
30°	10.4	11.1	12.0	12.9	13.7	14.1	13.9	13.2	12.4	11.5	10.6	10.2
40°	9.6	10.7	11.9	13.2	14.4	15.0	14.7	13.8	12.5	11.2	10.0	9.4
50°	8.6	10.1	11.8	13.8	15.4	16.4	16.0	14.5	12.7	10.8	9.1	8.1

Example: Calculate the potential evapotranspiration from an area in the month of November by Penman's formula. The following data are available:

Latitude	:	$28^0\ 4'N$
Elevation	:	230 m (above sea level)
Mean monthly temperature	:	$19\ ^0C$
Mean relative humidity	:	75%
Mean ovserved sunshine hours	:	9 h
Wind velocity at 2 m height	:	85 km / day
Nature of surface cover	:	Close-ground green crop

Solution : From table

$A = 1.00$ mm/ °C

$e_w = 16.50$ mm of Hg

From Table

$H_a = 9.506$ mm of water/day

From Table

$N = 10.716$ h

$n/N = 9/10.716 = 0.84$

From given data

$e_a = 16.50 \times 0.75 = 12.38$ mm of Hg

$a = 0.29 \cos 28^{0} 4' = 0.2559$

$b = 0.52$

$\sigma = 2.01 \times 10^{-9}$ mm/ day

$T_a = 273 + 19 = 292$ K

$\sigma T_a^4 = 14.6132$

r = albedo for close-ground green crop is taken as 0.25

Form Eq.

$$H_n = 9.506 \times (1-0.25) \times [0.2559 + (0.52\times0.84)]$$
$$- 14.613 \times (0.56-0.092\sqrt{12.38})\times (0.10 + (0.9 \times 0.84))$$
$$= 4.936 - 2.946$$
$$= 1.990 \text{ mm of water /day}$$

From Eq.

$$E_a = 0.35\times\left(1+\frac{85}{160}\right)\times(16.50-12.38)$$
$$= 2.208 \text{ mm/day}$$

From Eq. noting the value of $\gamma = 0.49$

$$\text{PET} = \frac{(1\times1.990)+(2.208\times0.49)}{(1.00+0.49)} = 2.06\,mm/day$$

Examle: Using the data of the above example, estimate the daily evaporation from a lake situated in that place.

Solution: For estimating the daily evaporation from a lake, Penman's equation is used with the albedo r=0.05.

Hence

$$H_n = (4.936) \times \frac{(1.0 - 0.05)}{(1.0 - 0.25)} - 2.946$$

$$= 6.252 - 2.946 = 3.306 \text{ mm of water/day}$$

$$E_a = 2.208 \text{ mm/day}$$

From Eq.

PET = Lake evaporation

$$= \frac{(1.0 \times 3.306) + (2.208 \times 0.49)}{(1.0 + 0.49)}$$

$$= 2.95 \text{ mm/day}$$

Infiltration

this movement of water through the soil surface is known as infiltration.

Infiltration process

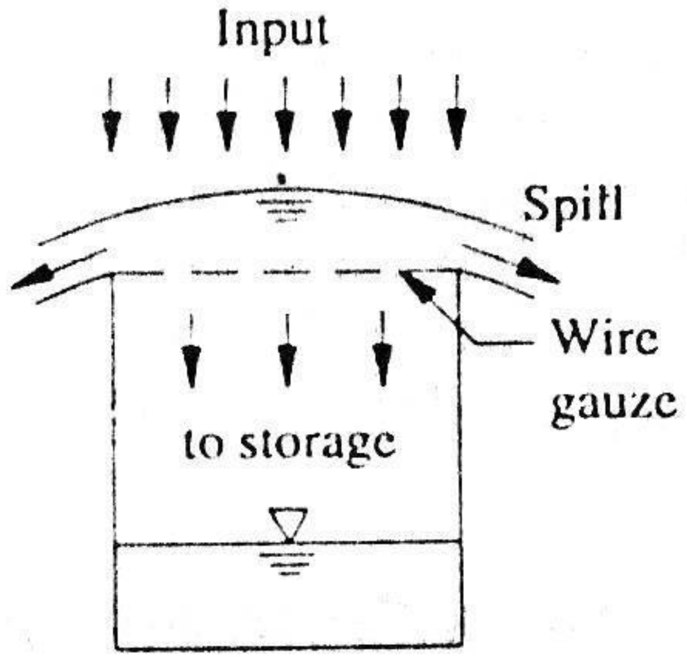


Fig. 3.8 An anology for infiltration

Infiltration Capacity

The infiltration rate at which a given soil at a given time can absorb water is defined as the infiltration capacity. It is designed as fc and is expressed in units of cm/h. The actual rate of infiltration f can be expressed as

$$f = f_c \quad \text{when } i \geq f_c$$

$$f = i \quad \text{when } i < f_c$$

Where i = intensity of rainfall

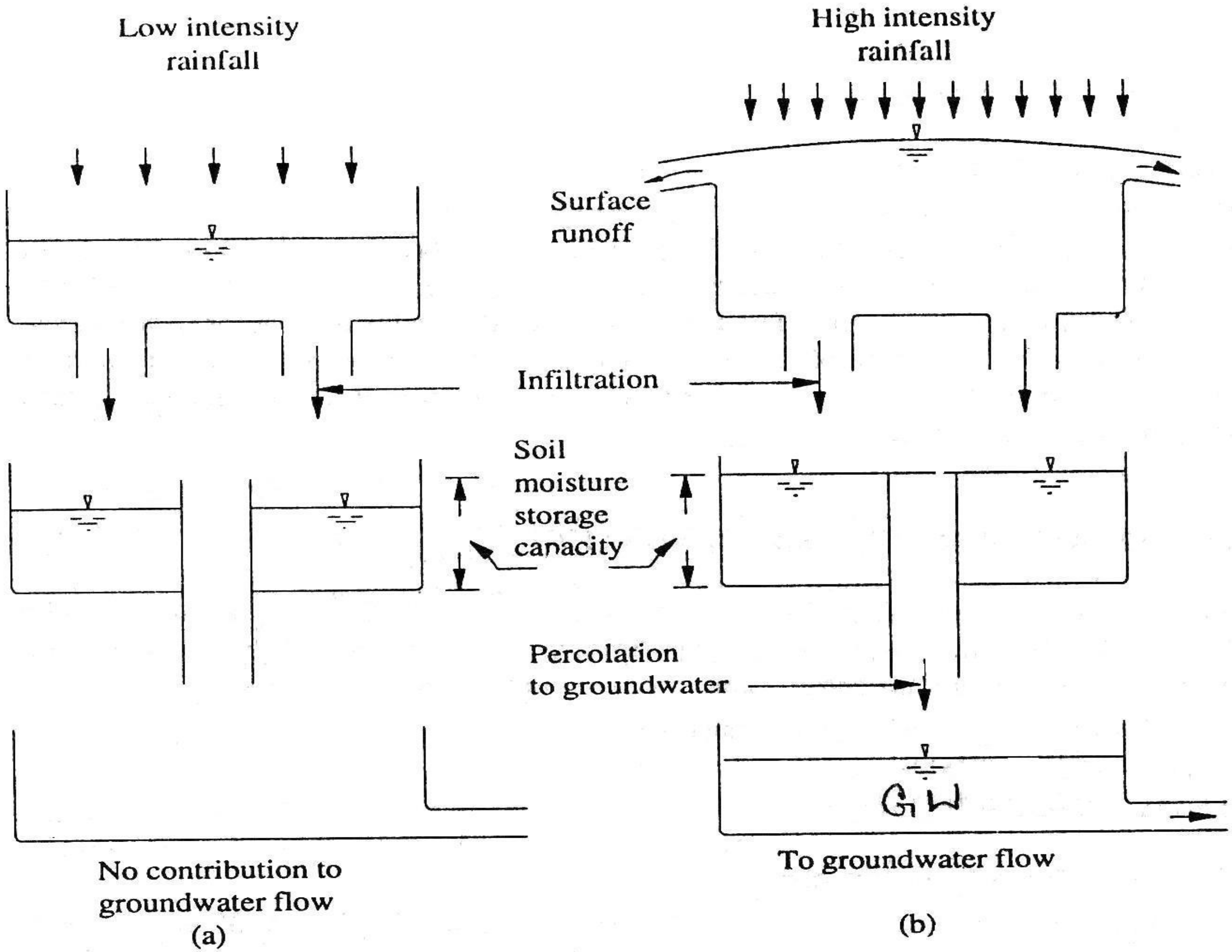


Fig. 3.9 An infiltration model

Measurement of Infiltration

There are two kinds of infiltrometers :

(i) Flooding - type infiltrometer, and

(ii) Rainfall simulator

(i) Flooding - type infiltrometer

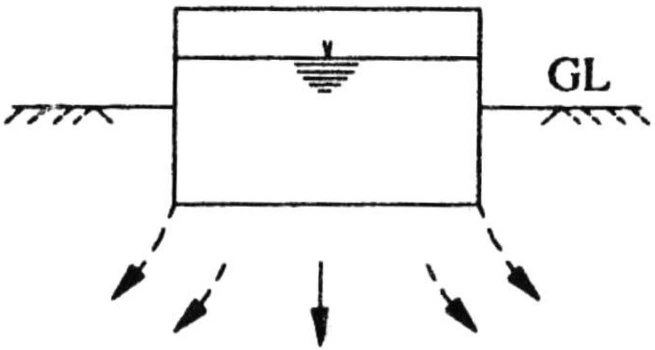


Fig. 3.10 Simple infiltrometer

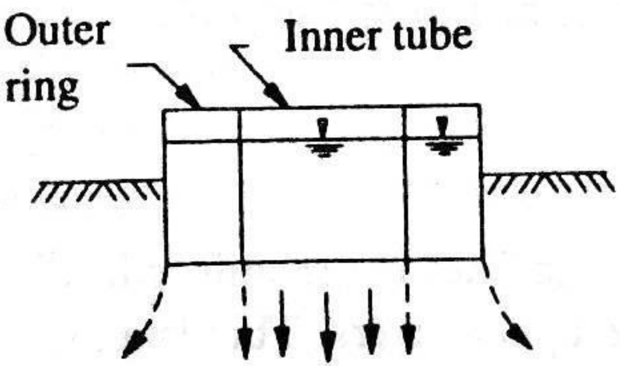


Fig. 3.11 Ring infiltrometer

(ii) Rainfall simulator

Infiltration Capacity values

Horton (1930) expressed the decay of the infiltration capacity with time as

$$f_{ct} = f_{cf} + (f_{co} - f_{cf})\, e^{-K_h t} \qquad \text{for } 0 \le t \le t_d$$

Where f_{ct} = infiltration capacity at any time t from start of the rainfall

f_{co} = initial infiltration capacity at t = 0

f_{cf} = final steady state value

t_d = duration of the rainfall and

k_h = constant depending upon the soil characteristics and vegetation cover

Initial Loss

In the precipitation reaching the surface of a catchment the major abstraction is from

the infiltration process. These are

(i) the interception process and

(ii) the depression storage,

and together they are called initial loss.

(i) Interception

The volume of water so caught is called interception. The intercepted precipitation may follow one of the three possible routes:

(a) obstructed by vegetation : interception loss

(b) contribution to surface flow through drip water : through fall

(c) flow through stem : stem flow

Interception loss is about 10 to 20%

The interception loss is estimated as

$$I_i = S_i + k_i E t$$

where I_i = interception loss in mm, S_i = interception storage whose value varies from 0.25 to 1.25 mm depending on the nature of vegetation, k_i = ratio of vegetal surface area to its projected area,

E = evaporation rate in mm/h during the precipitation and

t = duration of rainfall in hours

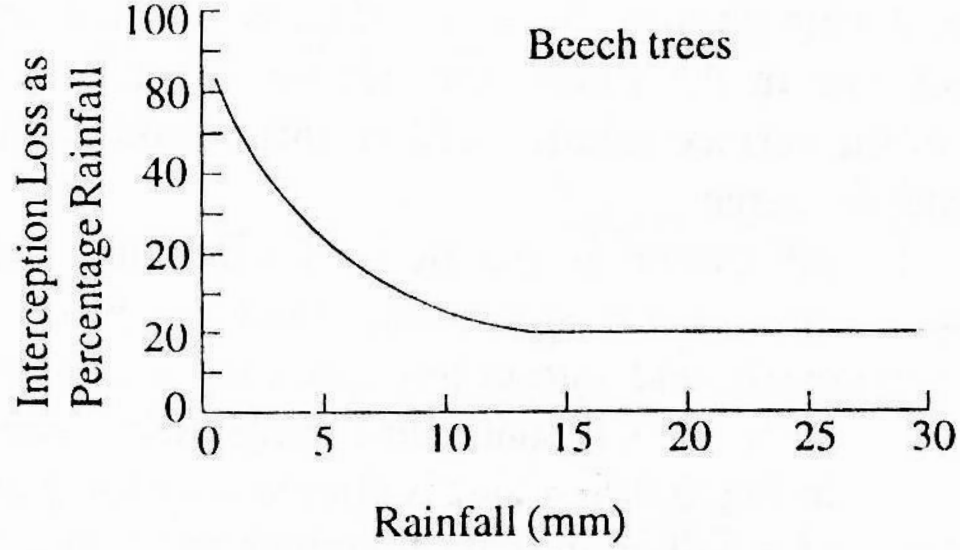


Fig. 3.7 Typical interception loss curve

(ii) Depression storage

the volume of water trapped in these depressions is called depression storage. Depression storage depends on a vast number of factors the chief of which are :

(a) the type of soil,

(b) the condition of the surface reflecting the amount and nature of depression,

(c) the slope of the catchment and

(d) the antecedent precipitation

values of 0.50 cm in sand, 0.4 cm in loam and 0.25 cm in clay

Φ – Index

The Φ index is the average rainfall above which the rainfall volume is equal to the runoff volume.

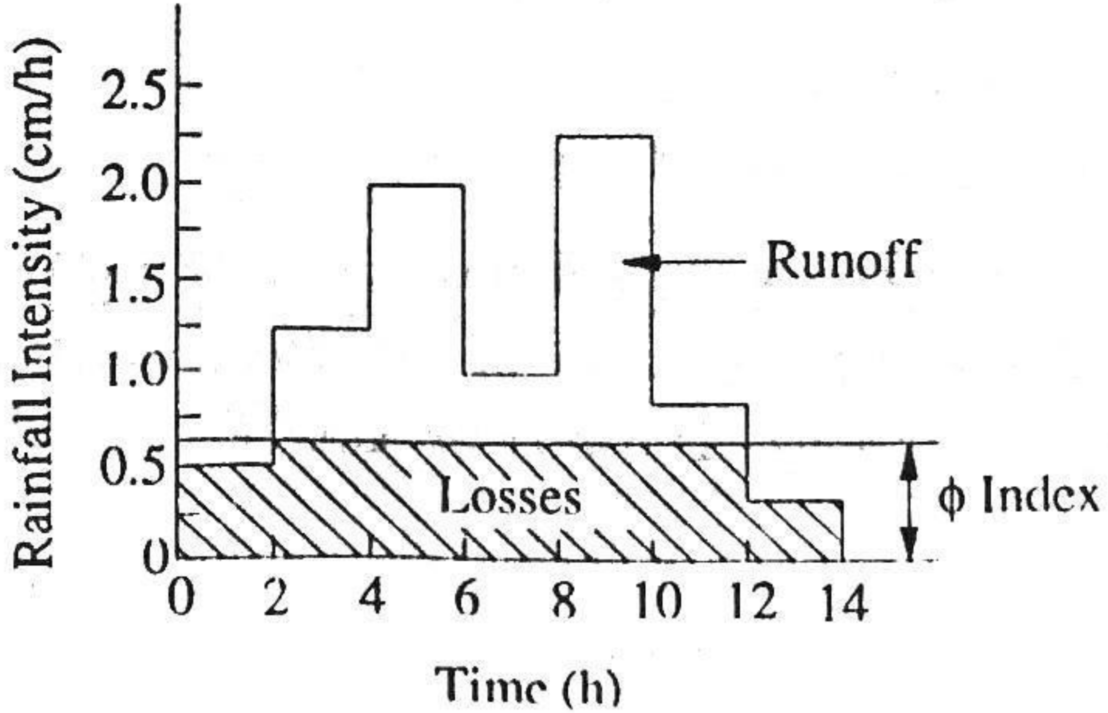


Fig. 3.13 ϕ Index

EXAMPLE 3.5 *A storm with 10.0 cm precipitation produced a direct runoff of 5.8 cm. Given the time distribution of the storm as below, estimate the ϕ index of the storm.*

Time from start (h)	1	2	3	4	5	6	7	8
Incremental rainfall in each hour (cm)	0.4	0.9	1.5	2.3	1.8	1.6	1.0	0.5

SOLUTION : Total infiltration = 10.0 – 5.8 = 4.2 cm

Assume t_e = time of rainfall excess = 8 h for the first trial.

Then

$$\phi = \frac{4.2}{8} = 0.525 \text{ cm/h}$$

But this value of ϕ makes the rainfalls of the first hour and eighth hour ineffective as their magnitude is less than 0.525 cm/h. The value of t_e is therefore modified.

Assume t_e = 6 h for the second trial

In this period,

$$\text{Infiltration} = (10.0\text{-}0.4\text{-}0.5\text{-}5.8)$$
$$= 3.3 \text{ cm}$$
$$\phi = \frac{3.3}{6} = 0.55 \text{ cm/h}$$

This value of ϕ is satisfactory as it gives t_e = 6 h and by calculating the rainfall excesses.

Time from start (h)	1	2	3	4	5	6	7	8
Rainfall excess (cm)	0	0.35	0.95	1.75	1.25	1.05	0.45	0

Total rainfall excess = 5.8 cm = total runoff.

Runoff

Introduction

Runoff means the draining or flowing off of precipitation from a catchment area through a surface channel. it thus represents the output from the catchment in a given unit of time.

The runoff is classified into two categories

i. Direct runoff and

ii. Base flow

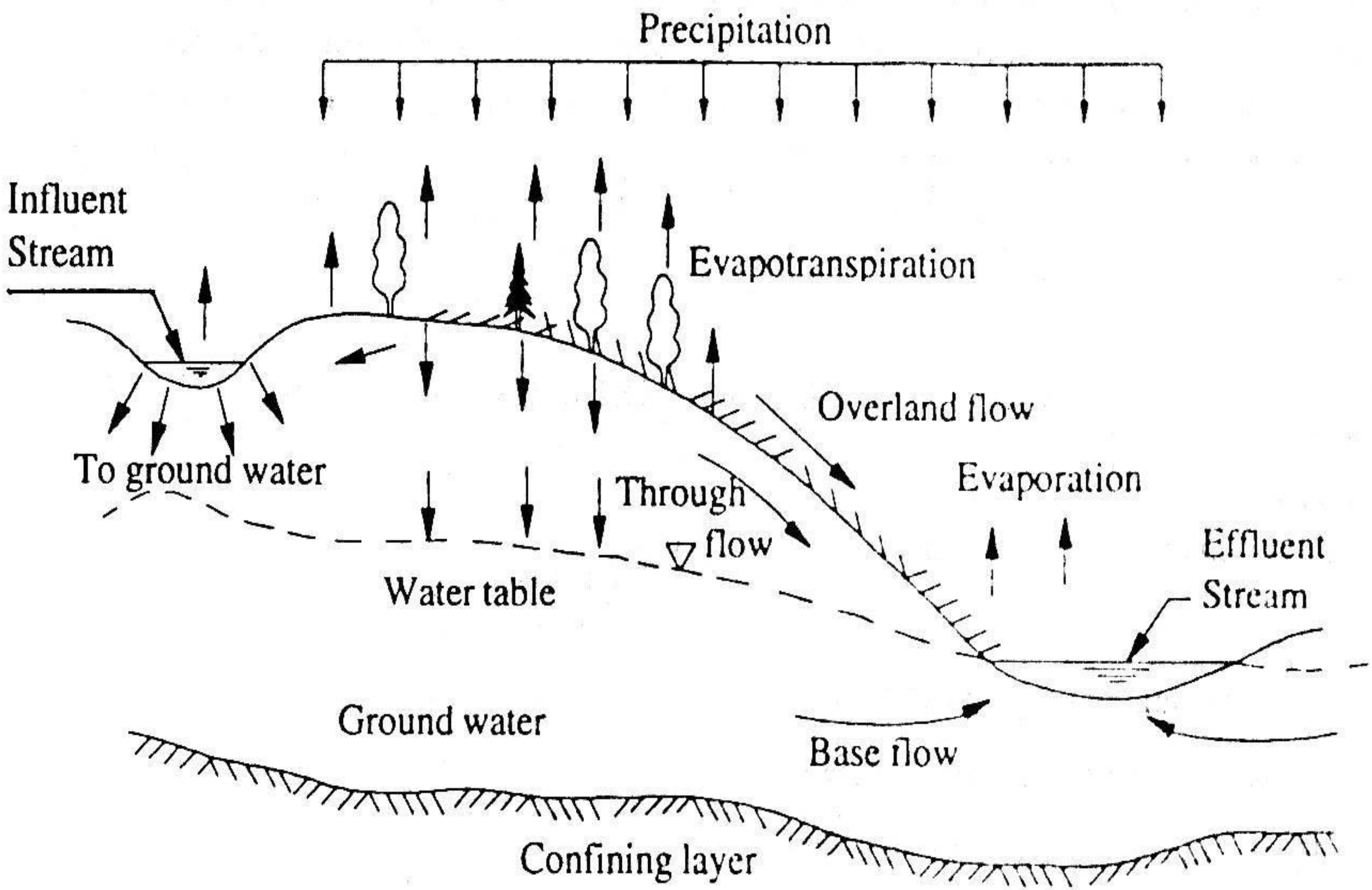


Fig. 5.1 Different routes of runoff

Runoff Characteristics of Streams

The streams is classified into three categories

i. perennial

ii. Intermittent and

iii. ephemeral

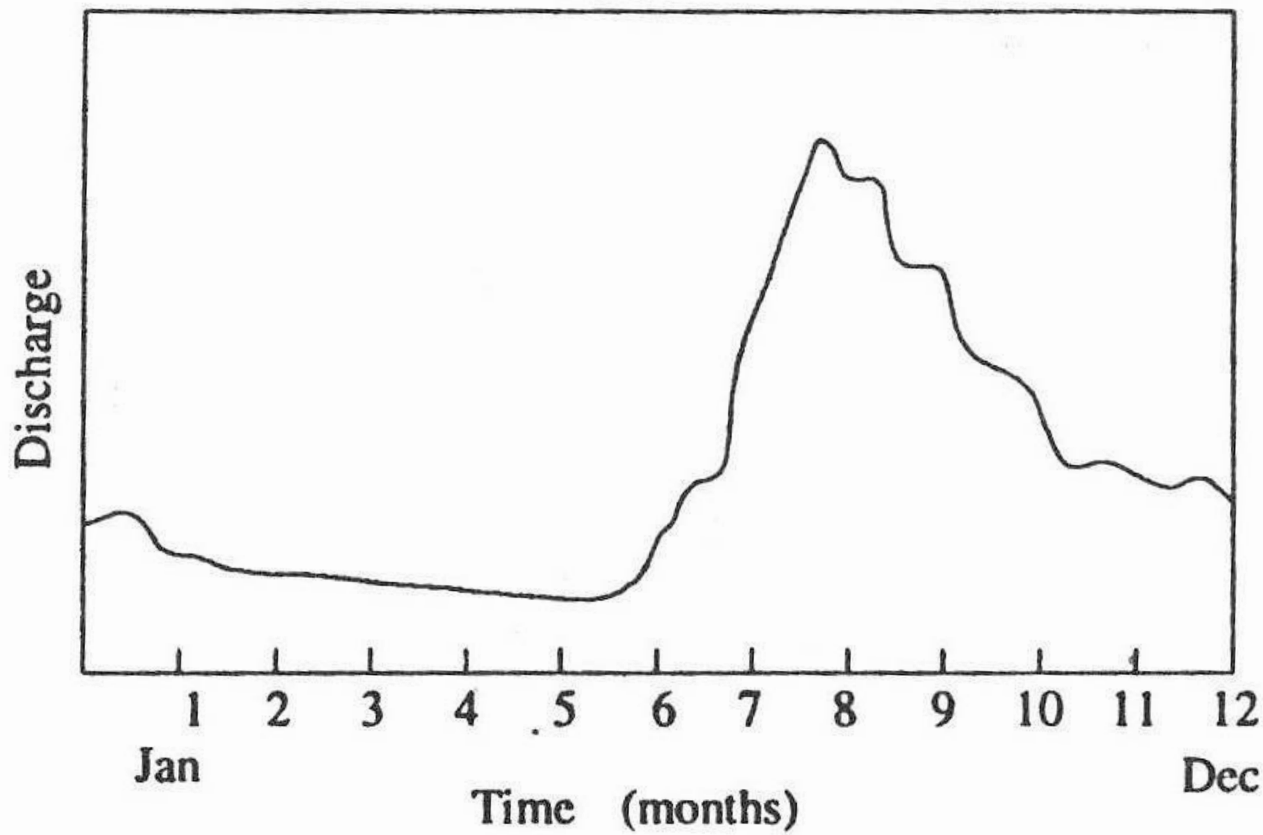


Fig. 5.2 Perennial stream

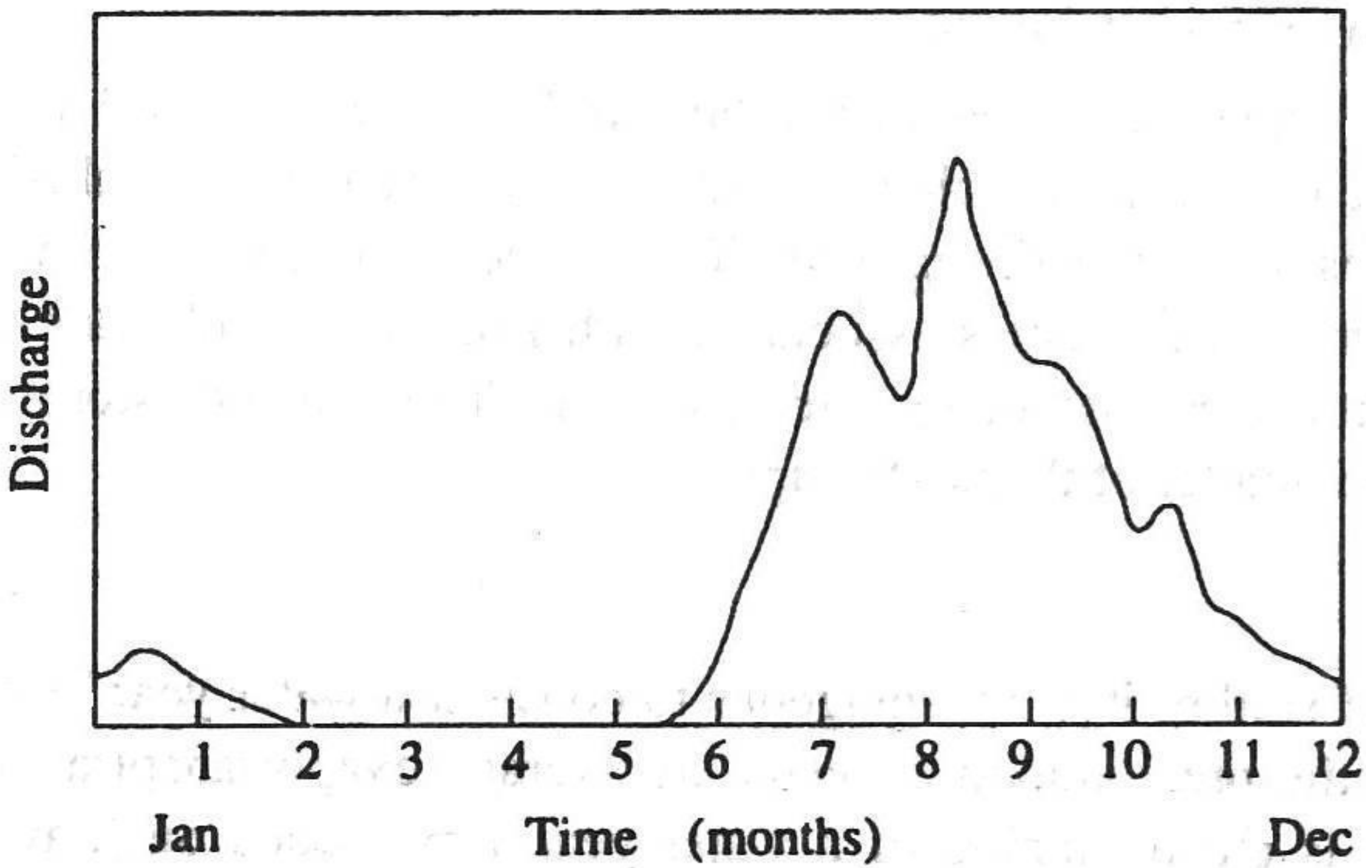


Fig. 5.3 Intermittent Stream

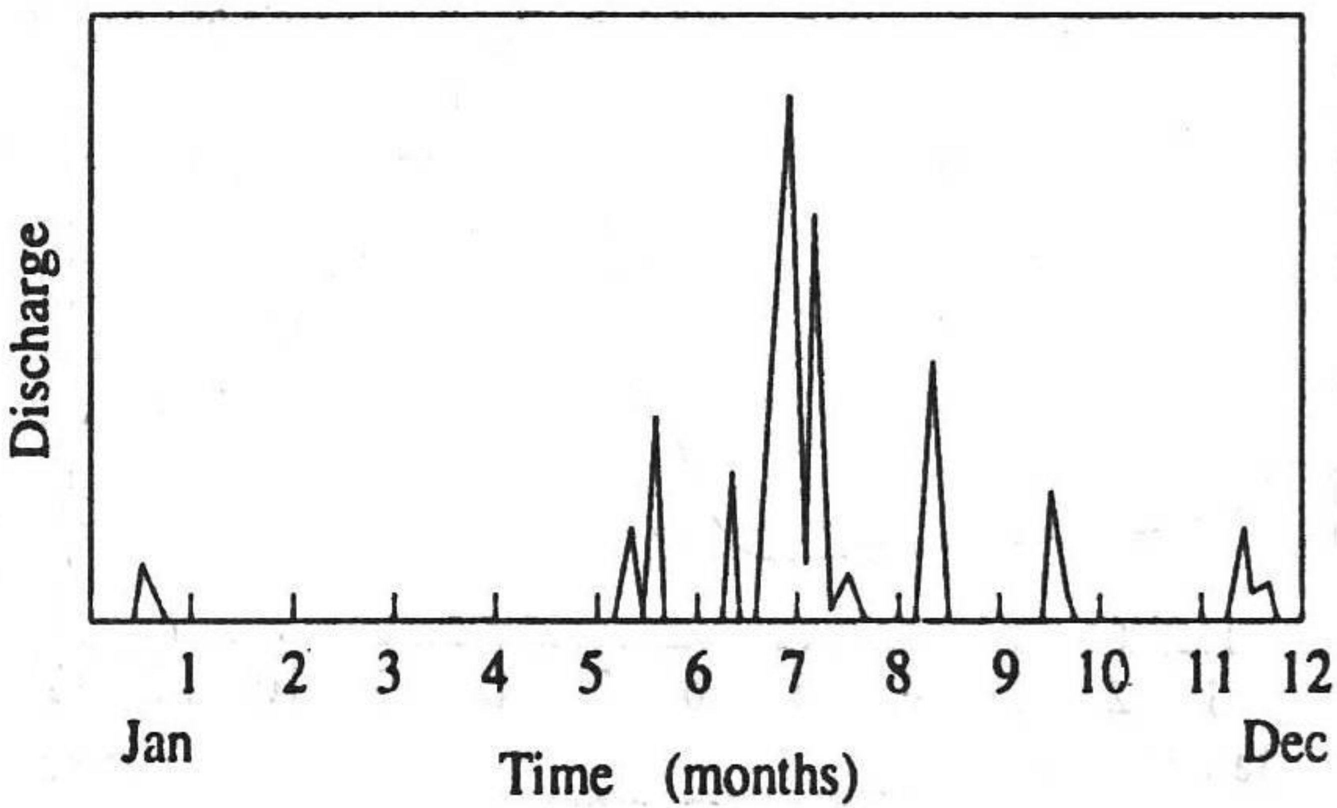


Fig. 5.4 Ephemeral stream

Rainfall – Runoff Correlation

$$R = a\,P + b$$

For large catchment, it is found advantageous to have an exponential relationship as

$$R = \beta\,P^{m}$$

Hydrographs

The hydrograph of this kind which results due to an isolated storm is typically single-peaked skew distribution of discharge and is known variously as storm hydrograph, flood hydrograph or simply hydrograph. It has three characteristics regions:

(i) the rising limb AB, joining point A, the starting point of the rising curve and point B, the point of inflection,

(ii) the crest segment BC between the two points of inflection with a peak P in between,

(iii) the falling limb or depletion curve CD starting from the second point of inflection C.

Lag Time

The time interval from the center of mass of rainfall to the center of mass of hydrograph called lag time T_L

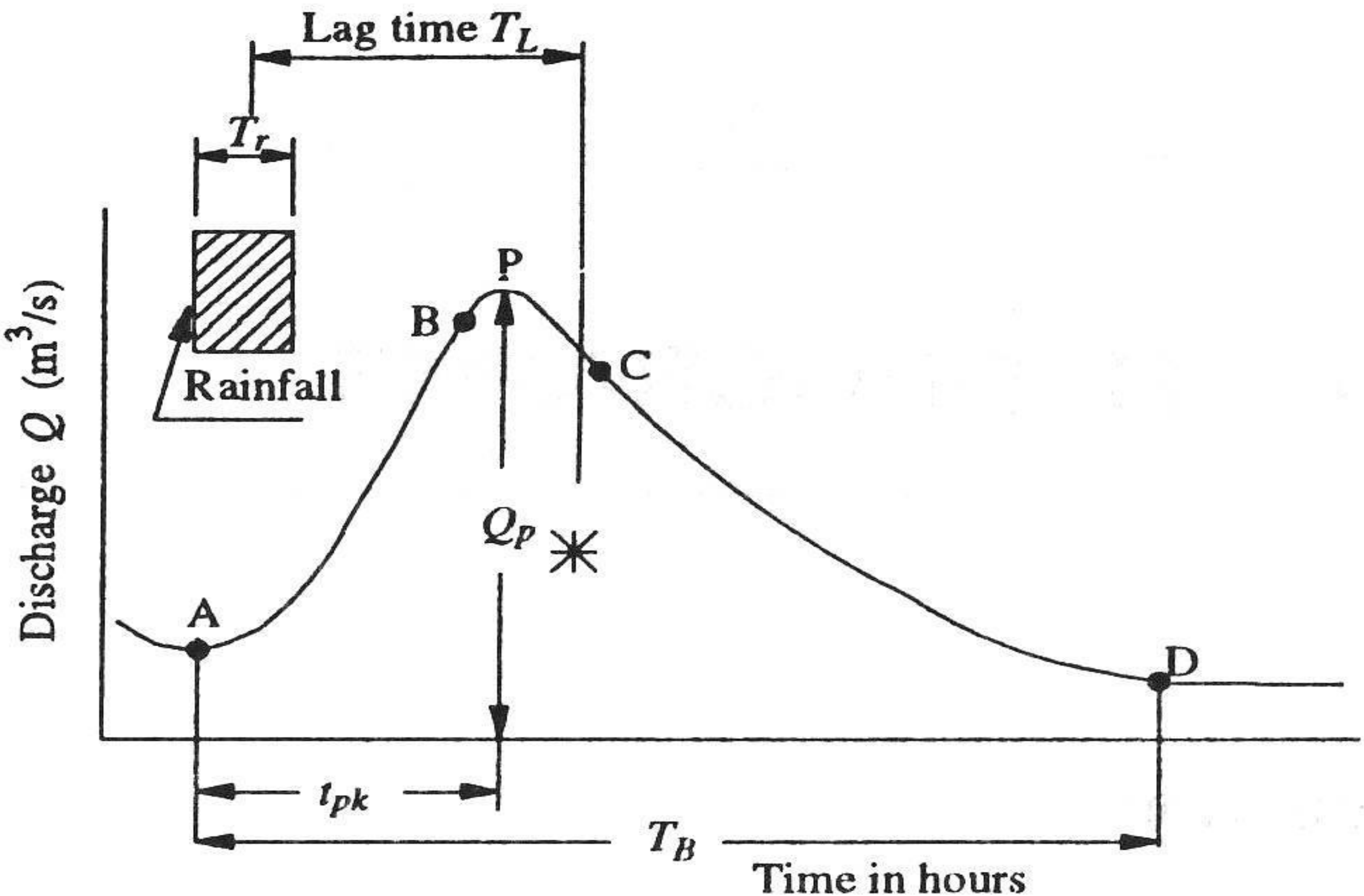


Fig. 6.1 Elements of a Flood Hydrograph

Factors affecting flood hydrograph

TABLE 6.1 FACTORS AFFECTING FLOOD HYDROGRAPH

Physiographic factors	Climatic factors
1. Basin characterstics : (a) Shape (b) size (c) slope (d) nature of the valley (e) elevation (f) drainage density	1. Storm characterstics : precipitation, intensity, duration, magnitude and movement of storm.
2. Infiltration characteristics : (a) land use and cover (b) soil type and geological conditions (c) lakes, swamps and other storage	2. Initial loss
3. Channel characteristics : cross-section, roughness and storage capacity.	3. Evapotranspiration

Shape of the basin

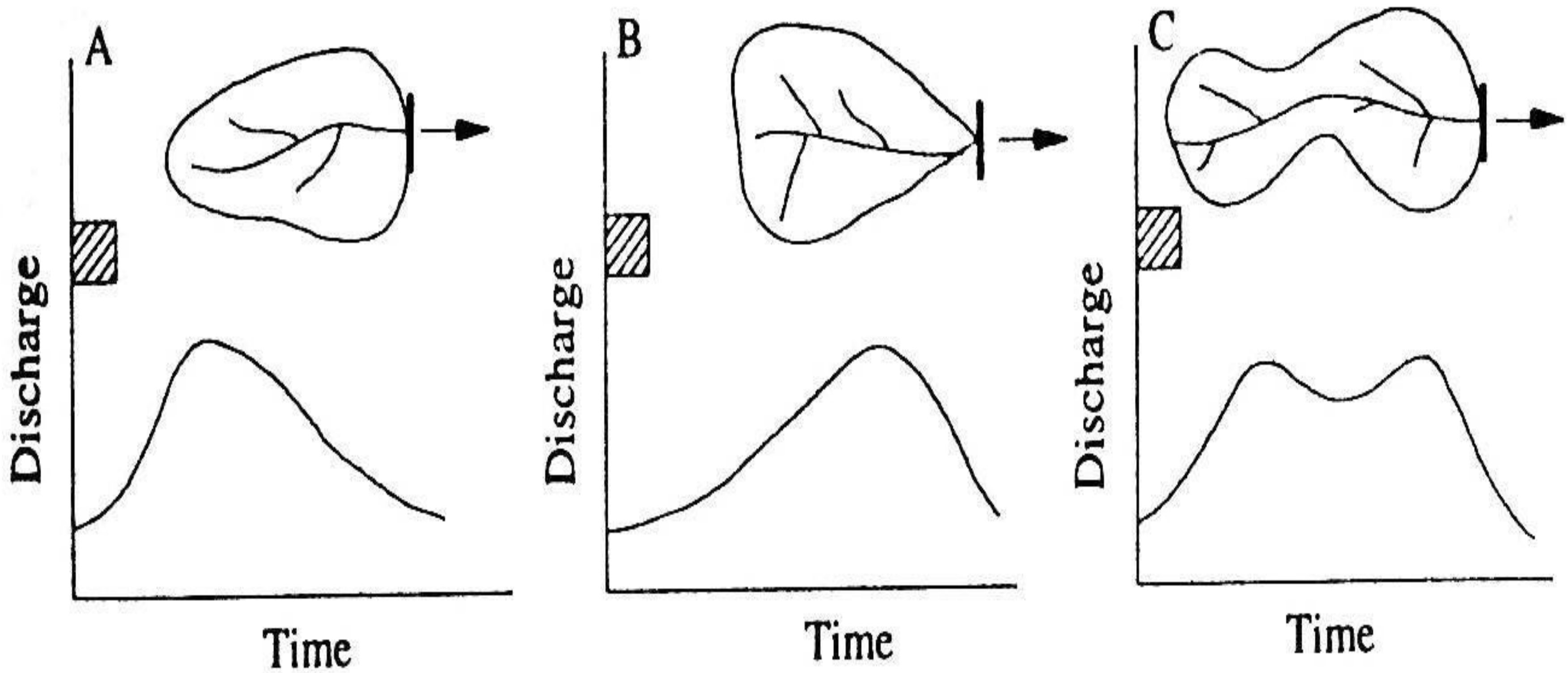


Fig. 6.2 Effect of catchment shape on the hydrograph

Drainage Density

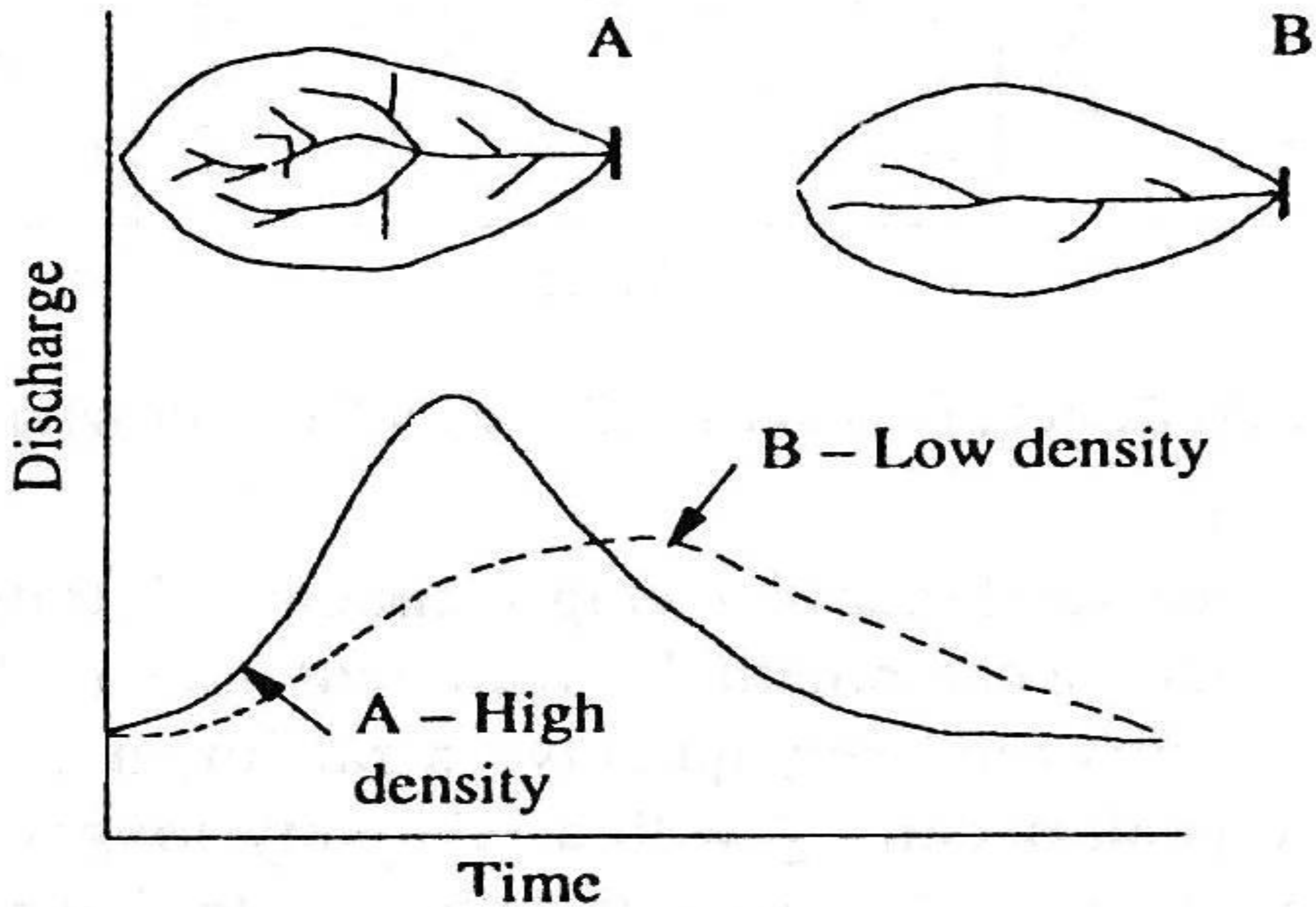


Fig. 6.3 Role of drainage density on the hydrograph

Components of a Hydrograph

(i) the rising limb,

(ii) the crest segment, and

(iii) the recession limb

Recession Limb

The storage of water in the basin exists as

(i) surface storage ,

(ii) interflow storage,

(iii) ground water storage, i.e. base- flow storage.

The recession of a storage can be expressed as

$$Q_t = Q_o \times K_r^t$$

$$\frac{Q_t}{Q_o} = K_r^t$$

$$\log\left(\frac{Q_t}{Q_o}\right) = \log K_r^t$$

$$\log\left(\frac{Q_t}{Q_o}\right) = t \log K_r$$

$$\log K_r = \frac{1}{t} \log\left(\frac{Q_t}{Q_o}\right)$$

The recession constant K_r can be considered to be made up of three components to take care of the three types of storage as

$$K_r = K_{rs} \cdot K_{ri} \cdot K_{rb}$$

Where K_{rs} = recession constant for surface storage,

K_{ri} = recession constant for interflow and

K_{rb} = recession constant for base flow

Typically the values of these recession constants, when t is in days, are

K_{rs} = 0.05 to 0.20

K_{ri} = 0.50 to 0.85

K_{rb} = 0.85 to 0.99

EXAMPLE 6.1 *The recession portion of a flood hydrograph is given below. The time is indicated from the arrival of peak. Assuming the interflow component to be negligible, calculate the baseflow and surface flow recession coefficients.*

Time from peak (days)	Discharge (m^3/s)	Time from peak (days)	Discharge (m^3/s)
0.0	90	3.5	5.0
0.5	66	4.0	3.8
1.0	34	4.5	3.0
1.5	20	5.0	2.6
2.0	13	5.5	2.2
2.5	9	6.0	1.8
3.0	6.7	6.5	1.6
		7.0	1.5

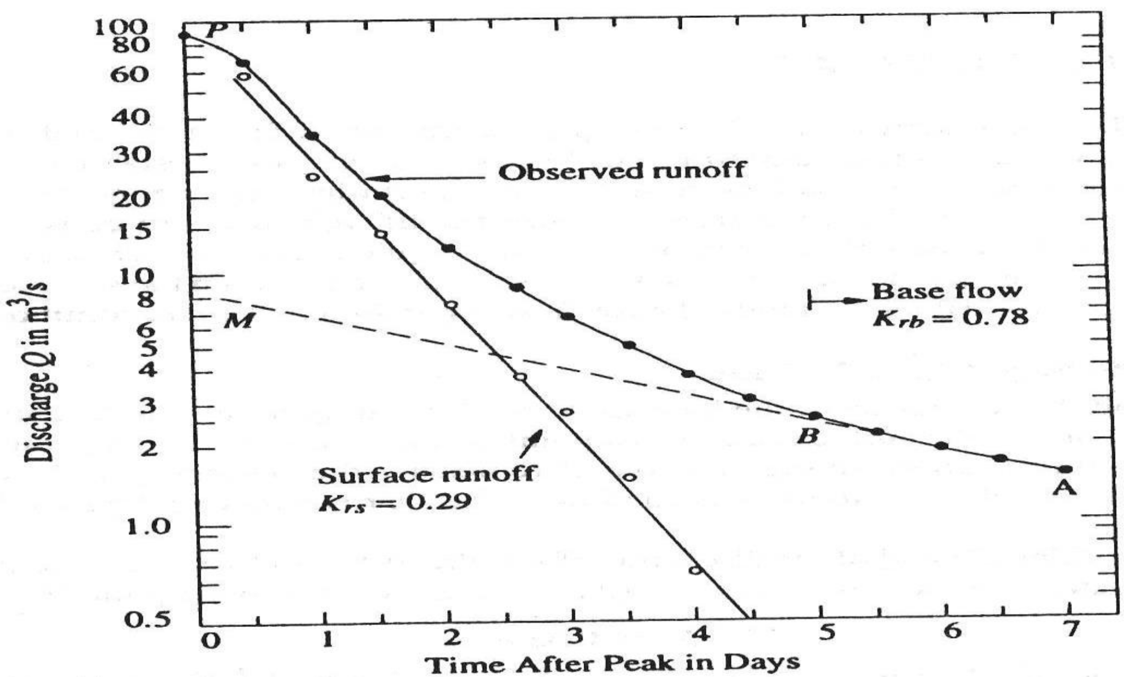


Fig. 6.4 Storage recession curve—Example 6.1

SOLUTION : The data are plotted on a semilog paper with discharge on the log-scale (Fig. 6.4). The part of the curve AB that plots as a straight line indicates the base flow. The surface flow terminates at point B, 5 days after the peak. From Eq. (6.1),

$$Q_t/Q_0 = K_{rb}^t$$

$$\log K_{rb} = \frac{1}{t}\log(Q_t/Q_0)$$

The base flow recession is shown by line ABM in Fig. 6.4.

In this figure by taking th initial discharge at 1 day after the peak as $Q_0 = 6.6\ m^3/s$ and t = time interval as 2 days, Q_t = discharge at a time interval of t = 2 days, i.e. 3 days after the peak = 40 m^3/s. This leads to

$$\log K_{rb} = \frac{1}{2}\log\left(\frac{4.0}{6.6}\right)$$

$$K_{rb} = 0.778 \quad \text{say } 0.78$$

From the curve PA, the base flow recession ABM is subtracted to get the surface runoff. Fig. 6.4. shows the surface runoff depletion plot as a straight line. Now by taking $Q = 26\ m^3/s$ and t = 2 days with correspondin $Q = 2.25\ m^3/s$ the recession constant for surface storage K_{rs} is given by

$$\log K_{rs} = \frac{1}{2}\log\left(\frac{2.25}{26.0}\right)$$

Thus $$K_{rs} = 0.29.$$

Base Flow Separation

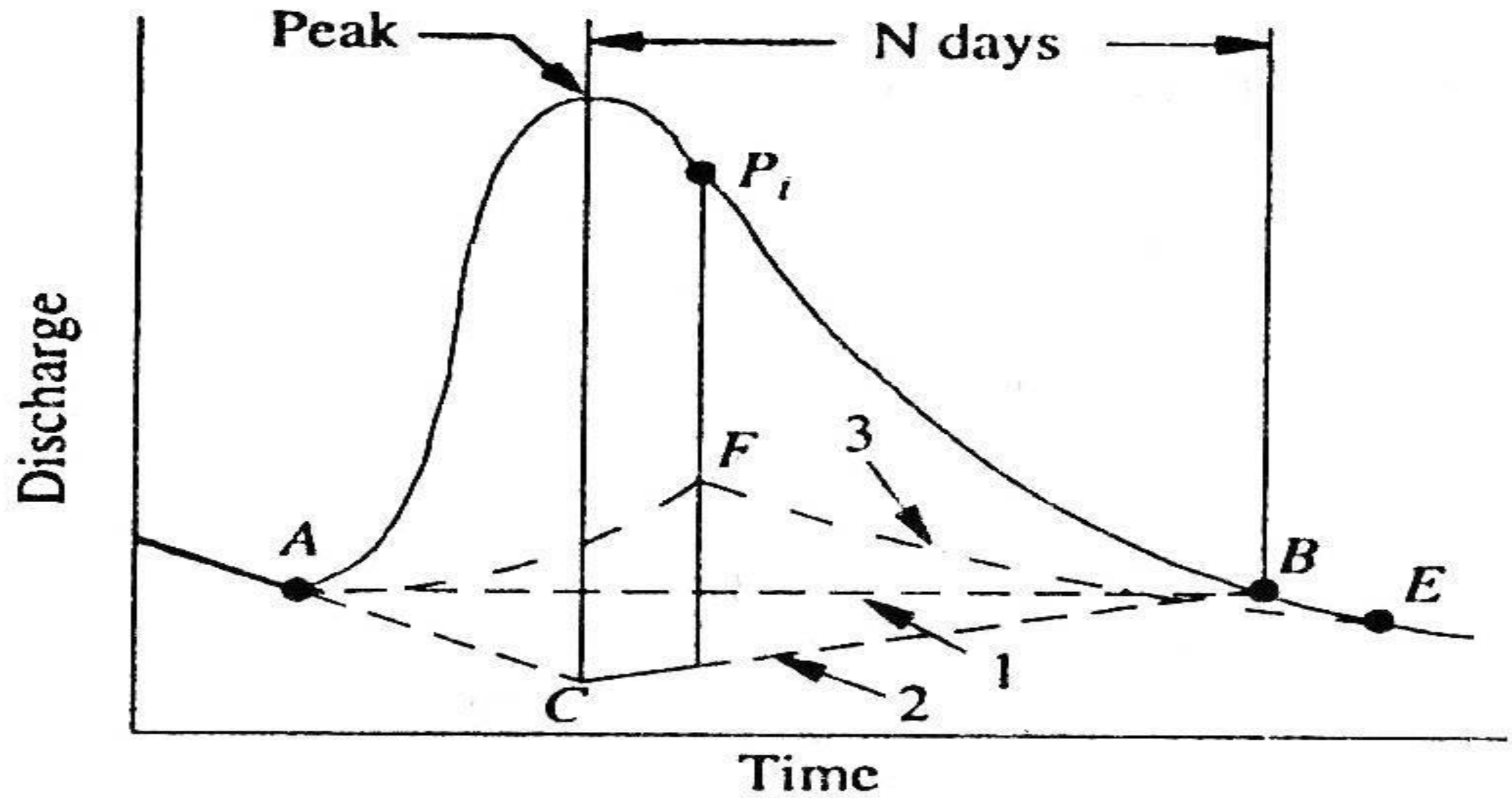


Fig. 6.5 Base flow seperation methods

Method I – Straight-Line Method

In this method the separation of the base flow is achieved by joining with a straight line the beginning of the surface runoff to a point on the recession limb representing the end of the direct runoff.

$$N = 0.83\, A^{0.2}$$

Where **A** = drainage area in Km^2 and **N** is in days

Method II

In this method the base flow curve existing prior to the commencement of the surface runoff is extended till it intersects the ordinate drawn at the pick (point C in Fig.). This point is joined to point B by a straight line.

Method III

In this method the base flow recession curve after the depletion of the flood water is extended backwards till it intersects the ordinate at the point of inflection (line EF in Fig.). Points A and F are joined by an arbitrary smooth curve.

Direct Runoff Hydrograph (DRH)

The surface runoff hydrograph obtained after the base-flow separation is also known as direct runoff hydrograph (DRH).

EXAMPLE 6.2. *Rainfall of magnitude 3.8 cm and 2.8 cm occurring on two consecutive 4-h durations on a catchment of area 27 km^2 produced the following hydrograph of flow at the outlet of the catchment. Estimate the rainfall excess and ϕ index.*

Time from start of rainfall (h)	6	0	6	12	18	24	30	36	42	48	54	60	66
Observed flow (m^3/s)	6	5	13	26	21	16	12	9	7	5	5	4.5	4.5

SOLUTION: The hydrograph is plotted to scale (Fig. 6.7). It is seen that the storm hydrograph has a base-flow component. For using the simple straight-line method of base-flow separation, by Eq. (6.4)

$$N = 0.83 \times (27)^{0.2} = 1.6 \text{ days} = 38.5 \text{ h}$$

However, by inspection, DRH starts at $t = 0$, has the peak at at $t = 12$ h and ends at $t = 48$ h (which gives a value of $N = 48 - 12 = 36$ h). As $N = 36$ h appears to be more satisfactory than $N = 38.5$ h, in the present case DRH is assumed to exist from $t = 0$ to 48 h. A straight line base flow separation gives a constant value of 5 m^3/s for the base flow.

$$\text{Area of DRH} = (6 \times 60 \times 60)\left[\frac{1}{2}(8) + \frac{1}{2}(8+21) + \frac{1}{2}(21+16) + \frac{1}{2}(16+11) + \frac{1}{2}(11+7)\right.$$
$$\left. + \frac{1}{2}(7+4) + \frac{1}{2}(4+2) + \frac{1}{2}(2)\right]$$
$$= 3600 \times 6 \times (8 + 21 + 16 + 11 + 7 + 4 + 2)$$
$$= 1.4904 \times 10^6 \text{ m}^3$$
$$= \text{total direct runoff due to storm}$$

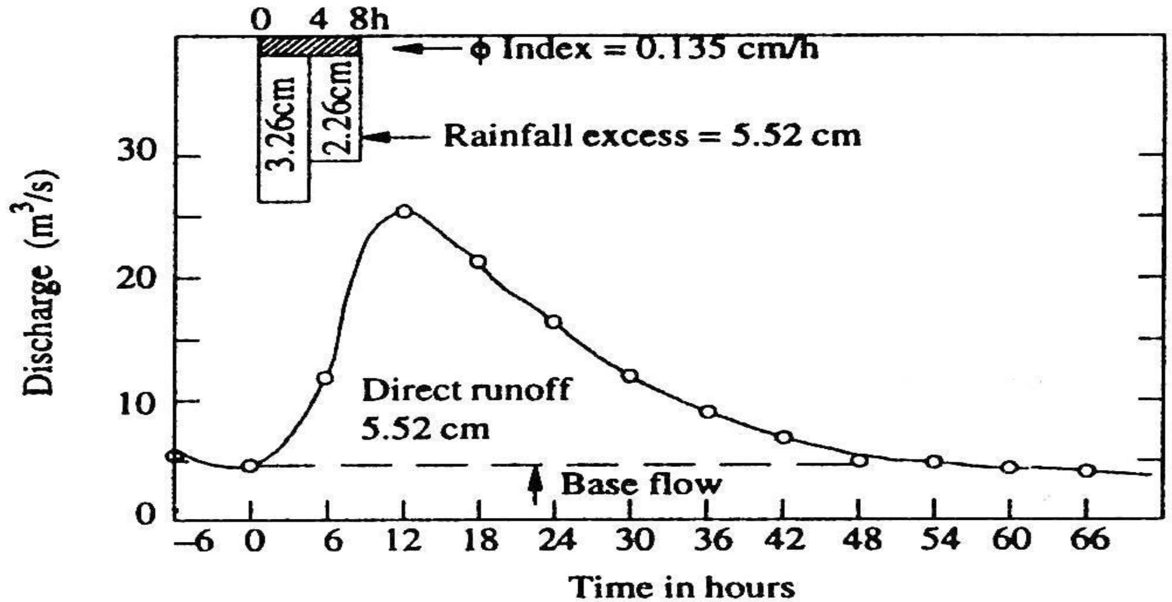


Fig. 6.7 Base flow seperation—Example 6.2

$$\text{Runoff depth} = \frac{\text{runoff volume}}{\text{catchment area}} = \frac{1.4904 \times 10^6}{27 \times 10^6}$$

$$= 0.0552 \text{ m}$$

$$= 5.52 \text{ cm} = \text{rainfall excess}$$

$$\text{Total rainfall} = 3.8 + 2.8 = 6.6 \text{ cm}$$

$$\text{Duration} = 8 \text{ h}$$

$$\phi \text{ index} = \frac{6.6 - 5.52}{8} = 0.135 \text{ cm/h}$$

Unit Hydrograph

A unit hydrograph is defined as the hydrograph of direct runoff resulting from one unit depth (1cm) of rainfall excess occurring uniformly over the basin and at a uniform rate for a specified duration (D hours).

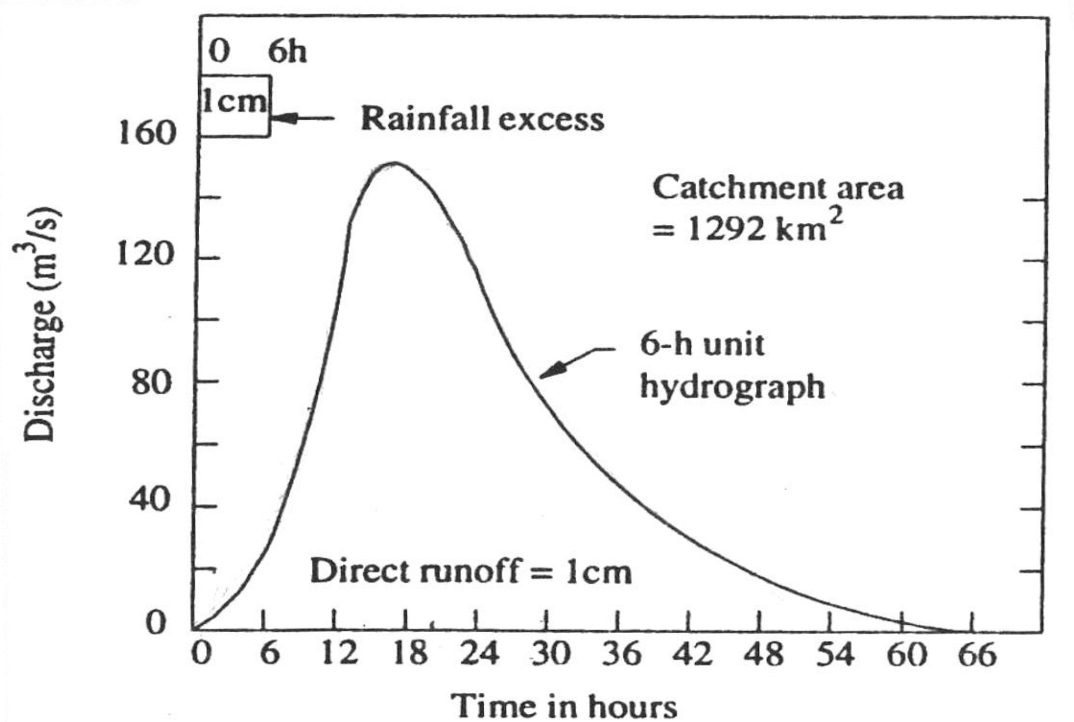


Fig. 6.9 Typical 6–h unit hydrograph

Two basic assumptions of unit hydrograph

(i) the time invariance

(ii) the linear response

EXAMPLE 6.4 *Given below are the ordinates of a 6-h unit hydrograph for a catchment. Calculate the ordinates of the DRH due to a rainfall excess of 3.5 cm occurring in 6 hr.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48	54	60	69
UH ordinate (m^3/s)	0	25	50	85	125	160	185	160	110	60	36	25	16	8	0

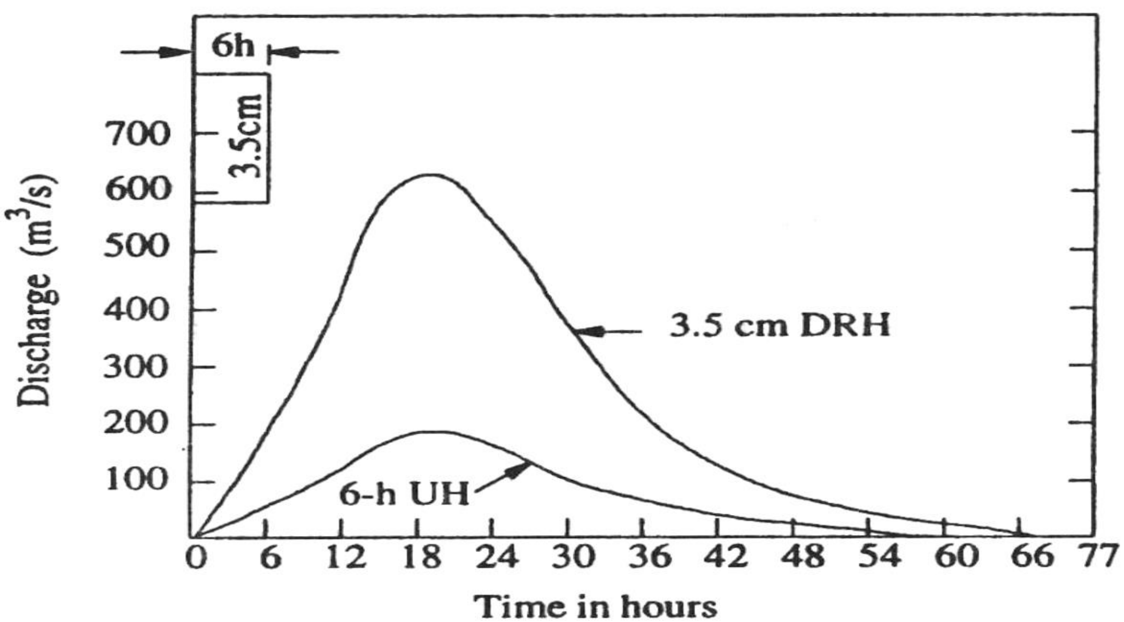


Fig. 6.10 (a) 3.5 cm DRH derived from 6–h unit hydrograph—Example 6.4

TABLE 6.3 CALCULATION OF DRH DUE TO 3.5 cm ER—*EXAMPLE 6.4*

Time (h)	Ordinate of 6-h unit hydrograph (m^3/s)	Ordinate of 3.5 cm DRH (m^3/s)
1	2	3
0	0	0
3	25	87.5
6	50	175.0
9	85	297.5
12	125	437.5
15	160	560.0
18	185	647.5
24	160	560.0
30	110	385.0
36	60	210.0
42	36	126.0
48	25	87.5
54	16	56.0
60	8	28.0
69	0	0

EXAMPLE 6.5 *Two storms each of 6-h duration and having rainfall excess values of 3.0 and 2.0 cm respectively occur successively. The 2-cm ER rain follows the 3-cm rain. The 6-h unit hydrograph for the catchment is the same as given in Example 6.4. Calculate the resulting DRH.*

TABLE 6.4 CALCULATION OF DRH BY METHOD OF SUPERPOSITION — *EXAMPLE 6.5*

Time (h)	Ordinate of 6-h UH (m^3/s)	Ordinate of 3-cm DRH (col.2) × 3	Ordinate of 2-cm DRH (col.2 lagged by 6 h) × 2	Ordinate of 5-cm DRH (col. 3 + col.4) (m^3/s)	Remarks
1	2	3	4	5	6
0	0	0	0	0	
3	25	75	0	75	
6	50	150	0	150	
9	85	255	50	305	
12	125	375	100	475	
15	160	480	170	650	
18	185	555	250	805	
(21)	(172.5)	(517.5)	(320)	(837.5)	Interpolated value
24	160	480	370	850	
30	110	330	320	650	
36	60	180	220	400	
42	36	108	120	228	
48	25	75	72	147	
54	16	48	50	98	
60	8	24	32	56	
(66)	(2.7)	(8.1)	(16)	(24.1)	Interpolated value
69	0	0	(10.6)	(10.6)	Interpolated value
75	0	0	0	0	

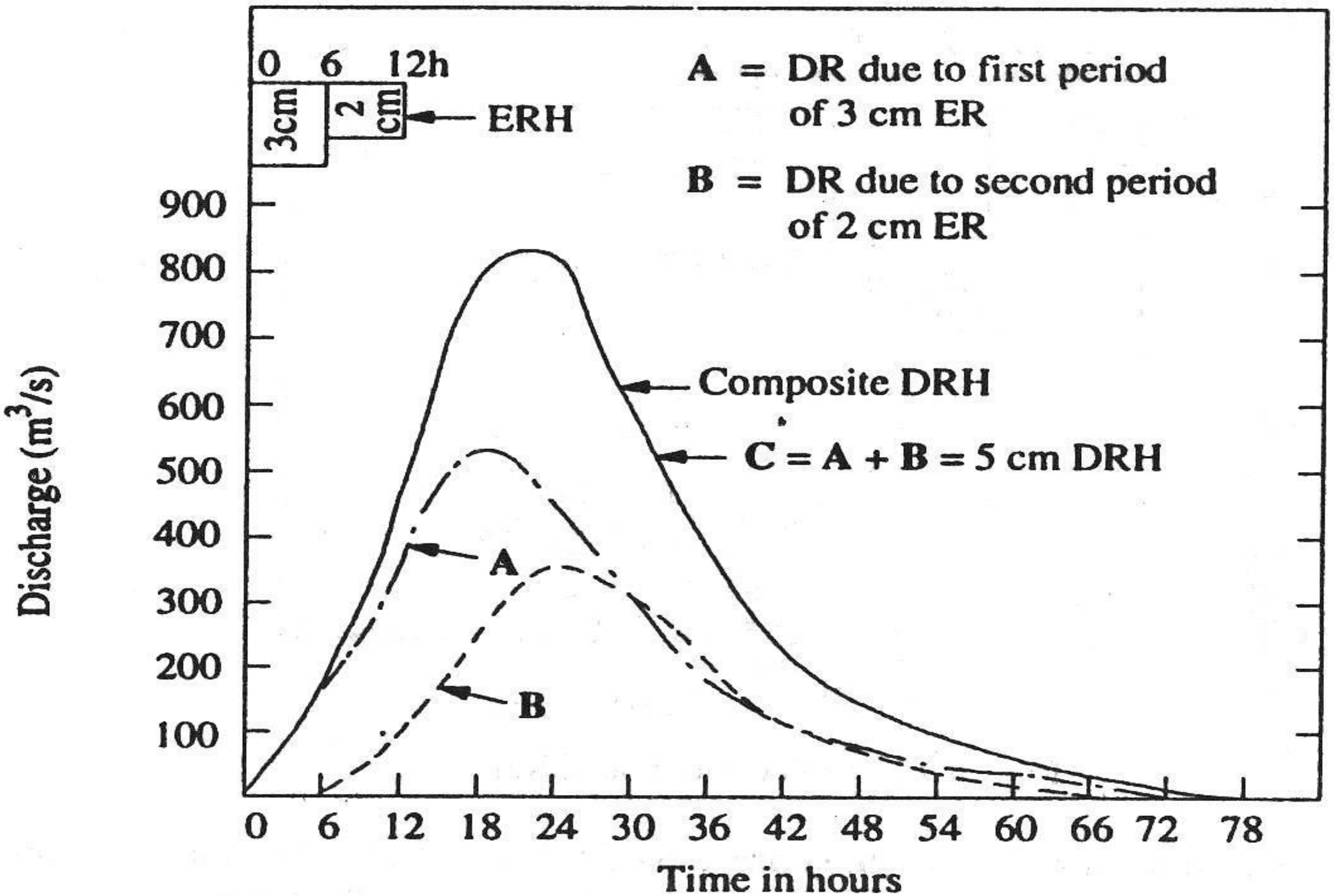


Fig. 6.10 (b) Principle of superposition—Example 6.5

Application of unit hydrograph

using the basic principles of the unit hydrograph, one can easily calculate the DRH in a catchment due to a given storm if an appropriate unit hydrograph was available

EXAMPLE 6.6 *The ordinates of a 6- hour unit hydrograph of a catchment is given below.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48	54	60	69
Ordinate of 6-h UH	0	25	50	85	125	160	185	160	110	60	36	25	16	8	0

Derive the flood hydrograph in the catchment due to the storm given below:

Time from start of storm (h)	0	6	12	18
Accumulated rainfall (cm)	0	3.5	11.0	16.5

The storm loss rate (ϕ – index) for the catchment is estimated as 0.25 cm/h. The base flow can be assumed to be 15 m^3/s *at the beginning and increasing by 2.0* m^3/s *for every 12 hours till the end of the direct–runoff hydrograph.*

at a given time interval added. The base flow is then added to obtain the flood hydrograph shown in Col. 8, Table 6.6.

Interval	1st 6 hours	2nd 6 hours	3rd 6 hours
Rainfall depth (cm)	3.5	(11.0-3.5) = 7.5	(16.5-11.0) = 5.5
Loss @ 0.25 cm/h for 6 h	1.5	1.5	1.5
Effective rainfall (cm)	2.0	6.0	4.0

TABLE 6.5 CALCULATION OF FLOOD HYDROGRAPH DUE TO A KNOWN ERH—*EXAMPLE 6.6*

Time	Ordinaters of U.H.	DRH due to 2 cm ER Col. 2 × 2.0	DRH due to 2 cm ER × 6.0 (Advanced by 6 h)	DRH due to 4 cm ER Col.2 × 4.0 (Advanced by 12 h)	Ordinates of final DRH (Col 3+ 4+5)	Base flow (m^3/s)	Ordinates of flood hydrograph (m^3/s) (Col. 6 + 7)
1	2	3	4	5	6	7	8
0	0	0	0	0	0	15	15
3	25	50	0	0	50	15	65
6	50	100	0	0	100	15	115
9	85	170	150	0	320	15	335
12	125	250	300	0	550	17	567
15	160	320	510	100	930	17	947
18	185	370	750	200	1320	17	1337
(21)	(172.5)	(345)	960	340	1645	(17)	1662
24	160	320	1110	500	1930	19	1949
(27)	(135)	(270)	(1035)	640	1945	19	1964
30	110	220	960	740	1920	19	1939
36	60	120	660	640	1420	21	1441
42	36	72	360	440	872	21	893
48	25	50	216	240	506	23	529
54	16	32	150	144	326	23	349
60	8	16	96	100	212	25	237
66	(2.7)	(5.4)	48	64	117	25	142
69	0	0	—	—	—	—	—
72		0	16	32	48	27	75
75		0	0	—	—	—	—
78		0	0	(10.8)	(11)	27	49
81				0	0	27	27
84						27	27

Derivation of Unit Hydrograph

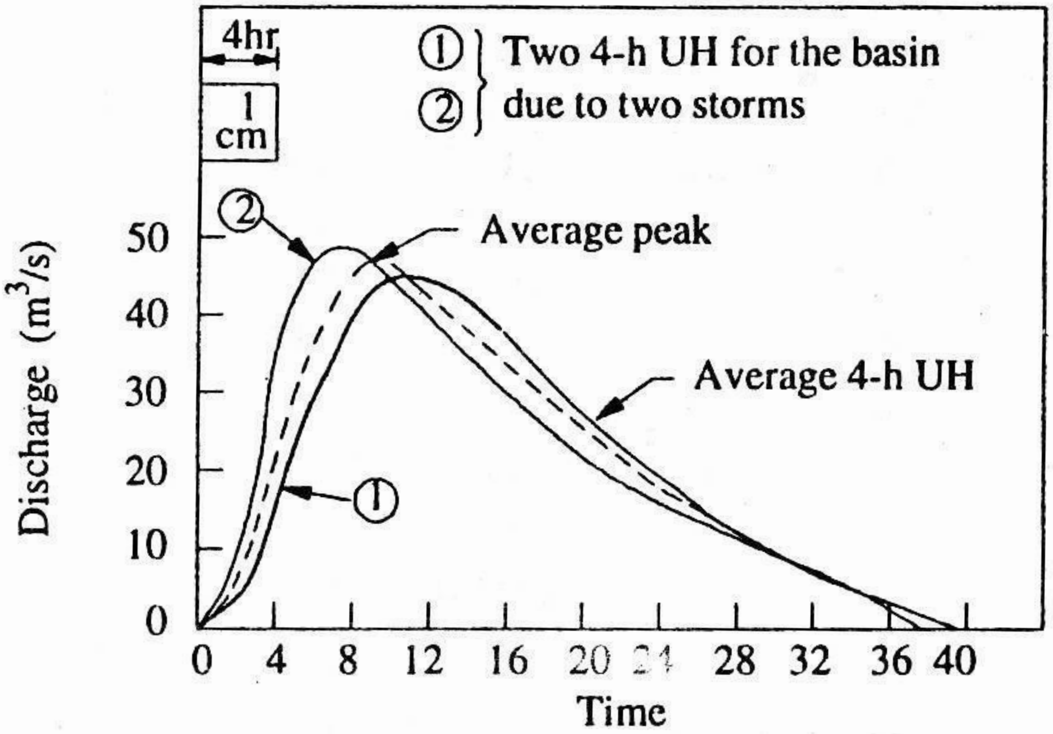


Fig. 6.12 Derivation of an average unit hydrograph

EXAMPLE 6.7 *Following are the ordinates of a storm hydrograph of a river draining a catchment area of 423 km^2 due to a 6-h isolated storm. Derive the ordinates of a 6-h unit hydrograph for the catchment.*

Time from start of storm (h)	—6	0	6	12	18	24	30	36	42	48
Discharge (m^3/s)	10	10	30	87.5	115.5	102.5	85.0	71.0	59.0	47.5
Time from start of storm (h)	54	60	66	72	78	84	90	96	102	
Discharge (m^3/s)	39.0	31.5	26.0	21.5	17.5	15.0	12.5	12.0	12.0	

SOLUTION: The storm hydrograph is plotted to scale (Fig. 6.13). Denoting the time from beginning of storm as t, by inspection of Fig. 6.12,

$$A = \text{beginning of DRH} \qquad t = 0$$

$$B = \text{end of DRH} \qquad t = 90 \text{ h}$$

$$P_m = \text{peak} \qquad t = 20 \text{ h}$$

Hence

$$N = (90 - 20) = 70 \text{ h} = 2.91 \text{ days}$$

By Eq. (6.4),

$$N = 0.83\ (423)^{0.2} = 2.78 \text{ days},$$

However, N = 2.91 days is adopted for convenience. A straight line joining A and B is taken as the divide line for base-flow separation. The ordinates of DRH are obtained by subtracting the base flow from the ordinates of the storm hydrograph. The calculations are shown in Table 6.6

$$\text{Volume of DRH} = 60 \times 60 \times 6 \times (\text{sum of DRH ordinates})$$

$$= 60 \times 60 \times 6 \times 587 = 12.68 \text{ Mm}^3$$

$$\text{Drainage area} = 423 \text{ km}^2 = 423 \text{ Mm}^2$$

$$\text{Runoff depth} = \text{ER depth} = \frac{12.68}{423} = 0.03 \text{ m} = 3 \text{ cm}.$$

The ordinates of DRH (col. 4) are divided by 3 to obtain the ordinates of the 6-h unit hydrograph,, see Table 6.6

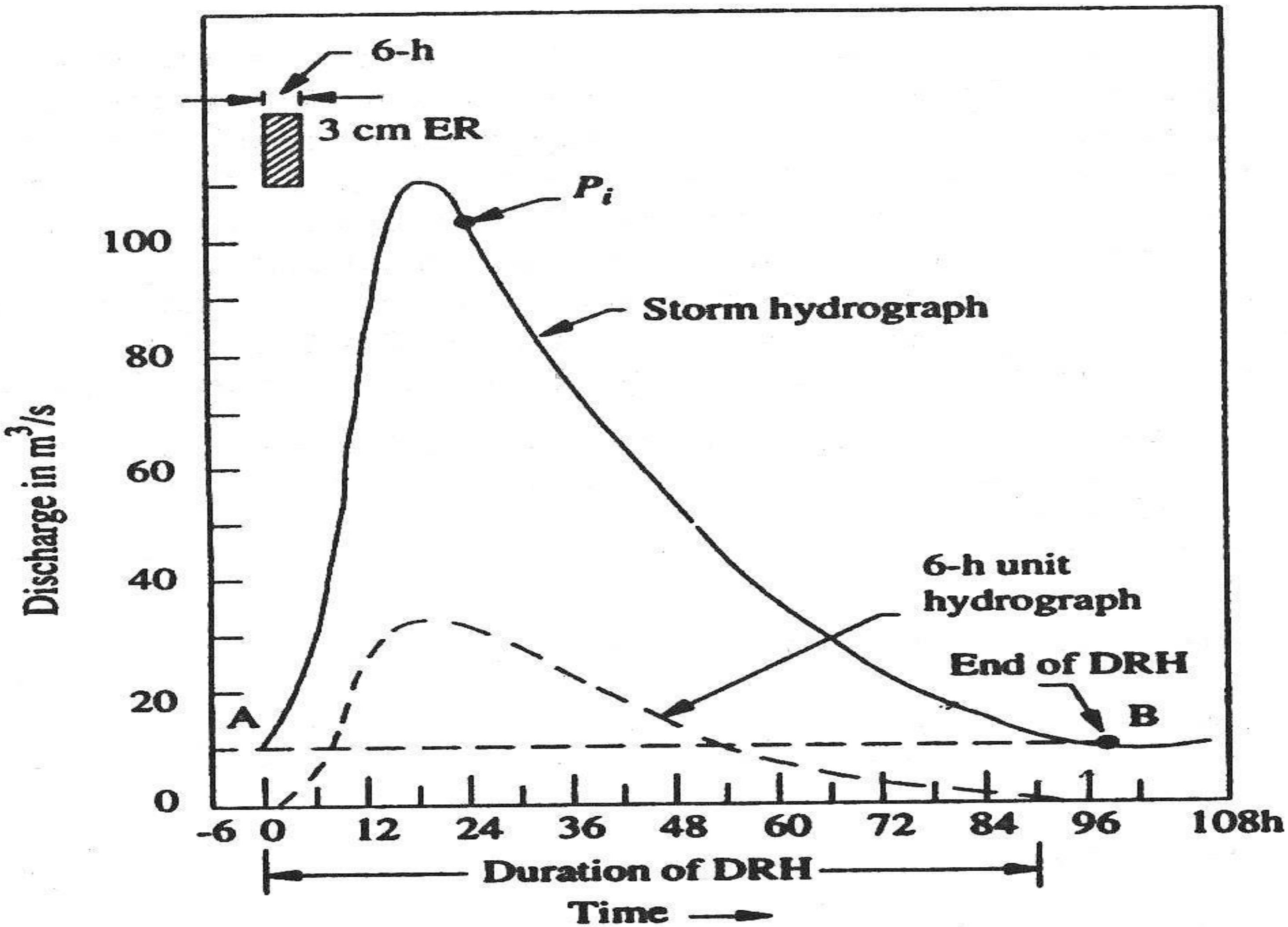


Fig. 6.13 Derivation of unit hydrograph from a storm hydrograph

TABLE 6.6 CALCULATION OF THE ORDINATES OF A 6–h UNIT HYDROGRAPH—*EXAMPLE 6.7*

Time from beginning of storm (h)	Ordinate of storm hydrograph (m^3/s)	Base flow (m^3/s)	Ordinate of DRH (m^3/s)	Ordinate of 6-h unit hydrograph (Col. 4 ÷ 3)
1	2	3	4	5
–6	10.0	10.0	0	0
0	10.0	10.0	0	0
6	30.0	10.0	20.0	6.7
12	87.5	10.5	77.0	25.7
18	111.5	10.5	101.0	33.7
24	102.5	10.5	101.0	33.7
30	85.0	11.0	74.0	24.7
36	71.0	11.0	60.0	20.0
42	59.0	11.0	48.0	16.0
48	47.5	11.5	36.0	12.0
54	39.0	11.5	27.5	9.2

TABLE 6.6 (Continued)

1	2	3	4	5
60	31.5	11.5	20.0	6.6
66	26.0	12.0	14.0	4.6
72	21.5	12.0	9.5	3.2
78	17.5	12.0	5.5	1.8
84	15.0	12.5	2.5	0.8
90	12.5	12.5	0	0
96	12.0	12.0	0	0
102	12.0	12.0	0	0

Unit hydrograph of Different Durations

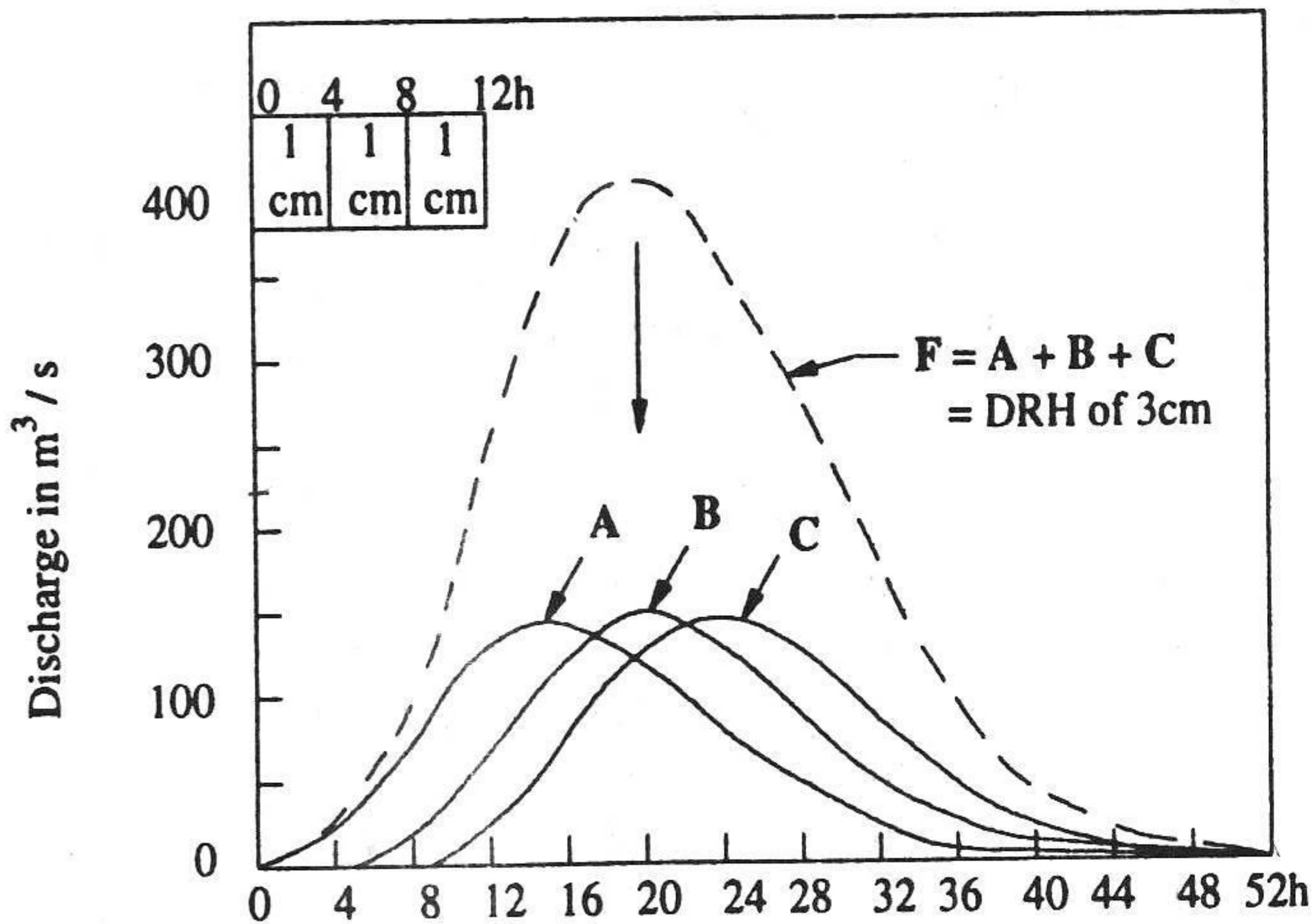
Two methods are available for this purpose

(i) Method of superposition, and

(ii) the S – curve

(i) Method of Superposition

If a D – h unit hydrograph is available, and it is desired to develop a unit hydrograph of nD h, where n is an integer.



0 4 8 12h
1 cm
1 cm
1 cm
F = A + B + C
= DRH of 3cm
A
B
C
Discharge in m³ / s
0
100
200
300
400
0 4 8 12 16 20 24 28 32 36 40 44 48 52h

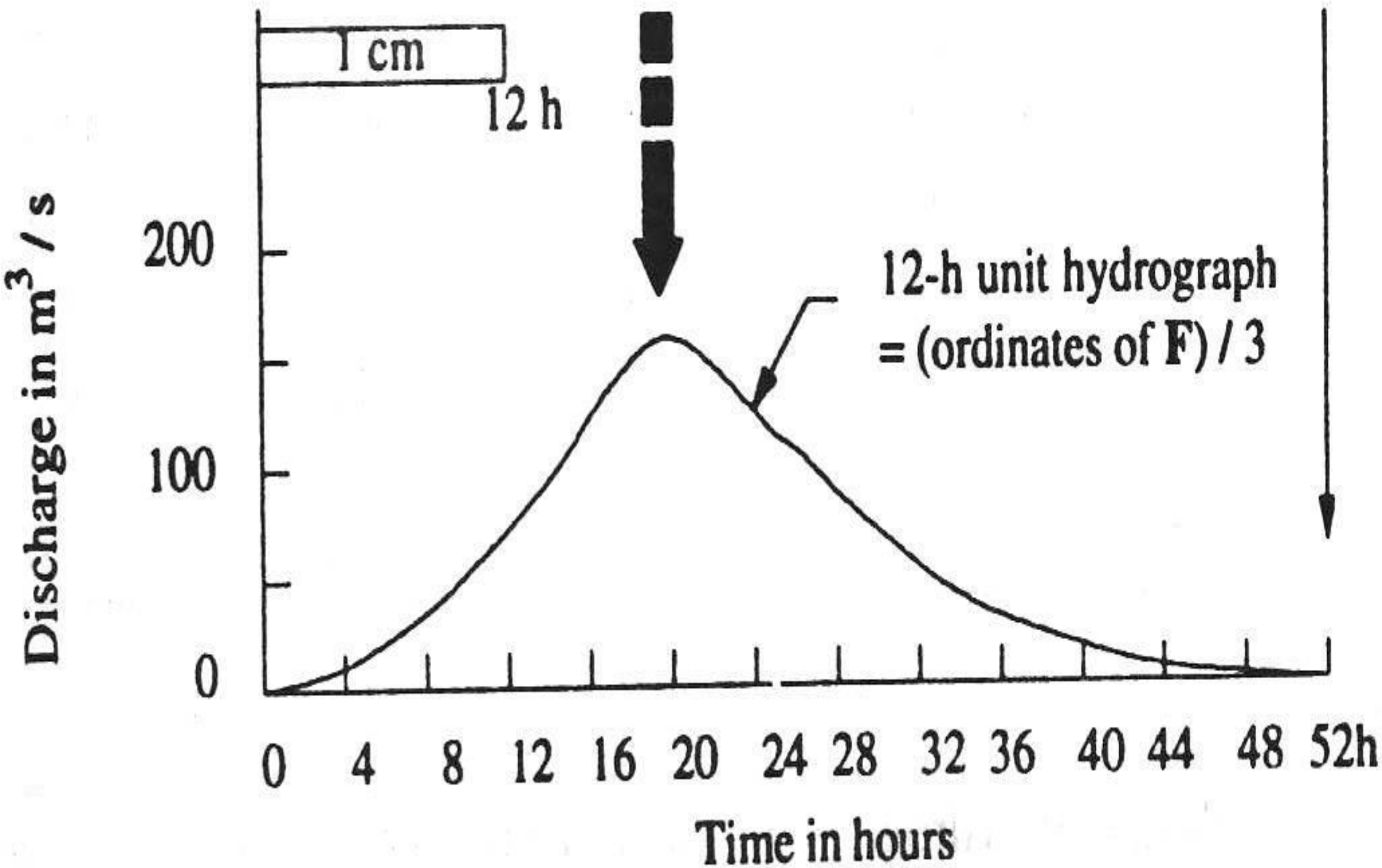


Fig. 6.15 Construction of a 12–h unit hydrograph from a 4–h unit hydrograph

EXAMPLE 6.9 *Given the ordinates of a 4-h unit hydrograph as below derive the ordinates of a 12-h unit hydrograph for the same catchment.*

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate of 4–h UH	0	20	80	130	150	130	90	52	27	15	5	0

SOLUTION: The calculations are performed in a tabular form in Table 6.7. In this

Column 3 = ordinates of 4-h UH lagged by 4 h

Column 4 = ordinates of 4-h UH lagged by 8 h

Column 5 = ordinates of DRH representing 3 cm ER in 12 h

Column 6 = ordinates of 12-h UH = (Column 5)/3

The 12-h unit hydrograph is shown in Fig. 6.15.

TABLE 6.7 CALCULATION OF A 12-h UNIT HYDROGRAPH FROM A 4-h UNIT HYDROGRAPH—*EXAMPLE 6.9*

Time (h)	Ordinates of 4-h UH (m^3/s) A	B Lagged by 4 h	C Lagged by 8 h	DRH of 3 cm in 12-h (m^3/s) (Col.2+3+4)	Ordinate of 12-h UH (m^3/s) (Col. 5)/3
1	2	3	4	5	6
0	0	—	—	0	0
4	20	0	—	20	6.7
8	80	20	0	100	33.3
12	130	80	20	230	76.7
16	150	130	80	360	120.0
20	130	150	130	410	136.7
24	90	130	150	370	123.3
28	52	90	130	272	90.7
32	27	52	90	169	56.3
36	15	27	52	94	31.3
40	5	15	27	47	15.7
44	0	5	15	20	6.7
48		0	5	5	1.7
52			0	0	0

(ii) The S – Curve

If it is desired to develop a unit hydrograph of duration mD, where m is a fraction, the method of superposition cannot be used.

Ordinates (S_A – S_B) = T h DRH of (1/D x T) cm ER

UH – T h = (ordinates of DRH)/ (1/D x T)

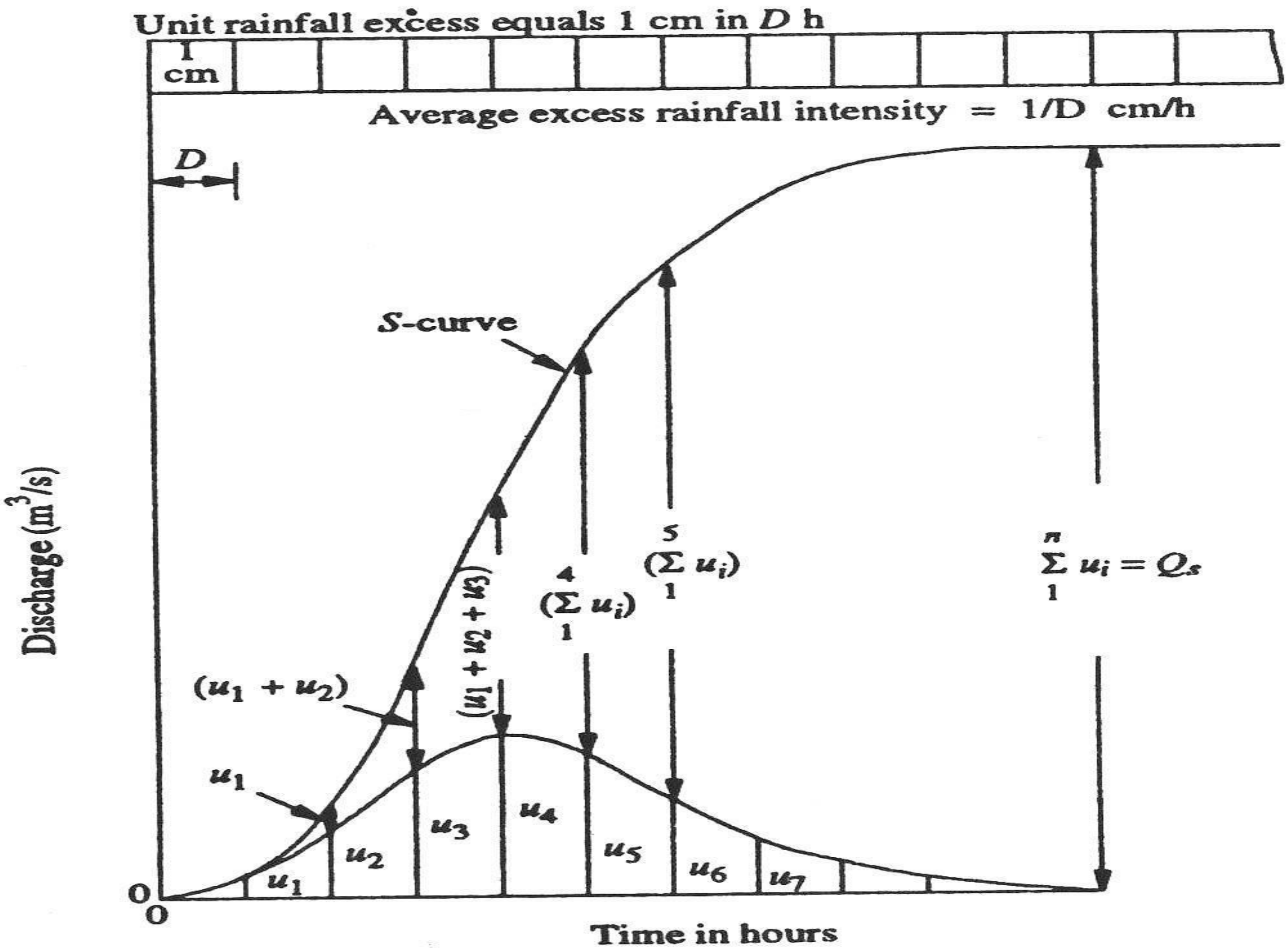


Fig. 6.16(a) S-curve

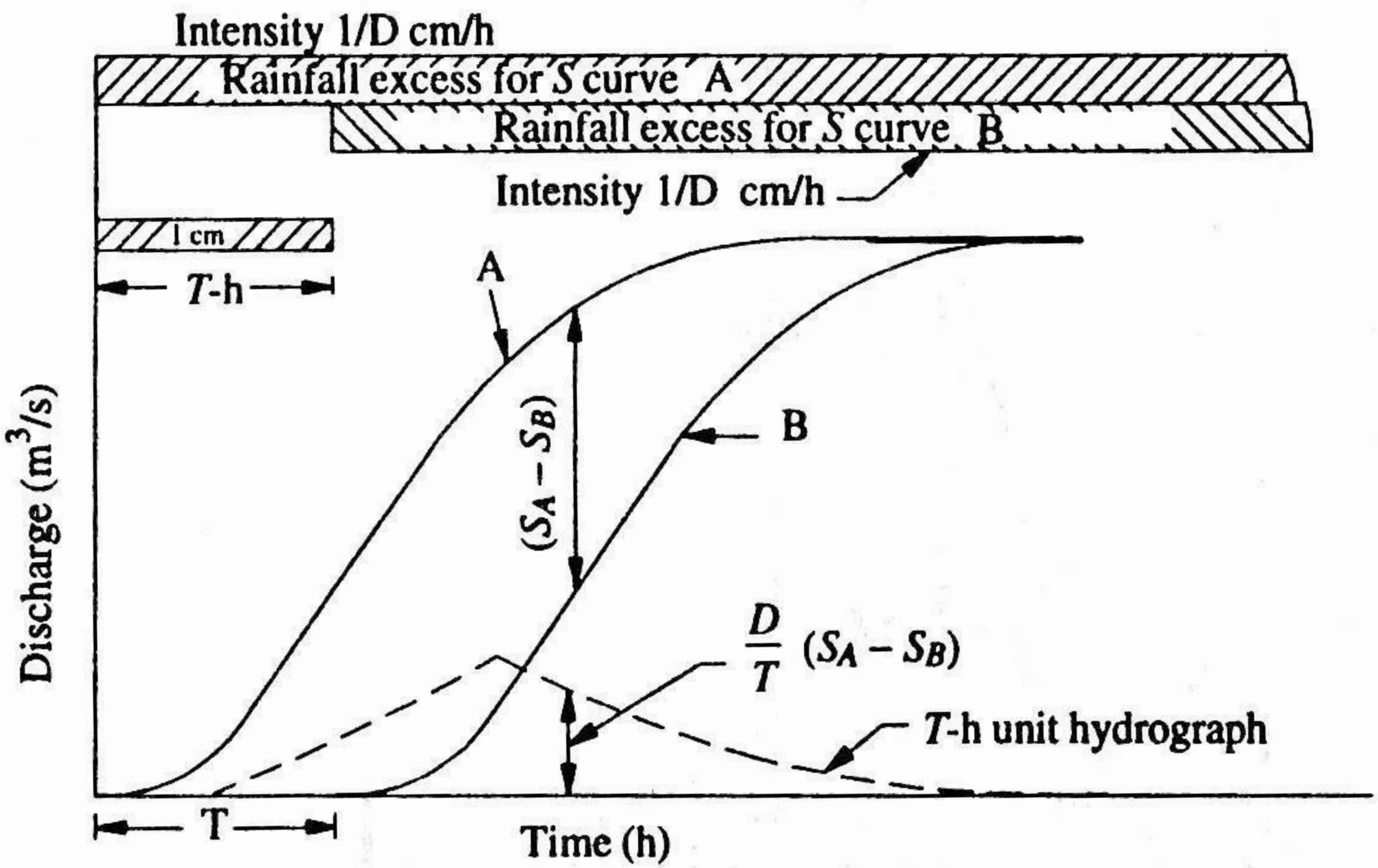


Fig. 6.16 (b) Derivation of a *T*-h unit hydrograph by *S*-curve lagging method

Example 6.10 – Solve example 6.9 by the S – curve method.

Solution:

TABLE 6.8 DETERMINATION OF A 12-h UNIT HYDROGRAPH BY *S*-CURVE METHOD—*EXAMPLE 6.10*

Time (h)	Ordinate of 4–h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (m^3/s) (Col.2 + Col. 3)	S-curve lagged by 12 h (m^3/s)	(Col. 4– Col. 5)	$\frac{\text{Col. 6}}{12/4}$ = 12–h UH ordinates (m^3/s)
1	2	3	4	5	6	7
0	0	—	0	—	0	0
4	20	0	20	—	20	6.7
8	80	20	100	—	100	33.3
12	130	100	230	0	230	76.7
16	150	230	380	20	360	120.0
20	130	380	510	100	410	136.7
24	90	510	600	230	370	123.3
28	52	600	652	380	272	90.7
32	27	652	679	510	169	56.3
36	15	679	694	600	94	31.3
40	5	694	699	652	47	15.7
44	0	699	699	679	20	6.7
48		699	699	694	5	1.7
52			699	699	0	0

EXAMPLE 6.11 *Ordinates of a 4-h unit hydrograph are given. Using this derive the ordinates of a 2-h unit hydrograph for the same catchment.*

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate or 4-h UH (m^3/s)	0	20	80	130	150	130	90	52	27	15	5	0

Solution:

TABLE 6.9 DETERMINATION OF 2-h UNIT HYDROGRAPH FROM A 4-h UNIT HYDROGRAPH—*EXAMPLE 6.11*

Time (h)	Ordinate of 4-h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (Col. (2) + (3)) (m^3/s)	S-curve lagged by 2 h	(Col. (4) – Col. (5))	2–h UH ordinates Col. (6)/(2/4) (m^3/s)
1	2	3	4	5	6	7
0	0	—	0	—	0	0
2	8	—	8	0	8	16
4	20	0	20	8	12	24
6	43	8	51	20	31	62
8	80	20	100	51	49	98
10	110	51	161	100	61	122
12	130	100	230	161	69	138
14	146	161	307	230	77	154
16	150	230	380	307	73	146
18	142	307	449	380	69	138

20	130	380	510	449	61	122
22	112	449	561	510	51	102
24	90	510	600	561	39	78
26	70	561	631	600	31	62
28	52	600	652	631	21	42
30	38	631	669	652	17	34
32	27	652	679	669	10	20
34	20	669	689	679	10	(20)15
36	15	679	694	689	5	(10)10
38	10	689	699	694	5	(10)6
40	5	694	699	699	(0)	(0)3
42	2	699	701	699	(2)	(4)0
44	0	699	699	701	(-2)	(-4)0

Synthetic Unit Hydrograph

Unit hydrographs derived from such relationships are known as synthetic unit hydrograph.

Snyder’s Method

The first of the Snyder’s equation relates the basin lag tp, defined as the time interval from the mid-point of the unit rainfall excess to the peak of the unit hydrograph (Fig. 6.18), to the basin characteristics as

$$t_p = C_t\ (L\ L_{ca})^{0.3}$$

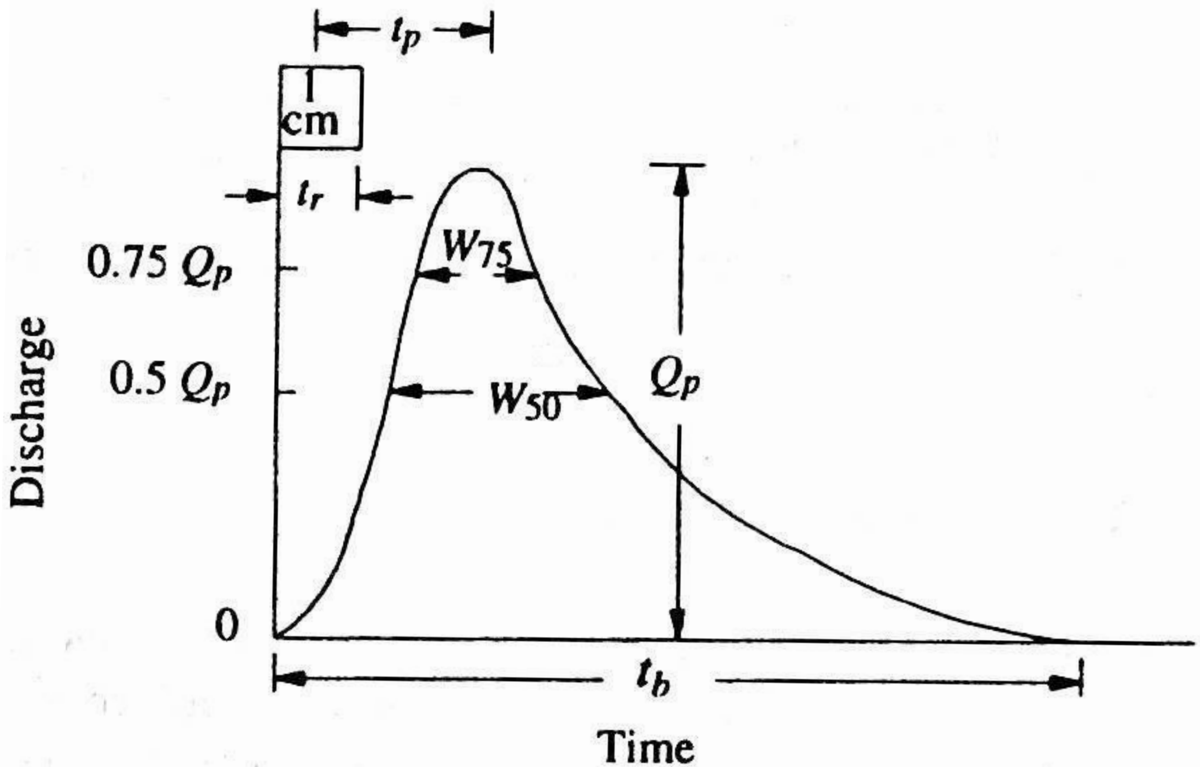


Fig. 6.18 Elements of a synthetic unit hydrograph

Where t_p is in hour

L = basin length measured along the water course from the basin divide to the gauging station in km

L_{ca} = distance along the main water course from the gauging station to a point opposite the watershed centroid in km

C_t = a regional constant representing watershed slope and storage

The value of C_t in Synder's study ranged from 1.35 to 1.65. However, studies by many investigators have shown that C_t depends upon the region under study and wide variations with the value of C_t ranging from 0.3 to 6.0 have been reported.

Snyder adopted a standard duration t_r hours of effective rainfall given by

$$t_r = t_p/5.5$$

the peak discharge Q_{ps} (m^3/s) of a unit hydrograph of standard duration t_r h is given by snyder as

$$Q_{ps} = (2.78\ C_p\ A) / t_p$$

Where A = catchment area in Km^2 and

C_p = a regional constant

If a non – standard rainfall duration tr h is adopted

$$t'_p = t_p + (t_R - t_r) / 4$$

$$= 21/22\ t_p + t_R / 4$$

Where t'_p = basin lag in hours for an effective duration of t_R h and t_p

peak discharge for a nonstandard ER of duration t_R is in m^3/s

$$Q_p = (2.78\ C_p\ A) / t'_p$$

Note that when $t_R = t_r$

$$Q_p = Q_{ps}$$

The time basin of a unit hydrograph given by snyder as

$$t_b = 3 + t'_p / 8 \text{ days}$$

$$= (72 + 3\, t'_p) \text{ hours}$$

where t_b = time basin

Taylor and Schwartz recommended

$$t_b = 5\,(t'_p + t_R/2)$$

$$W_{50} = 5.87 / q^{1.08}$$

$$W_{75} = W_{50} / 1.75$$

where W_{50} = width of unit hydrograph in h at 50% peak discharge

W_{75} = width of unit hydrograph in h at 75% peak discharge

$q = Q_p/A$ = peak discharge per unit catchment area in $m^3/s/Km^2$

EXAMPLE 6.13 *Two catchments A and B are considered meteorologically similar. Their catchment characteristics are given below.*

Catchment A	Catchment B
$L = 30$ km	$L = 45$ km
$L_{ca} = 15$ km	$L_{ca} = 25$ km
$A = 250$ km^2	$A = 400$ km^2

For catchment A, a 2-h unit hydrograph was developed and was found to have a peak discharge of 50 m^3/s. The time to peak from the beginning of the rainfall excess in this unit hydrograph was 9.0 h. Using Snyder's method, develop a unit hydrograph for catchment B.

SOLUTION: *For Catchment A:*

$$t_R = 2.0 \text{ h}$$

Time to peak from beginning of ER $$= \frac{t_R}{2} + t'_p = 9.0 \text{ h}$$

$$\therefore \quad t'_p = 8.0 \text{ h}$$

From Eq. (6.12),

$$t'_p = \frac{21}{22} t_p + \frac{t_R}{4}$$

$$= \frac{21}{22} t_p + 0.5 = 8.0$$

$$t_p = \frac{7.5 \times 22}{21} = 7.857 \text{ h}$$

from Eq. (6.8),

$$t_p = C_t (L\,L_{ca})^{0.3}$$

$$7.857 = C_t (30 \times 15)^{0.3}$$

$$C_t = 1.257$$

From Eq. (6.11a),

$$Q_p = 2.78\, C_p\, A/t'_p$$

$$50 = 2.78 \times C_p \times 250/8.0$$

$$C_p = 0.576$$

For Catchment B: Using the values of $C_t = 1.257$ and $C_p = 0.576$ in catchment B, the parameters of the synthetic-unit hydrograph for catchment B are determined.
From Eq. (6.8),

$$t_p = 1.257\,(45 \times 25)^{0.3} = 10.34 \text{ h}$$

By Eq. (6.10),

$$t_r = \frac{10.34}{5.5} = 1.88 \text{ h}$$

Using $t_R = 2.0$ h, i.e. for a 2-h unit hydrograph, by Eq. (6.12),

$$t'_p = 10.34 \times \frac{21}{22} + \frac{2.0}{4} = 10.37 \text{ h}$$

By Eq. (6.11a),

$$Q_p = \frac{2.78 \times 0.576 \times 400}{10.37}$$

$$= 61.77 \text{m}^3/\text{s}, \text{ say } 62 \text{ m}^3/\text{s}$$

From Eq. (6.15),

$$W_{50} = \frac{5.87}{(62/400)^{1.08}} = 44 \text{ h}$$

By Eq. (6.16),

$$W_{75} = \frac{44}{1.75} = 25 \text{ h}$$

Time base : From Eq. (6.13), $t_b = 72 + (3 \times 10.37) = 103$ h

From Eq. (6.14), $t_b = 5\,(10.37 + 1\ ) \approx 58$ h

Considering the values of W_{50} and W_{75} and noting that the area of catchment B is rather samll, $t_b \approx 58$ h is more appropriate in this case.

FLOOD ROUTING

Flood routing is the technique of determining the flood hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream sections. The hydrologic analysis of problems such as flood forecasting, flood protection, reservoir design and spillway design invariably include flood routing. In these applications two broad categories of routing can be recognized. These are:

1. Reservoir routing, and
2. Channel routing

A variety of routing methods are available and they can be broadly classified into two categories as:

1. Hydrologic routing and
2. hydraulic routing

Hydrologic-routing methods employ essentially the equation of continuity. Hydraulic methods, on the other hand, employ the continuity equation together with the equation of motion of unsteady.

BASIC EQUATIONS

The equation of continuity used in all hydrologic routing as the primary equation states that the difference between the inflow and outflow rate is equal to the rate of change of storage, i.e.

$$I - Q = \frac{dS}{dt}$$

where I = inflow rate, Q = outflow rate and S = storage. Alternatively, in a small time interval Δt the difference between the total inflow volume and total outflow volume in a reach is equal to the change in storage in that reach

$$\bar{I}\,\Delta t - \bar{Q}\,\Delta t = \Delta S$$

where $\bar{I}$ = average inflow in time Δt, $\bar{Q}$ = average outflow in time Δt and ΔS = change in storage. By taking $\bar{I} = (I_1 + I_2)/2$, $\bar{Q} = (Q_1 + Q_2)/2$ and $\Delta S = S_2 - S_1$ with suffixes 1 and 2 to denote the beginning and end of time interval Δt Eq.

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t = S_2 - S_1$$

The time interval Δt should be sufficiently short so that the inflow and outflow hydrographs can be assumed to be straight lines in that time interval.

How to Choose Δt ?

- Should as small as possible as that variation of flow can be assumed linear.
- Should not be greater than flood period.
- Normally 20 to 40% of this to read peak.

  a) $0.2\ tp = \Delta t$

  b) $0.4\ tp = \Delta t$

HYDROLOGIC STORAGE ROUINGT (Level pool Routing)

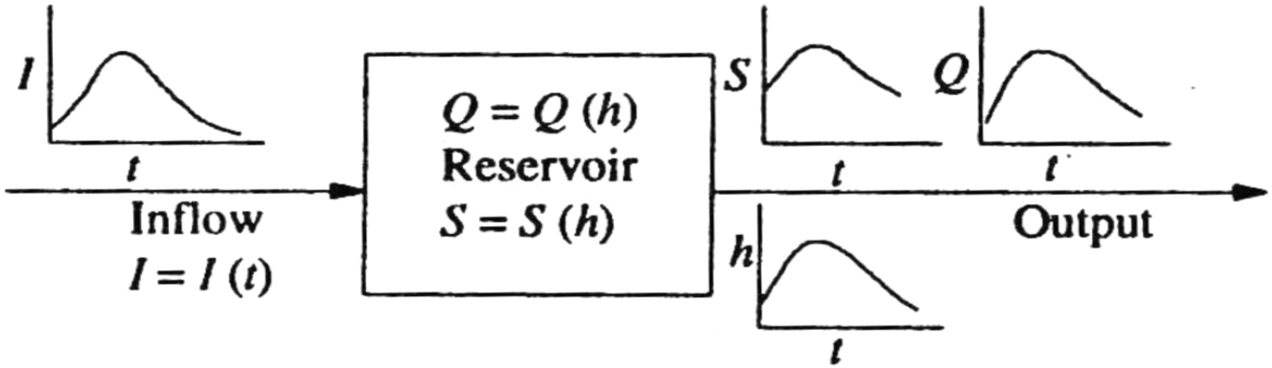


Fig. 8.1 Storage routing (Schematic)

For reservoir routing, the following data have to be known:

1. Storage volume *vs* elevation for the reservoir;
2. Water-surface elevation *vs* outflow and hence storage *vs* outflow discharge;
3. Inflow hydrograph, $I = I(t)$; and
4. Initial values of S, I and Q at time $t = 0$.

As the horizontal water surface is assumed in the reservoir, the storage routing is also known as Level Pool Routing.

Modified Pul's Method

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t = S_2 - S_1$$

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1}{2}\right)\Delta t - \left(\frac{Q_2}{2}\right)\Delta t = S_2 - S_1$$

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t + S_1 - \left(\frac{Q_1}{2}\right)\Delta t = S_2 + \left(\frac{Q_2}{2}\right)\Delta t$$

Here Δt is any chosen interval, approximately 20 to 40% of the time of rise of the inflow hydrograph.

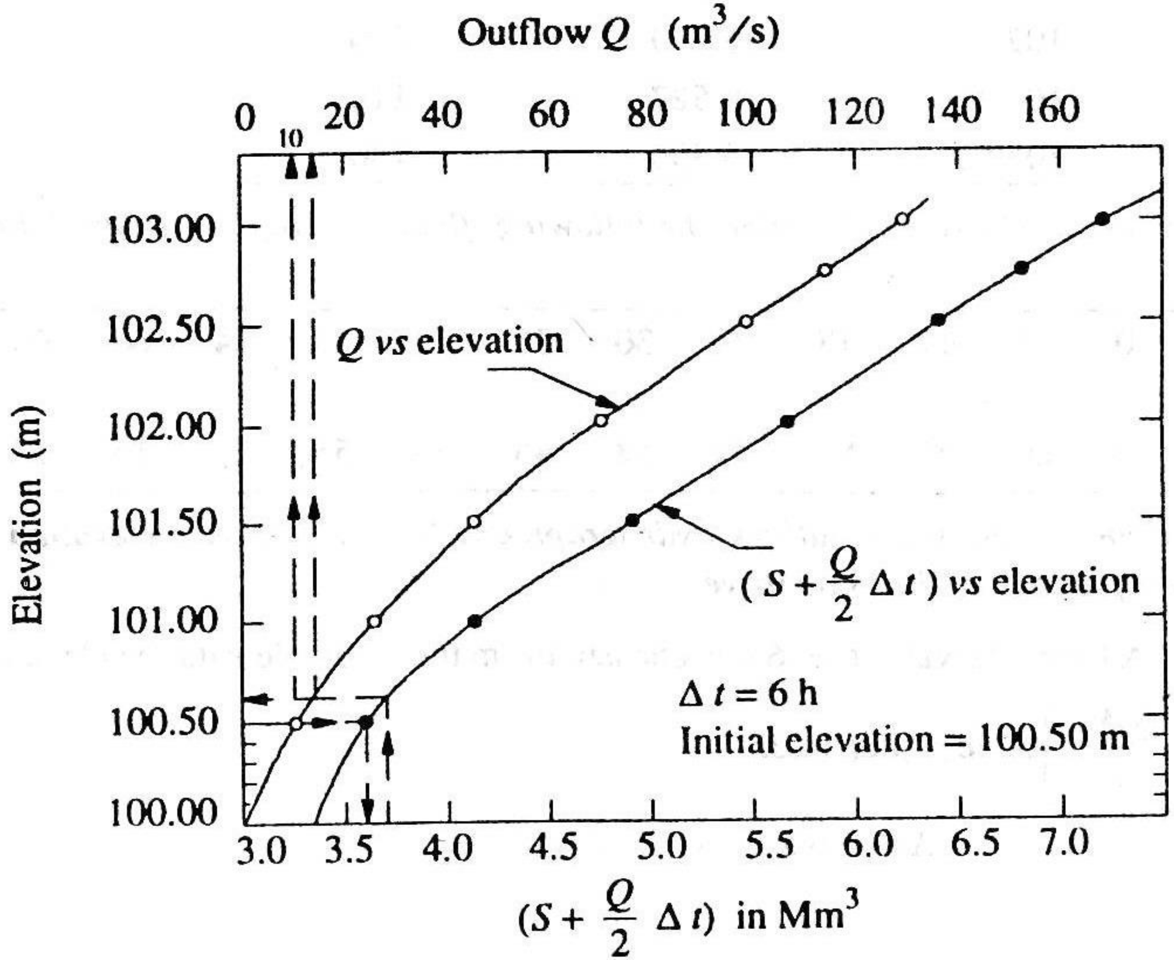


Fig. 8.2 Modified Pul's method of storage routing

EXAMPLE 8.1: *A reservoir has the following elevation, discharge and storage relationships:*

Elevattion (m)	Storage (10^6 m^3)	Outflow dischange (m^3/s)
100.00	3.350	0
100.50	3.472	10
101.00	3.880	26
101.50	4.383	46
102.00	4.882	72
102.50	5.370	100
102.75	5.527	116
103.00	5.856	130

When the reservoir level was at 100.50m, the following flood hydrograph entered the reservoir.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
Discharge (m^3/s)	10	20	55	80	73	58	46	36	55	20	15	13	11

Route the flood and obtain(i) the outflow hydrograph and (ii) the reservoir elevation vs time curve during the passage of the flood wave.

SOLUTION: A time interval $\Delta t = 6$ h is chosen. From the available data the elevation–discharge—$\left(S+\frac{Q\Delta t}{2}\right)$ table is prepared.

$$\Delta t = 6\times 60\times 60 = 0.0216\times 10^6\ s$$

Elevation (m)	100.00	100.50	101.00	101.50	102.00	102.50	102.75	103.00
Dischange Q (m^3/s)	0	10	26	46	72	100	116	130
$\left(S+\frac{Q\Delta t}{2}\right)$ (Mm^3)	3.35	3.58	4.16	4.88	5.66	6.45	6.78	7.26

A graph of Q vs elevation and $\left(S+\frac{Q\Delta t}{2}\right)$ vs elevation is prepared (Fig. 8.2). At the start of routing, elevation = 100.50 m, Q = 10.0 m^3/s, and $\left(S-\frac{Q\Delta t}{2}\right)$ = 3.362 Mm^3. Starting from this value of $\left(S-\frac{Q\Delta t}{2}\right)$, Eq. (8.6) is used to get $\left(S+\frac{Q\Delta t}{2}\right)$ at the end of first time step of 6 h as

$$\left(S+\frac{Q\Delta t}{2}\right)_2 = (I_1+I_2)\frac{\Delta t}{2} + \left(S-\frac{Q\Delta t}{2}\right)_1$$

$$= (10+20)\times\frac{0.0216}{2} + (3.362) = 3.686\ Mm^3.$$

Looking up in Fig. 8.2, the water-surface elevation corresponding to $\left(S+\frac{Q\Delta t}{2}\right) = 3.686$ Mm^3 is 100.62 m and the corresponding outflow discharge Q is 13 m^3/s, For the next step,

Initial value of $\left(S-\frac{Q\Delta t}{2}\right) = \left(S+\frac{Q\Delta t}{2}\right)$ of the previous step $- Q\Delta t$

$$= (3.686 - 13\times 0.0216) = 3.405\ Mm^3$$

The process is repeated for the entire duration of the inflow hydrograph in a tabular form as shown in Table 8.1.

TABLE 8.1 FLOOD ROUTING THROUGH A RESERVOIIR—*EXAMPLE 8.1* —Modified Pul's method.

$\Delta t = 6$ h $= 0.0216$ Ms, $\bar{I} = (I_1 + I_2)/2$

Time (h)	Inflow I (m^3/s)	$\bar{I}$ (m^3/s)	$\bar{I}.\Delta t$ (Mm^3)	$S - \frac{\Delta t Q}{2}$ (Mm^3)	$S + \frac{\Delta t Q}{2}$ (Mm^3)	Elevation (m)	Q (m^3/s)
1	2	3	4	5	6	7	8
0	10					100.50	10
		15.00	0.324	3.362	3.636		
6	20					100.62	13
		37.50	0.810	3.405	4.215		
12	55					101.04	27
		67.50	1.458	3.632	5.090		
18	80					101.64	53
		76.50	1.652	3.945	5.597		
24	73					101.96	69
		65.50	1.415	4.107	5.522		
30	58					101.91	66
		52.00	1.123	4.096	5.219		
36	46					101.72	57
		41.00	0.886	3.988	4.874		

TABLE 8.1 (Continued)

1	2	3	4	5	6	7	8
42	36					101.48	48
		31.75	0.686	3.902	4.588		
48	27.5					101.30	37
		23.75	0.513	3.789	4.302		
54	20					100.10	25
		17.50	0.378	3.676	4.054		
60	15					100.93	23
		14.00	0.302	3.557	3.859		
66	13					100.77	18
		12.00	0.259	3.470	3.729		
72	11					100.65	14
				3.427			

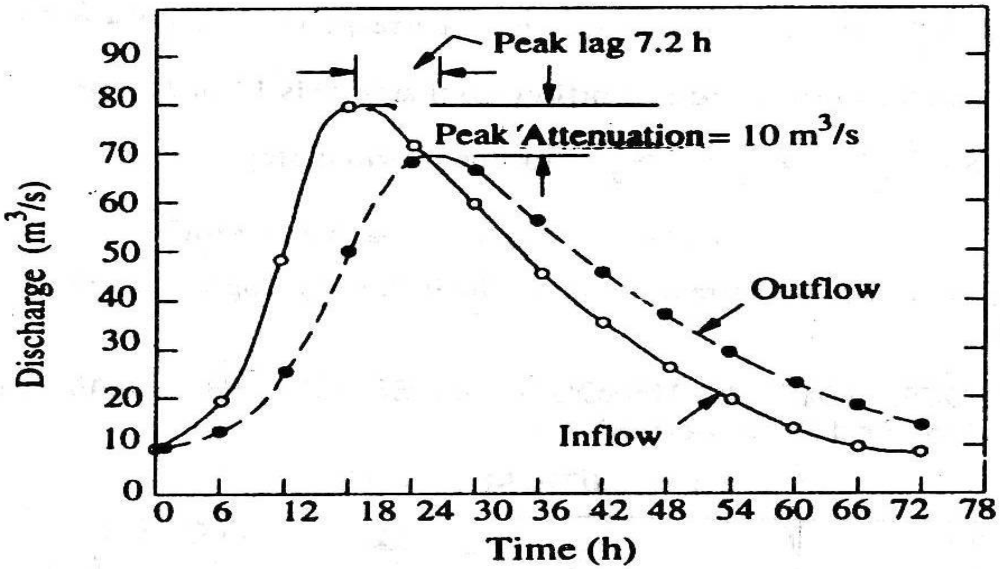


Fig. 8.3 Variation of inflow and outflow discharges—Example 8.1

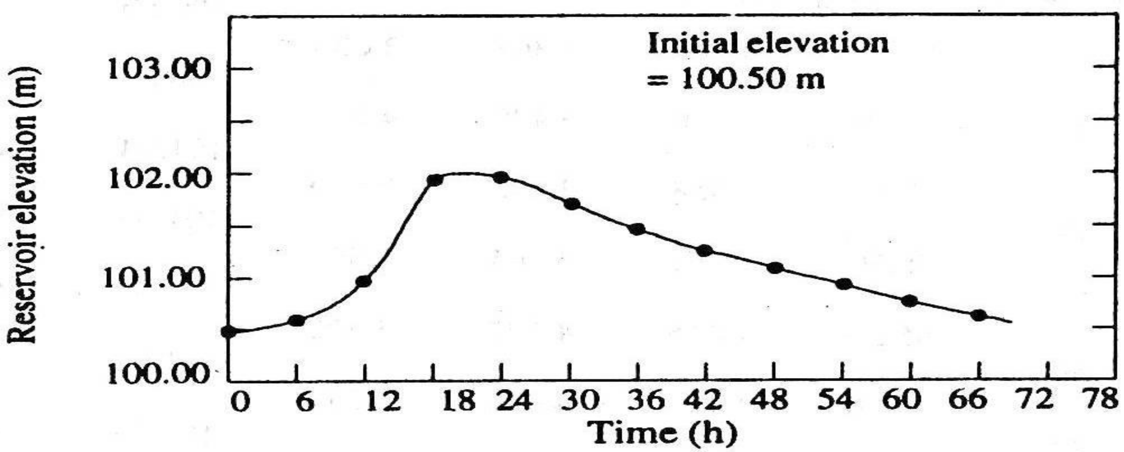


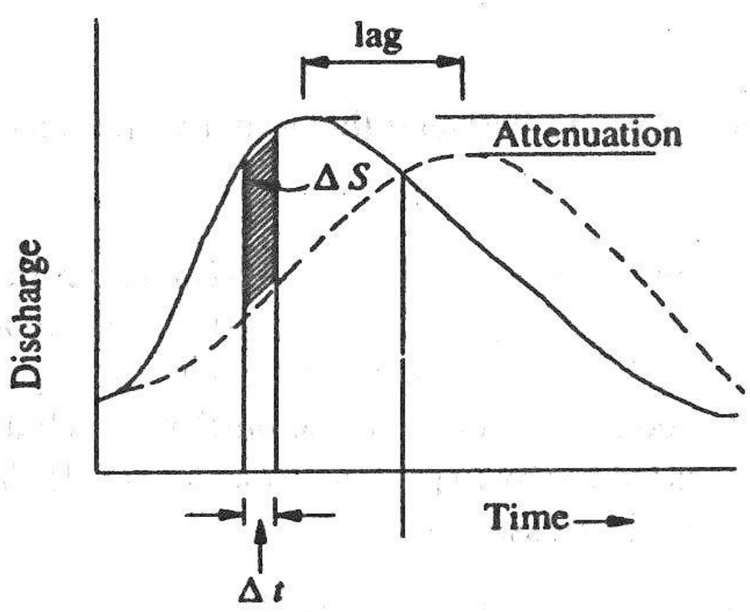
Fig. 8.4 Variation of reservoir elevation with time—Example 8.1

ATENUATION

The peak of the outflow hydrograph will be smaller than of the inflow hydrograph. This reduction in the peak value is called attenuation.

TIME LAG

The peak of the outflow occurs after the peak of the inflow; the time difference between the two peaks is known as lag. The attenuation and lag of a flood hydrograph at a reservoir are two very important aspects of a reservoir operating under a flood-control criteria.



HYDROLOGIC CHANNEL ROUTING

Channel routing the storage is a function of both outflow and inflow discharges. The total volume in storage can be considered under two categories as:

1. Prism storage, and
2. Wedge storage.

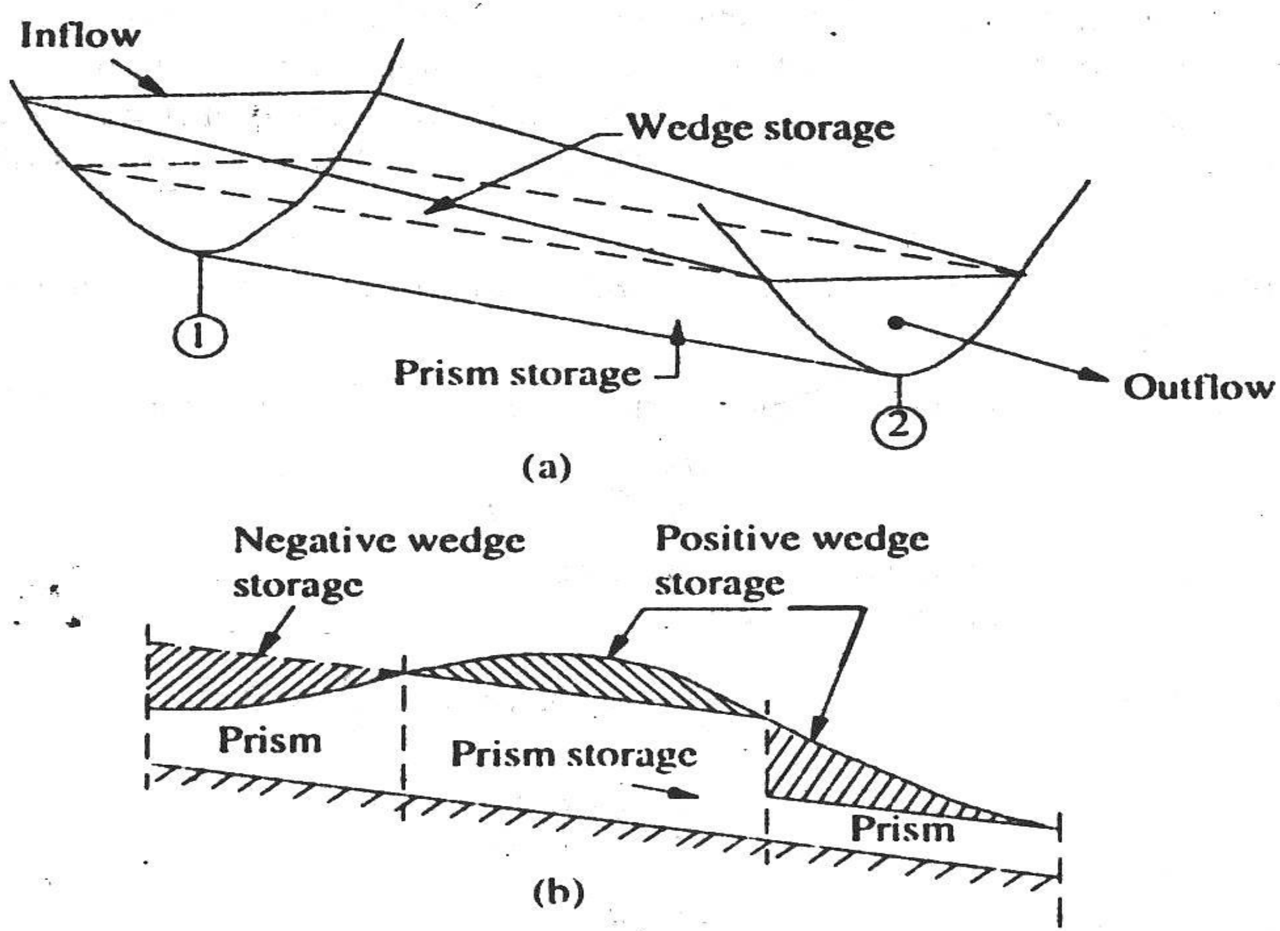


Fig. 8.7 Storage in a channel reach

The total storage in the channel reach can then be expressed as

$$S = K\,(x\,I^m + (1 - x)\,Q^m)$$

Where K and x are coefficients and m = a constant exponent. It has been found that the value of m varies from 0.6 for rectangular channels to a value of about 1.0 for natural channels.

Muskingum Equation

Using m = 1.0 reduces to a linear relationship for S in terms of I and Q as

$$S = K\,(x\,I + (1 - x)\,Q)$$

And this relationship is known as the Muskingum equation. In this the parameter x is known as weighting factor and takes a value between 0 and 0.5. When x = 0,

$$S = K\,Q \qquad \text{(This is reservoir routing)}$$

$$\text{or } S = f\,(Q)$$

When x = 0.5

$$S = K\,(0.5\,I + 0.5\,Q) \qquad \text{(This is channel routing)}$$

$$\text{or } S = f\,(I, Q)$$

The coefficient K is known as storage – time constant.

Estimation of K and x

Figure 8.8 shows a typical inflow and outflow hydrograph through a channel reach. Note that the outflow peak does not occur at the point of intersection of the inflow and outflow hydrographs. Using the continuity equation [Eq. (8.3)],

$$(I_1 + I_2)\frac{\Delta t}{2} - (Q_1 + Q_2)\frac{\Delta t}{2} = \Delta S$$

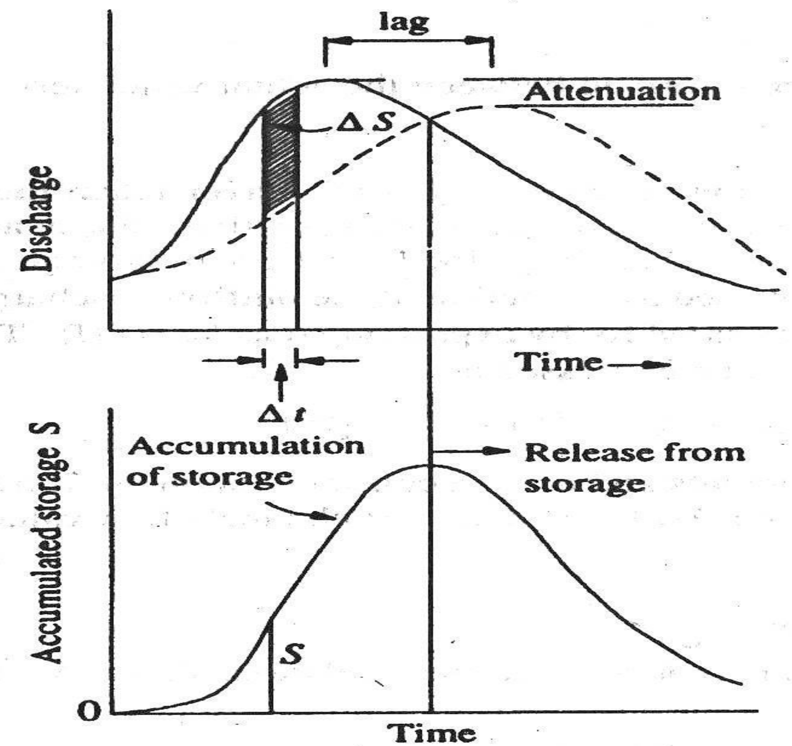


Fig. 8.8 Hydrographs and storage in channel routing

EXAMPLE 8.4. *The following inflow and outflow hydrographs were observed in a river reach. Estimate the values of* K *and* x *applicable to this reach for use in the Muskingum equation.*

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (m^3/s)	5	20	50	50	32	22	15	10	7	5	5	5
Outflow (m^3/s)	5	6	12	29	38	35	29	23	17	13	9	7

SOLUTION: Using a time increment $\Delta t = 6$ h, the calculations are performed in a tabular manner as in Table 8.3. The incremental storage ΔS and S are calculated in columns 6 and 7 respectively. It is advantageous to use the units [(m^3/s).h] for storage terms.

TABLE 8.3 DETERMINATION OF K AND x—*EXAMPLE 8.4*

$\Delta t = 6$ h, Storage in (m^3/s). h

Time (h)	I (m^3/s)	Q (m^3/s)	$(I-Q)$	Average $(I-Q)$	ΔS = Col. 5 $\times \Delta t$ $(m^3/s \cdot h)$	$S = \Sigma \Delta S$ $(m^3/s \cdot h)$	$[xI+(1-x)Q]$ (m^3/s) $x = 0.35$	$x = 0.30$	$x = 0.25$
1	2	3	4	5	6	7	8	9	10
0	5	5	0			0	5.0	5.0	5.0
				7.0	42				
6	20	6	14			42	10.9	10.2	9.5
				26.0	156				
12	50	12	38			198	25.3	23.4	21.5
				29.5	177				
18	50	29	21			375	36.4	35.3	34.3
				7.5	45				
24	32	38	−6			420	35.9	36.2	36.5
				−9.5	−57				
30	22	35	−13			363	30.5	31.1	31.8
				−13.5	−81				
36	15	29	−14			282	24.1	24.8	25.5
				−13.5	−81				

42	10	23	−13			201	18.5	19.1	19.8
				−11.5	−69				
48	7	17	−10			132	13.5	14.0	14.5
				−9.0	−54				
54	5	13	−8			78	10.2	10.6	11.0
				−6.0	−36				
60	5	9	−4			42	7.6	7.8	8.0
				−3.0	−18				
66	5	7	−2			24	6.3	6.4	6.5

As a first trial $x = 0.35$ is selected and the value of $[xI + (1 - x)\,Q]$ evaluated (column 8) and plotted against S in Fig. 8.9. Since a looped curve is obtained, further trails are performed with $x = 0.30$ and 0.25. It is seen from Fig. 8.9 that for $x = 0.25$ the data very nearly describe a straight line and as such $x = 0.25$ is taken as the appropriate value for the reach.

From Fig. 8.9, $K = 13.3$ h

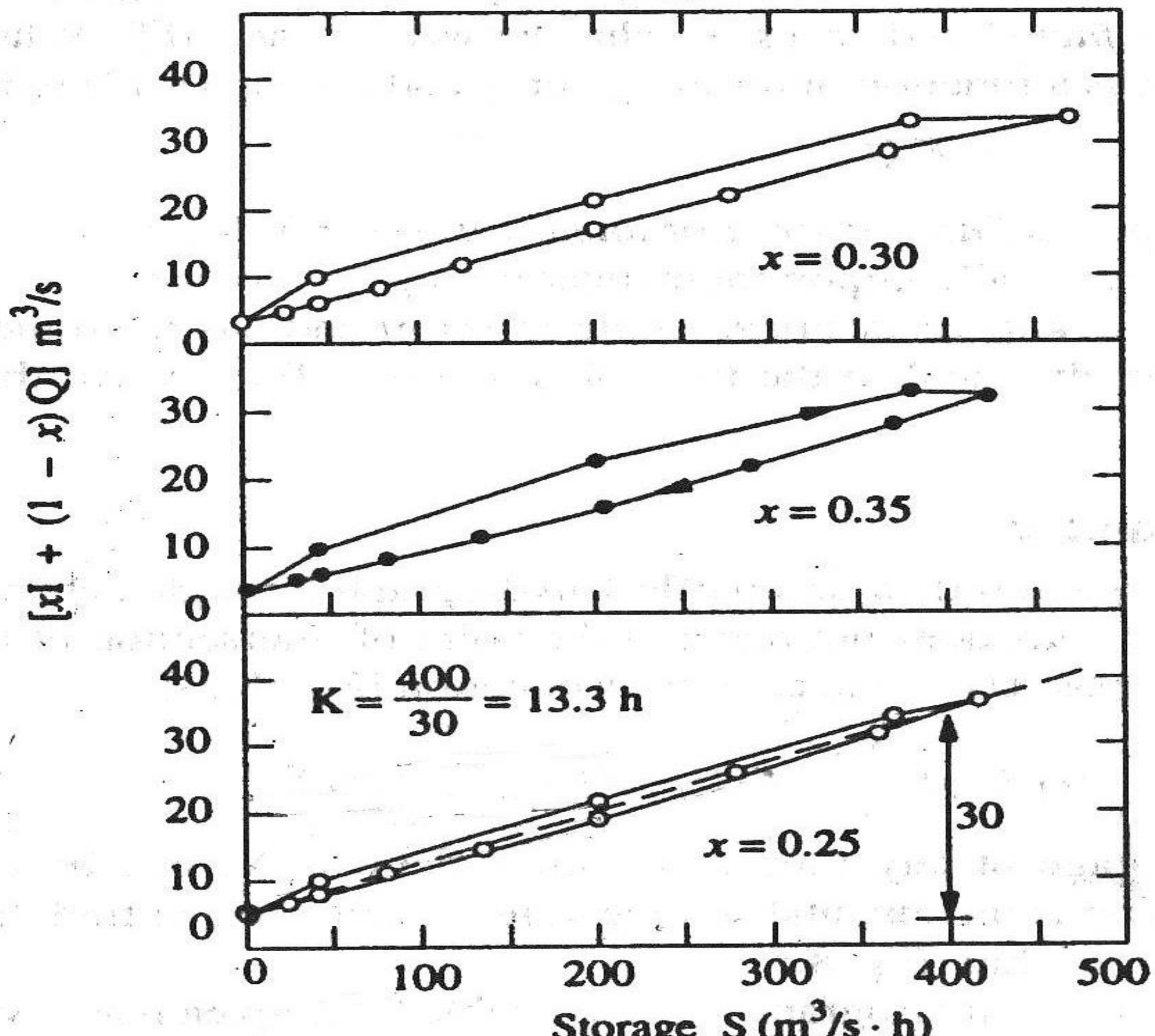


Fig. 8.9 Determination of K and x for a channel reach

Muskingum Method of Routing

For a given channel reach by selecting a routing interval Δt and using the Muskingum equation, the change in storage is

$$S_2 - S_1 = k[x(I_2 - I_1) + (1 - x)(Q_2 - Q_1)] \qquad \text{......................(i)}$$

$$S_2 - S_1 = \frac{I_1 + I_2}{2}\Delta t - \frac{Q_1 + Q_1}{2}\Delta t \qquad \text{......................(ii)}$$

From Eq^n (i) and Eq^n (ii)

$$\Rightarrow \quad \frac{I_1 + I_2}{2}\Delta t - \frac{Q_1 + Q_1}{2}\Delta t = k[x(I_2 - I_1) + (1 - x)(Q_2 - Q_1)]$$

$$\Rightarrow \quad \frac{I_1 + I_2}{2}\Delta t - \frac{Q_1}{2}\Delta t - \frac{Q_2}{2}\Delta t = kx(I_2 - I_1) + k(1 - x)Q_2 - k(1 - x)Q_1$$

$$\Rightarrow \quad -k(1 - x)Q_2 - \frac{Q_2}{2}\Delta = kxI_2 - kxI_1 - kQ_1 + kxQ_1 - \frac{I_1 + I_2}{2}\Delta t + \frac{Q_1}{2}\Delta t$$

$$\Rightarrow \quad k(1-x)Q_2 + \frac{Q_2}{2}\Delta = -kxI_2 + kxI_1 + kQ_1 - kxQ_1 + \frac{I_1 + I_2}{2}\Delta t - \frac{Q_1}{2}\Delta t$$

$$\Rightarrow \quad \frac{Q_2}{2}\Delta t + kQ_2 - kxQ_2 = -kxI_2 + kxI_1 + kQ_1 - kxQ_1 + \frac{I_1}{2}\Delta t + \frac{I_2}{2}\Delta t - \frac{Q_1}{2}\Delta t$$

$$\Rightarrow \quad \frac{Q_2}{2}\Delta t + kQ_2 - kxQ_2 = kxI_1 + \frac{I_1}{2}\Delta t - kxI_2 + \frac{I_2}{2}\Delta t + kQ_1 - kxQ_1 - \frac{Q_1}{2}\Delta t$$

$$\Rightarrow \quad Q_2\left(\frac{1}{2}\Delta t + k - kx\right) = I_1\left(kx + \frac{1}{2}\Delta t\right) - I_2\left(kx + \frac{1}{2}\Delta t\right) + Q_1\left(k - kx - \frac{1}{2}\Delta t\right)$$

$$\Rightarrow \quad Q_2 = \frac{\left(kx + \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)} I_1 + \frac{-\left(kx + \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)} I_2 + \frac{\left(k - kx - \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)} Q_1$$

Here,

$$C_0 = \frac{\left(kx + \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)} \qquad C_1 = \frac{\left(kx + \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)} \qquad C_2 = \frac{\left(k - kx - \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)}$$

$$\therefore \; Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$$

EXAMPLE 8.5: *Route the following hydrograph through a river reach for which K = 12.0 h and x = 0.20. At the start of the inflow flood, the outflow discharge is 10 m³/s.*

Time (h)	0	6	12	18	24	30	36	42	48	54
Inflow (m^3/s)	10	20	50	60	55	45	35	27	20	15

SOLUTION: Since $K = 12$ h and $2Kx = 2 \times 12 \times 0.2 = 4.8$ h, Δt should be such that $12\ h > \Delta t > 4.8\ h$. In the present case $\Delta t = 6$ h is selected to suit the given inflow hydrograph ordinate interval.

Using Eqs. (8.16–a, b & c) the coefficients C_0, C_1 and C_2 are calculated as

$$C_0 = \frac{-12 \times 0.20 + 0.5 \times 6}{12 - 12 \times 0.2 + 0.5 \times 6} = \frac{0.6}{12.6} = 0.048$$

$$C_1 = \frac{12 \times 0.2 + 0.5 \times 6}{12.6} = 0.429$$

$$C_2 = \frac{12 - 12 \times 0.2 - 0.5 \times 6}{12.6} = 0.523$$

For the first time interval, 0 to 6 h,

$I_1 = 10.0$ $\qquad$ $C_1 I_1 = 4.29$

$I_2 = 20.0$ $\qquad$ $C_0 I_2 = 0.96$

$Q_1 = 10.0$ $\qquad$ $C_2 Q_1 = 5.23$

From Eq. (8.16) $Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1 = 10.48\ \text{m}^3/\text{s}$

$\Delta t = 6$ h

Time (h)	I (m^3/s)	$0.048\ I_2$	$0.429\ I_1$	$0.523\ Q_1$	Q (m^3/s)
1	2	3	4	5	6
0	10				10.00
		0.96	4.29	5.23	
6	20				10.48
		2.40	8.58	5.48	
12	50				16.46
		2.88	21.45	8.61	
18	60				32.94
		2.64	25.74	17.23	
24	55				45.61
		2.16	23.60	23.85	
30	45				49.61
		1.68	19.30	25.95	
36	35				46.93
		1.30	15.02	24.55	
42	27				40.87
		0.96	11.58	21.38	
48	20				33.92
		0.72	8.58	17.74	
54	15				27.04

Floods

To estimate the magnitude of a flood peak the following alternative are available

1. Rational Method
2. Empirical Method
3. Unit-Hydrograph technique, and
4. Flood-Frequency Studies

1. Rational Method

$$Q_p = C A i \; ; \; \text{for } t \geq t_c$$

where C = coefficient of runoff = (runoff/ rainfall), A = area of the catchment and i = intensity of rainfall. This is the basic equation of the *rational method*.

Rainfall Intensity ($i_{tc,p}$)

$$i_{tc,p} = \frac{K\,T^{x}}{(t_c + a)^{m}}$$

In which K, a, x and m are constant

Time of Concentration (tc)

Runoff Coefficient (C)

The coefficient C represents the integrated effect of the catchment looses and hence depends upon the nature of the surface. Some typical values of C are indicated in Table 7.1

TABLE 7.1 VALUE OF THE COEFFICIENT C IN EQ. (7.2)

Type of area		Value of C
A. *Urban area* (P = 0.05 to 0.10)		
Lawns:	Sandy-soil, flat, 2%	0.05–0.10
	Sandy soil, steep, 7%	0.15–0.20
	Heavy soil, average, 2.7%	0.18–0.22
Residential areas:		
	Single family areas	0.30–0.50
	Multi units, attached	0.60–0.75
Industrial:		
	Light	0.50–0.80
	Heavy	0.60–0.90
Streets		0.70–0.95
B. *Agricultural Area*		
Flat:	Tight clay; cultivated	0.50
	woodland	0.40
	Sandy loam; cultivated	0.20
	woodland	0.10
Hilly:	Tight clay; cultivated	0.70
	woodland	0.60
	Sandy loam; cultivated	0.40
	woodland	0.30

2. Empirical Method

Dickens Formula (1865)

$$Q_p = C_D A^{3/4}$$

where Q_p = maximum flood discharge (m^3/s)

A = catchment area (km^2)

C_D = Dickens constant with value between 6 to 30

The following are some guidelines in selecting the value of C_D :

	Value of C_D
North–Indian plains	6
North–Indian hilly regions	11–14
Central India	14–28
Coastal Andhra and Orissa	22–28

3. Unit- Hydrograph Technique

4. Flood Frequency Studies

$$P = \frac{m}{N+1}$$

where m = order number of the event and N = total number of events in the data. The recurrence interval, T (also called the *return period* or *frequency*) is calculated as

$$T = 1/P$$

Chow (1951) has shown that most frequency–distribution functions applicable in hydrologic studies can be expressed by the following equation known as the *general equation of hydrologic frequency analysis:*

$$x_T = \bar{x} + K\sigma$$

where x_T = value of the variate X of a random hydrologic series with a return period T, $\bar{x}$ = mean of the variate, σ = standard deviation of the variate, K = frequency factor which depends upon the return period, T and the assumed frequency distribution. Some of the commonly used frequency distribution functions for the predication of extreme flood values are :

1. Gumbel's extreme–value distribution,
2. log–Pearson Type III distribution, and
3. log normal distribution.

Gumbel's Method

$$x_T = \bar{x} + K\,\sigma_{n-1}$$

where σ_{n-1} = standard deviation of the sample of size N

$$= \sqrt{\frac{\Sigma\,(x-\bar{x})^2}{N-1}}$$

K = frequency factor expressed as

$$K = \frac{y_T - \bar{y}_n}{S_n}$$

in which y_T = reduced variate, a function of T and is given by

$$y_T = -\left[\ln . \ln \frac{T}{T-1}\right]$$

TABLE 7.3 REDUCED MEAN $\bar{y}_n$ IN GUMBEL'S EXTREME VALUE DISTRIBUTION

N = sample size

N	0	1	2	3	4	5	6	7	8	9
10	0.4952	0.4996	0.5035	0.5070	0.5100	0.5128	0.5157	0.5181	0.5202	0.5220
20	0.5236	0.5252	0.5268	0.5283	0.5296	0.5309	0.5320	0.5332	0.5343	0.5353
30	0.5362	0.5371	0.5380	0.5388	0.5396	0.5402	0.5410	0.5418	0.5424	0.5430
40	0.5436	0.5442	0.5448	0.5453	0.5458	0.5463	0.5468	0.5473	0.5477	0.5481
50	0.5485	0.5489	0.5493	0.5497	0.5501	0.5504	0.5508	0.5511	0.5515	0.5518
60	0.5521	0.5524	0.5527	0.5530	0.5533	0.5535	0.5538	0.5540	0.5543	0.5545
70	0.5548	0.5550	0.5552	0.5555	0.5557	0.5559	0.5561	0.5563	0.5565	0.5567
80	0.5569	0.5570	0.5572	0.5574	0.5576	0.5578	0.5580	0.5581	0.5583	0.5585
90	0.5586	0.5587	0.5589	0.5591	0.5592	0.5593	0.5595	0.5596	0.5598	0.5599
100	0.5600									

TABLE 7.4 REDUCED STANDARD DEVIATION S_n IN GUMBEL'S EXTREME VALUE DISTRIBUTION

N = sample size

N	0	1	2	3	4	5	6	7	8	9
10	0.9496	0.9676	0.9833	0.9971	1.0095	1.0206	1.0316	1.0411	1.0493	1.0565
20	1.0628	1.0696	1.0754	1.0811	1.0864	1.0915	1.0961	1.1004	1.1047	1.1086
30	1.1124	1.1159	1.1193	1.1226	1.1255	1.1285	1.1313	1.1339	1.1363	1.1388
40	1.1413	1.1436	1.1458	1.1480	1.1499	1.1519	1.1538	1.1557	1.1574	1.1590
50	1.1607	1.1623	1.1638	1.1658	1.1667	1.1681	1.1696	1.1708	1.1721	1.1734
60	1.1747	1.1759	1.1770	1.1782	1.1793	1.1803	1.1814	1.1824	1.1834	1.1844
70	1.1854	1.1863	1.1873	1.1881	1.1890	1.1898	1.1906	1.1915	1.1923	1.1930
80	1.1938	1.1945	1.1953	1.1959	1.1967	1.1973	1.1980	1.1987	1.1994	1.2001
90	1.2007	1.2013	1.2020	1.2026	1.2032	1.2038	1.2044	1.2049	1.2055	1.2060
100	1.2065									

Gumbel Probability Paper

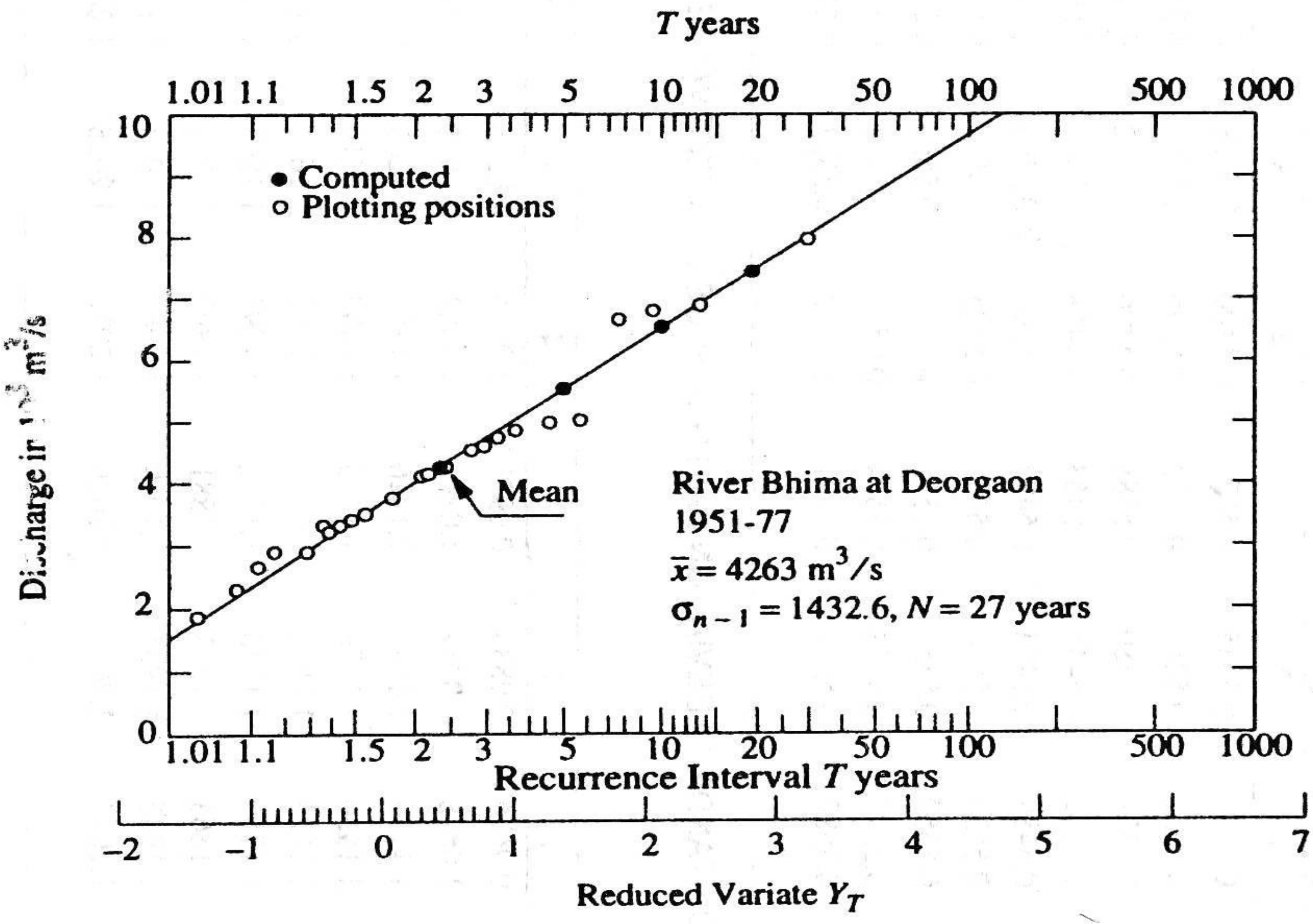


Fig. 7.3 Flood probability analysis by Gumbel's distribution

EXAMPLE 7.3. *Annual maximum recorded floods in the river Bhima at Deorgaon, a tributary of the river Krishna, for the period 1951 to 1977 is given below. Verify whether the Gumbel extreme-value distribution fit the recorded values. Estimate the flood discharge with recurrence interval of (i) 100 years and (ii) 150 years by graphical extrapolation.*

Year	1951	1952	1953	1954	1955	1956	1957	1958	1959
Max. flood (m^3/s)	2947	3521	2399	4124	3496	2947	5060	4903	3757
Year	1960	1961	1962	1963	1964	1965	1966	1967	1968
Max. flood (m^3/s)	4798	4290	4652	5050	6900	4366	3380	7826	3320
Year	1969	1970	1971	1972	1973	1974	1975	1976	1977
Max. flood (m^3/s)	6599	3700	4175	2988	2709	3873	4593	6761	1971

SOLUTION: The flood discharge values are arranged in descending order and the plotting position recurrence interval T_p for each discharge is obtained as

$$T_p = \frac{N+1}{m} = \frac{28}{m}$$

where m = order number. The discharge magnitude Q are plotted against the corresponding T_p on a Gumbel extreme probability paper (Fig. 7.3).

The statistics $\bar{x}$ and σ_{n-1} for the series are next calculated and are shown in Table 7.5. Using these the discharge x_T for some chosen recurrence interval is calculated by using Gumbel's formulae [Eqs. (7.22), (7.21) and (7.20)].

TABLE 7.5 CALCULATION OF T_p FOR OBSERVED DATA —*EXAMPLE 7.3*

Order number m	Flood discharge x (m^3/s)	T_p (years)	Order number m	Flood discharge x (m^3/s)	T_p (years)
1	7826	28.00	15	3873	1.87
2	6900	14.00	16	3757	1.75
3	6761	9.33	17	3700	1.65
4	6599	7.00	18	3521	1.56
5	5060	5.60	19	3496	1.47
6	5050	4.67	20	3380	1.40
7	4903	4.00	21	3320	1.33
8	4798	3.50	22	2988	1.27
9	4652	3.11	23	2947	—
10	4593	2.80	24	2947	1.17
11	4366	2.55	25	2709	1.12
12	4290	2.33	26	2399	1.08
13	4175	2.15	27	1971	1.04
14	4124	2.00			

$N = 27$ years, $\bar{x} = 4263\ m^3/s$, $\sigma_{n-1} = 1432.6\ m^3/s$

From Tables 7.3 and 7.4, for $N = 27$, $\bar{y}_n = 0.5332$ and $S_n = 1.1004$.
Choosing $T = 10$ years, by Eq. (7.22),

$$y_T = -[\ln . \ln (10/9)] = 2.25037$$

$$K = \frac{2.25307 - 0.5332}{1.1004} = 1.56$$

$$x_T = 4263 + (1.56 \times 1432.6)$$

$$= 6499 \text{ m}^3/\text{s}$$

Similarly, values of x_T are calculated for two more T values as shown below.

T (years)	x_T [obtaained by Eq. (7.20)] (m^3/s)
5.0	5522
10.0	6499
20.0	7436

By extrapolation of the theoretical x_T *vs* T relationship, from Fig. 7.3,

At $T = 100$ years, $x_T = 9600 \text{ m}^3/\text{s}$

At $T = 150$ years, $x_T = 10{,}700 \text{ m}^3/\text{s}$

[By using Eq. (7.20) to (7.22), $x_{100} = 9558 \text{ m}^3/\text{s}$ and $x_{150} = 10088 \text{ m}^3/\text{s}$.]

EXAMPLE 7.4. *Flood–frequency computations for the river Chambal at Gandhisagar dam, by using Gumbel's method, yielded the following results:*

Return period T (years)	Peak flood (m^3/s)
50	40,809
100	46,300

Estimate the flood magnitude in this river with a return period of 500 years.

SOLUTION : By Eq. (7.20),

$$x_{100} = \bar{x} + K_{100}\,\sigma_{n-1}$$

$$x_{50} = \bar{x} + K_{50}\,\sigma_{n-1}$$

$$(K_{100} - K_{50})\,\sigma_{n-1} = x_{100} - x_{50} = 46300 - 40809 = 5491$$

But
$$K_T = \frac{y_T}{S_n} - \frac{\bar{y}_n}{S_n}$$

where S_n and $\bar{y}_n$ are constants for the given data series.

$$\therefore \qquad (y_{100} - y_{50}) \frac{\sigma_{n-1}}{S_n} = 5491$$

By Eq. (7.22)

$$y_{100} = -[\ln . \ln (100/99)] = 4.60015$$

$$y_{50} = -[\ln . \ln (50/49)] = 3.90194$$

$$\frac{\sigma_{n-1}}{S_n} = \frac{5491}{(4.60015 - 3.90194)} = 7864$$

For $T = 500$ years, by Eq. (7.22),

$$y_{500} = -[\ln . \ln (500/499)] = 6.21361$$

$$(y_{500} - y_{100}) \frac{\sigma_{n-1}}{S_n} = x_{500} - x_{100}$$

$$(6.21361 - 4.60015) \times 7864 = x_{500} - 46300$$

$$x_{500} = 58988, \quad \text{say } 59{,}000 \text{ m}^3/\text{s}$$

Confidence Limits

For a confidence probability c, the confidence interval of the variate XT is bounded by values X1 and X2 given by

$$x_{1/2} = x_T \pm f(c)\, S_e$$

where $f(c)$ = function of the confidence probability c determined by using the table of normal variates as

c in per cent	50	68	80	90	95	99
$f(c)$	0.674	1.00	1.282	1.645	1.96	2.58

$$S_e = \text{probable error} = b\,\frac{\sigma_{n-1}}{\sqrt{N}}$$

$$b = \sqrt{1 + 1.3\,K + 1.1\,K^2}$$

K = frequency factor give by Eq. (7.21)

σ_{n-1} = standard deviation of the sample

N = sample size.

EXAMPLE 7.5 *Data covering a period of 92 years for the river Ganga at Raiwala yielded the mean and standard derivation of the annual flood series as 6437 and 2951* m^3/s *respectively. Using Gumbel's method estimate the flood discharge with a return period of 500 years. What are the (a) 95% and (b) 80% confidence limits for this estimate.*

SOLUTION: From Table 7.3 for $N = 92$ years, $\bar{y}_n = 0.5589$
and $S_n = 1.2020$ from Table 7.4.

$$y_{500} = -[\ln . \ln (500/499)]$$

$$= 6.21361$$

$$K_{500} = \frac{6.21361 - 0.5589}{1.2020} = 4.7044$$

$$x_{500} = 6437 + 4.7044 \times 2951 = 20320 \text{ m}^3/\text{s}$$

From Eq. (7.33 a)

$$b = \sqrt{1 + 1.3\,(4.7044) + 1.1\,(4.7044)^2}$$

$$= 5.61$$

$$S_e = \text{probable error} = 5.61 \times \frac{2951}{\sqrt{92}} = 1726$$

(a) For 95% confidence probability $f(c) = 1.96$ and by Eq. (7.23)

$$x_{1/2} = 20320 \pm (1.96 \times 1726)$$

$$x_1 = 23703 \text{ m}^3/\text{s} \quad \text{and} \quad x_2 = 16937 \text{ m}^3/\text{s}$$

Thus estimated discharge of 20320 m^3/s has a 95% probability of lying between 23700 and 16940 m^3/s

(b) For 80% confidence probability, $f(c) = 1.282$ and by Eq. (7.23)

$$x_{1/2} = 20320 \pm (1.282 \times 1726)$$

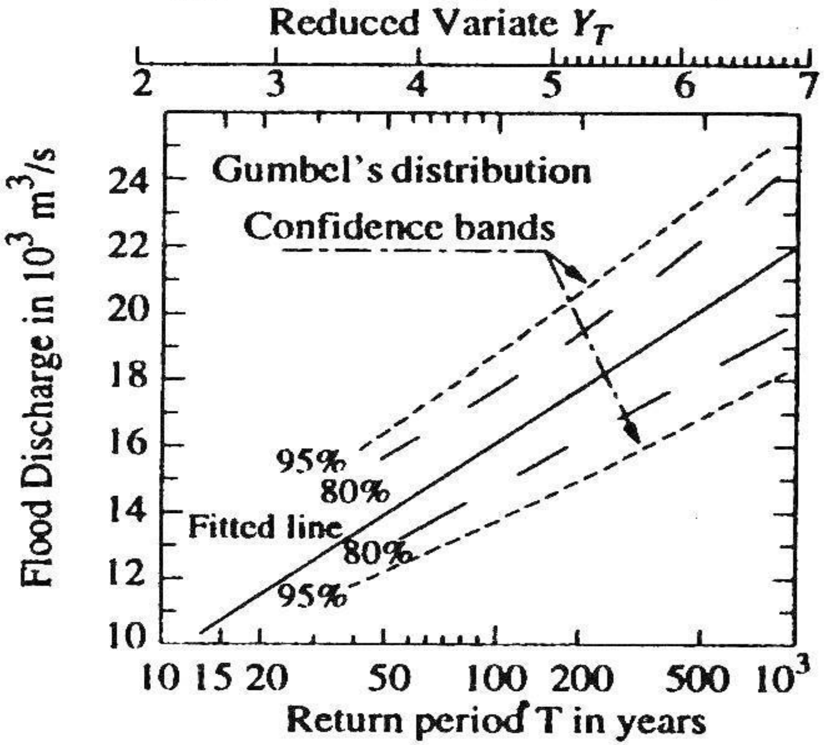


Fig. 7.4 Confidence bands for Gumbels distribution–Example 7.5

$$x_1 = 22533 \text{ m}^3/\text{s} \quad \text{and} \quad x_2 = 18107 \text{ m}^3/\text{s}$$

The estimated discharge of 20320 m^3/s has a 80% probability of lying between 22530 and 18110 m^3/s.

Streamflow Measurement

Streamflow measurement techniques can be broadly classified into two categories as (i) direct determination and (ii) indirect determination.

1. Direct determination of stream discharge:
   (a) Area-velocity methods,
   (b) dilution techniques,
   (c) electromagnetic method, and
   (d) ultrasonic method.

2. Indirect determination of stream flow:
   (a) Hydraulic structures, such as weirs, flumes and gated structures and
   (b) Slope area method

Measurement of Stage

(a) Staff Gauge

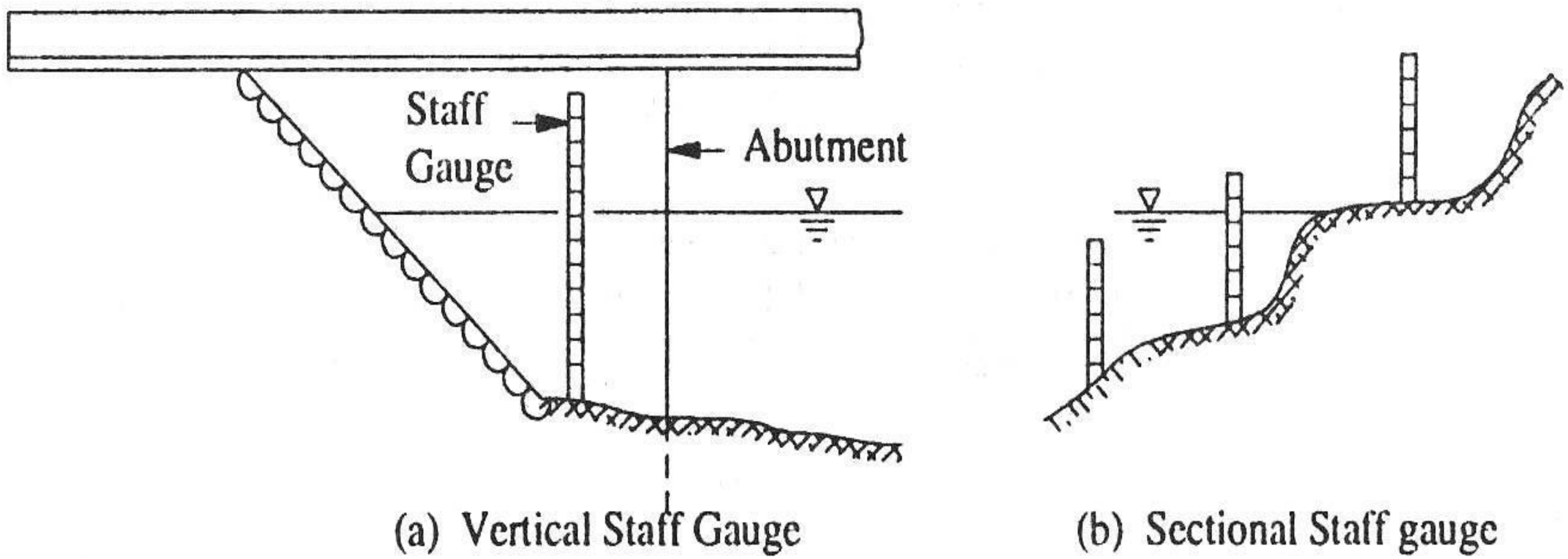


Fig. 4.1 Staff gauge

(b) Wire Gauge

(c) Automatic Stage Recorders

Two typical automatic recorders are below

(i) Float-Gauge Recorder

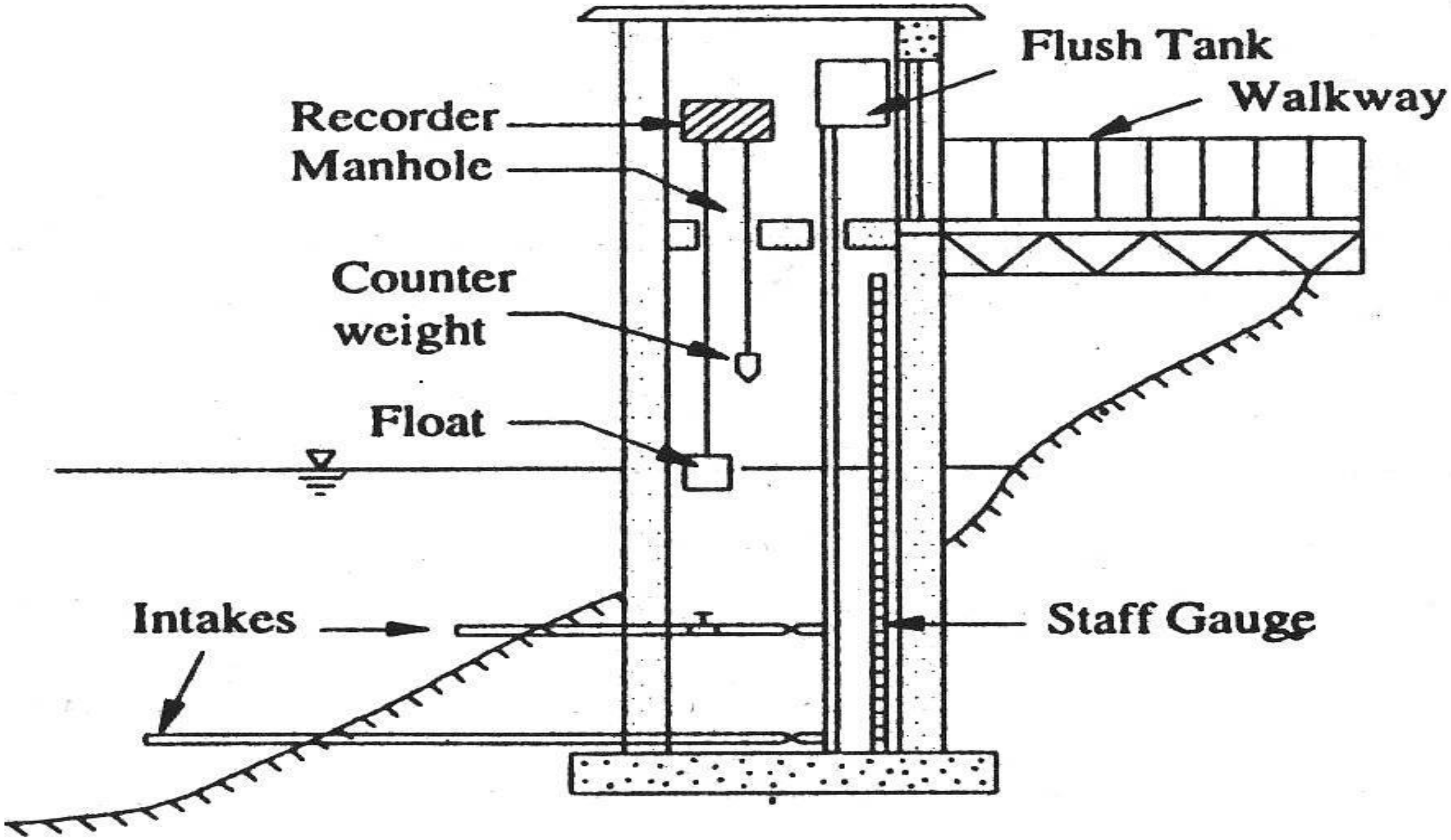


Fig. 4.2 Stilling well installation

(ii) Bubble Gauge

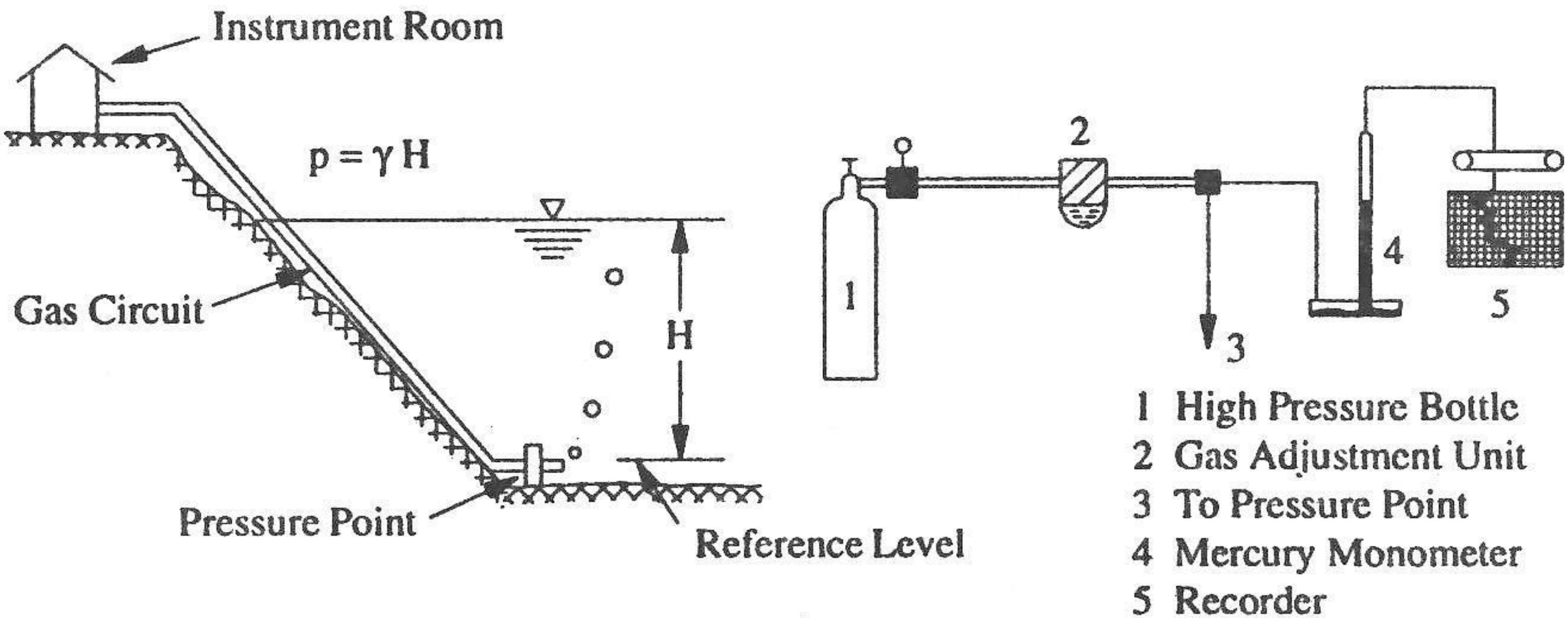


Fig. 4.4 Bubble guage

The bubble gauge has certain specific advantages over a float operated water stage recorder and these can be listed as under:

1. There is no need for costly stilling wells;
2. a large change in the stage, as much as 30 m, can be measured;
3. the recorder assembly can be quite far away from the sensing point; and
4. due to constant bleeding action there is less likelihood of the inlet getting blocked or choked.

Measurement of Velocity

The measurement of velocity is an important aspect of many direct stream flow measurement techniques. A mechanical device, called current meter.

Current Meters

There are two main types of current meter

1. Vertical – axis meters, and
2. Horizontal – axis meters

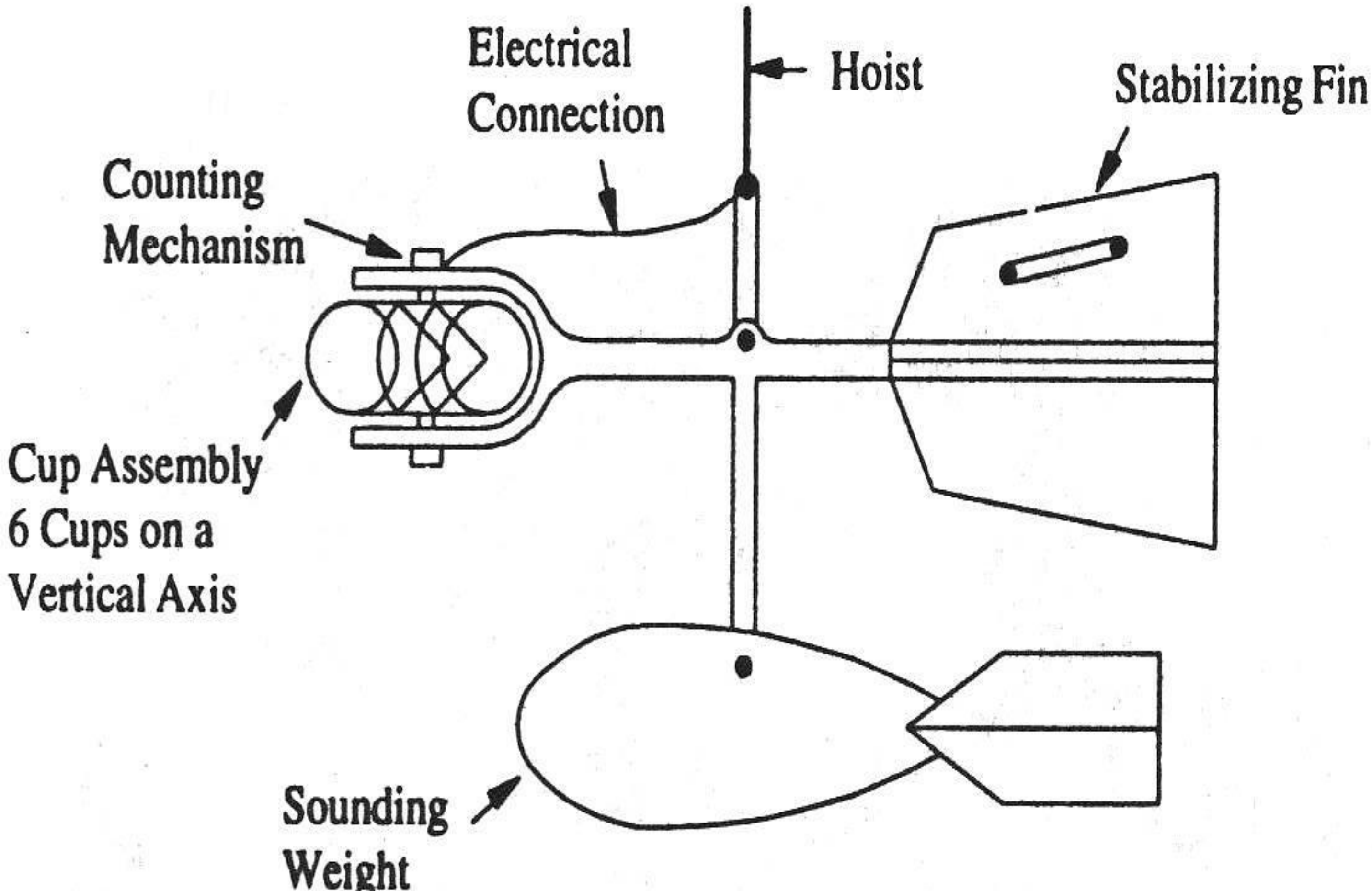


Fig. 4.8 Vertical-axis current meter

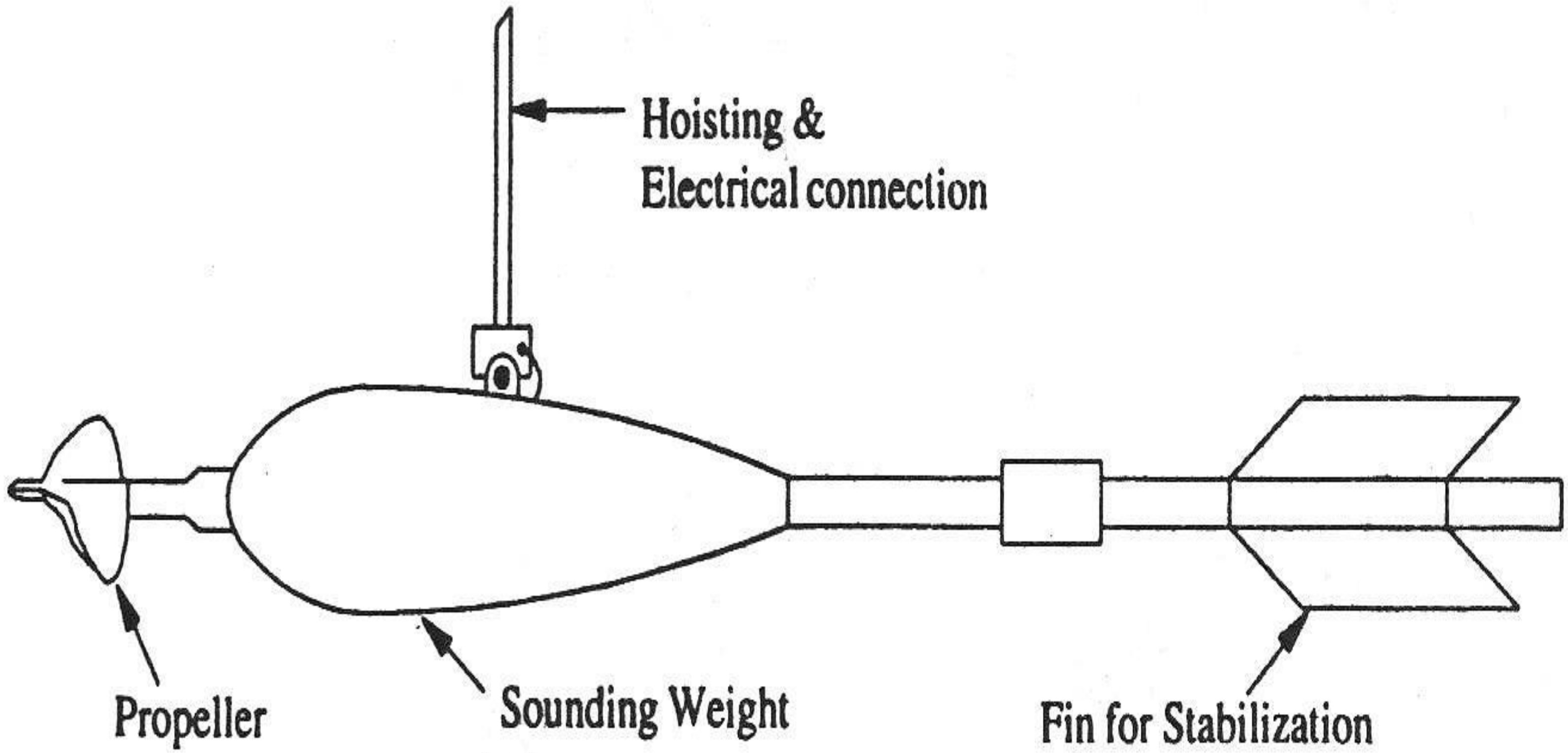


Fig. 4.11 Horizontal-axis current meter

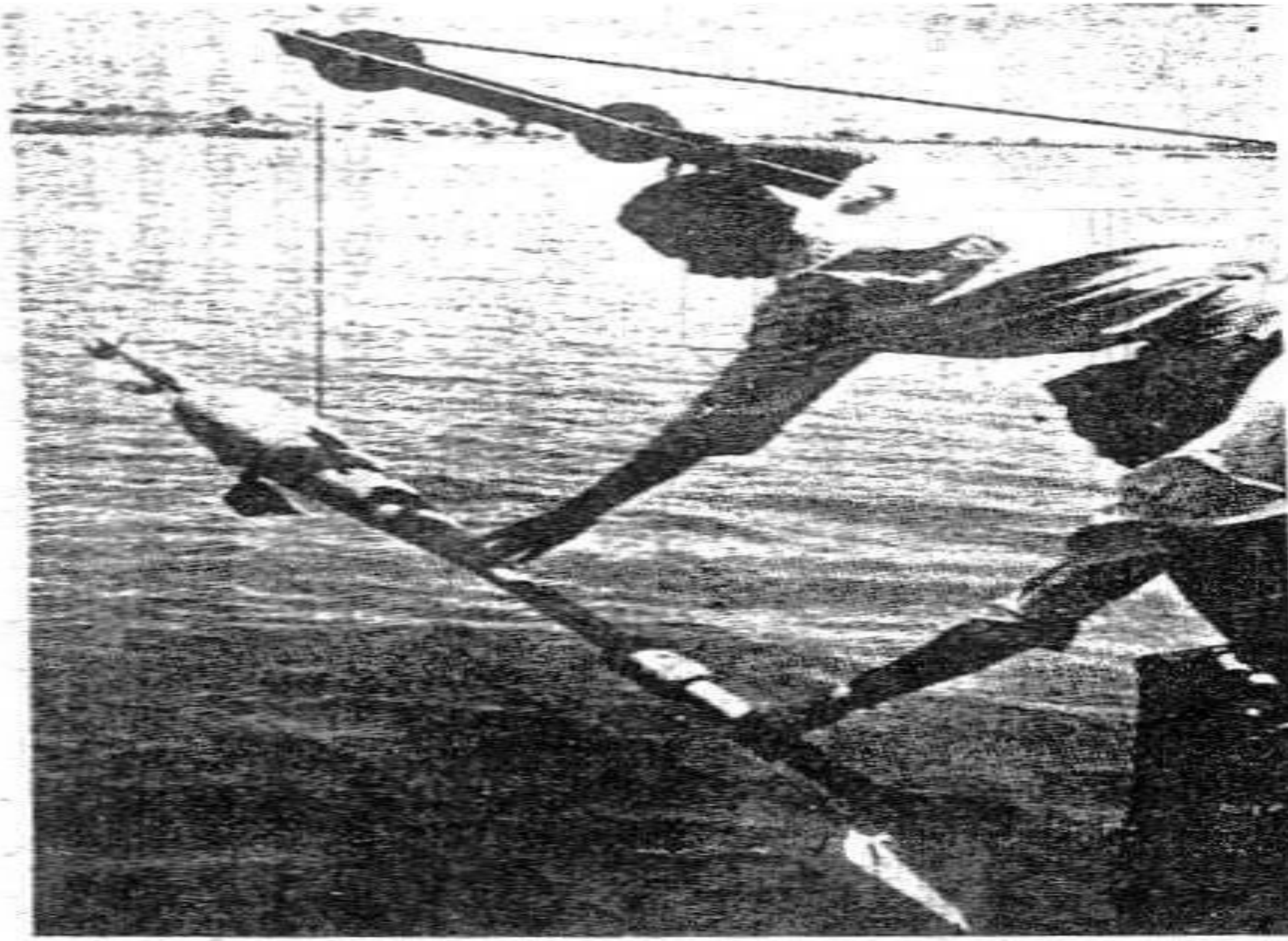


Fig. 4.10 Propeller-type current meter — Neyrtec type with sounding weight

A current meter is so designed that its rotation speed varies linearly with the stream velocity v at the location of the instrument. A typical relationship is

$$V = a N_s + b$$

Where v = stream velocity at the instrument location in m/s

Ns = revolutions per second of the meter and

a, b = constants of the meter

Sounding Weights

Current meter are weighted down by lead weights called sounding weights

$$W = 50 \, v \, d$$

Where w = minimum weight in N

v = average stream velocity in the vertical in m/s

d = depth of flow at the vertical in m

Velocity Measurement by Floats

A floating object on the surface of a stream when timed can yield the surface velocity by the relation

$$v_s = \frac{S}{t}$$

Where S = distance travelled in time t.

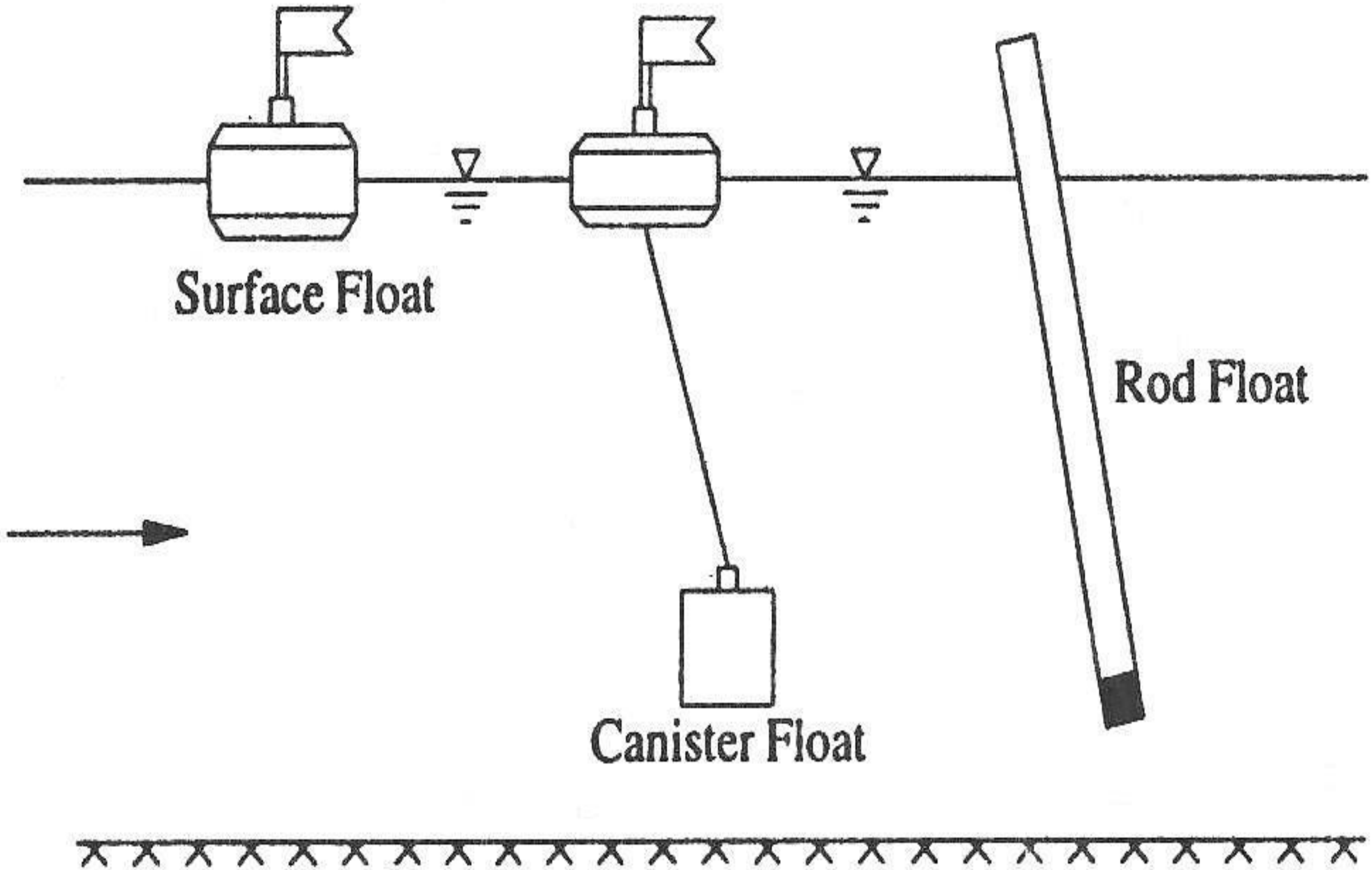


Fig. 4.13 Floats

Area – Velocity Method

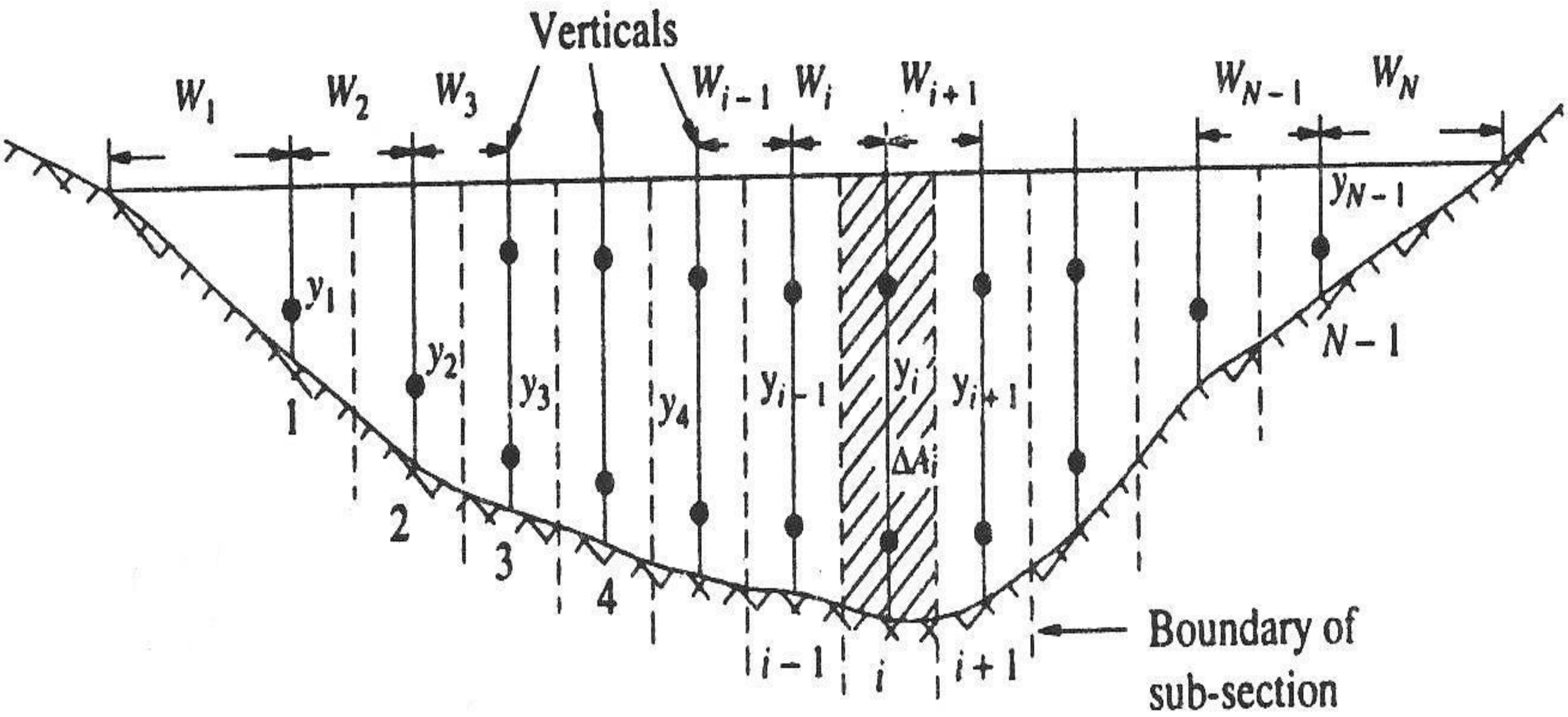


Fig. 4.14 Stream section for area-velocity method

Calculation of Discharge

Figure (4.14) shows the cross section of a river in which N–1 verticals are drawn. The velocity averaged over the vertical at each section is known. Considering the total area to be divided into N–1 segments, the total discharge is calculated by the *method of mid-sections* as follows:

$$Q = \sum_{i=1}^{N-1} \Delta Q_i$$

where ΔQ_i = discharge in the i th segment

= (depth at the ith segment) × ($\frac{1}{2}$ width to the left

+ $\frac{1}{2}$ width to right) × (average velocity at the ith vertical)

$$\Delta Q_i = y_i \times \left(\frac{W_i}{2} + \frac{W_{i+1}}{2} \right) \times v_i \qquad \text{for } i = 2 \text{ to } (N-2)$$

For the first and last sections, the segments are taken to have triangular areas and area calculated as

$$\Delta A_1 = \overline{W}_1 \, y_1$$

where $$\overline{W}_1 = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2\,W_1}$$

and $$\Delta A_N = \overline{W}_{N-1}\, y_{N-1}$$

where $$\overline{W}_{N-1} = \frac{\left(W_N + \frac{W_{N-1}}{2}\right)^2}{2\,W_N}$$

to get

$$\Delta Q_1 = \overline{v}_1 \,.\, \Delta A_1 \quad \text{and} \quad \Delta Q_{N-1} = \overline{v}_{N-1}\, \Delta A_{N-1}$$

EXAMPLE 4.1 *The data pertaining to a stream-gauging operation at a gauging site are given below.*
The rating equation of the current meter is $v = 0.51\, N_s + 0.03$ m/s
Calculate the discharge in the stream.

Distance from left water edge (m)	0	1.0	3.0	5.0	7.0	9.0	11.0	12.0
Depth (m)	0	1.1	2.0	2.5	2.0	1.7	1.0	0
Revolutions of a current meter kept at 0.6 depth	0	39	58	112	90	45	30	0
Duration of observation (s)	0	100	100	150	100	100	100	0

SOLUTION : The calculations are performed in a tabular form.
For the first and last sections,

$$\text{Average width,} \quad \overline{W} = \frac{\left(1+\frac{2}{2}\right)^2}{2 \times 1} = 2.0 \text{ m}$$

For the rest of the segments,

$$\overline{W} = \left(\frac{2}{2}+\frac{2}{2}\right) = 2.0 \text{ m}$$

Since the velocity is measured at 0.6 depth, the measured velocity is the average velocity at that vertical ($\bar{v}$).

The calculation of discharge by the mid-section method is shown in tabular form below:

Distance from left water edge (m)	Average width $\overline{W}$ (m)	Depth y (m)	Velocity $\bar{v}$ (m/s)	Segmental discharge ΔQ_i (m^3/s)
0	0	0	—	—
1	2.00	1.1	0.229	0.504
3	2.00	2.0	0.326	1.304
5	2.00	2.5	0.411	2.055
7	2.00	2.0	0.336	1.344
9	2.00	1.7	0.260	0.884
11	2.00	1.0	0.183	0.366
12	0	0		—
			$\Sigma \Delta Q_i =$	6.457

Total discharge $Q = 6.457\ m^3/s$

$$v_b = v_R \cos\theta \qquad \text{and} \qquad v_f = v_R \sin\theta$$

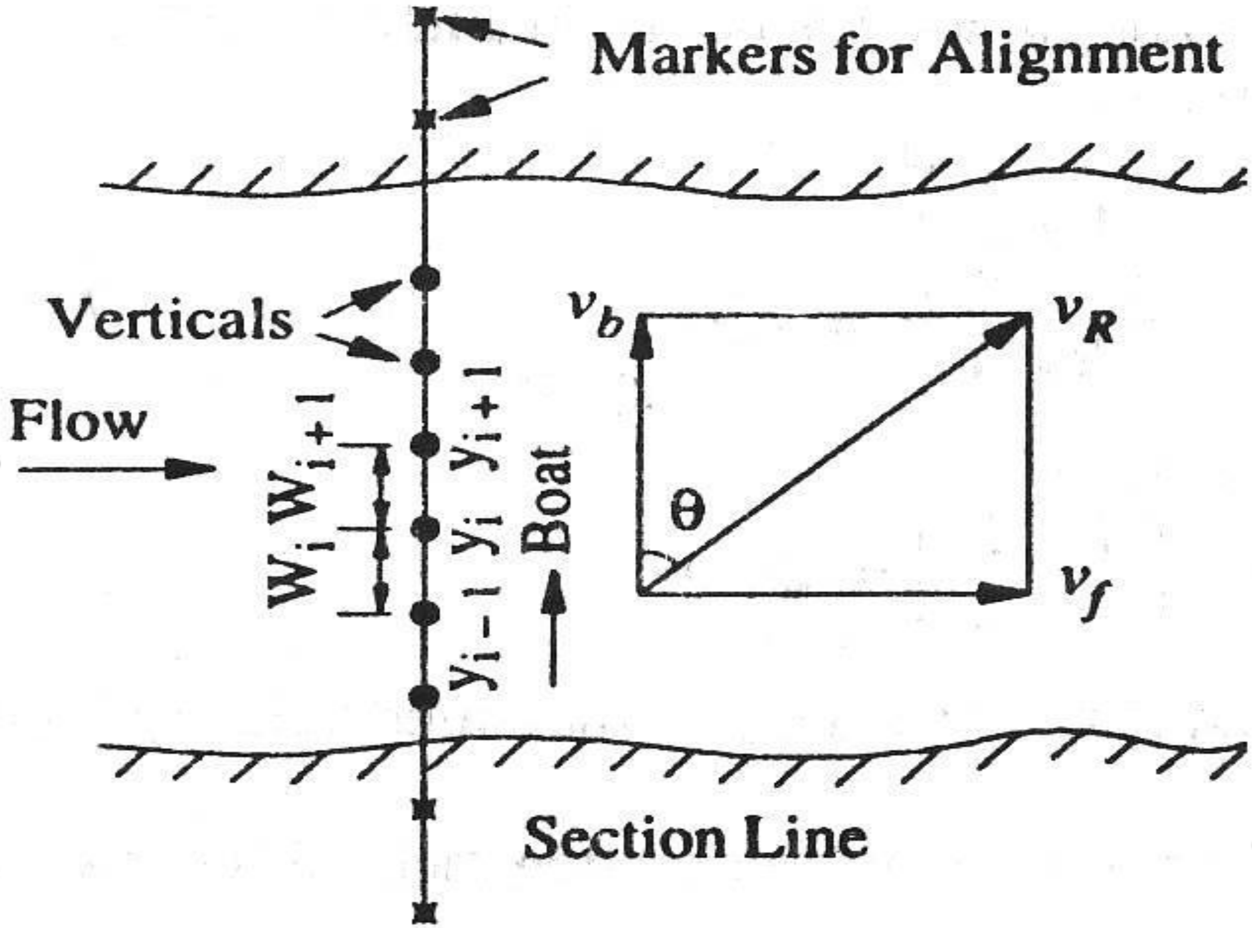


Fig. 4.15 Moving-boat method

If the time of transit between two verticals is Δt, then the width between the two verticals (Figure 4.15) is

$$W = v_b \Delta t$$

The flow in the sub-area between two verticals i and $i+1$ where the depths are y_i and y_{i+1} respectively, by assuming the current meter to measure the average velocity in the vertical, is

$$\Delta Q_i = \left(\frac{y_i + y_{i+1}}{2} \right) W_{i+1} v_f$$

i.e.

$$\Delta Q_i = \left(\frac{y_i + y_{i+1}}{2} \right) v_R^2 \sin \theta \cdot \cos \theta \cdot \Delta t$$

Thus by measuring the depths y_i, velocity v_R and θ in a reach and the time taken to cross the reach Δt, the discharge in the sub-area can be determined. The summation of the partial discharges ΔQ_i over the whole width of the stream gives the stream discharge

$$Q = \Sigma \Delta Q_i$$

Dilution Technique of Streamflow Measurement

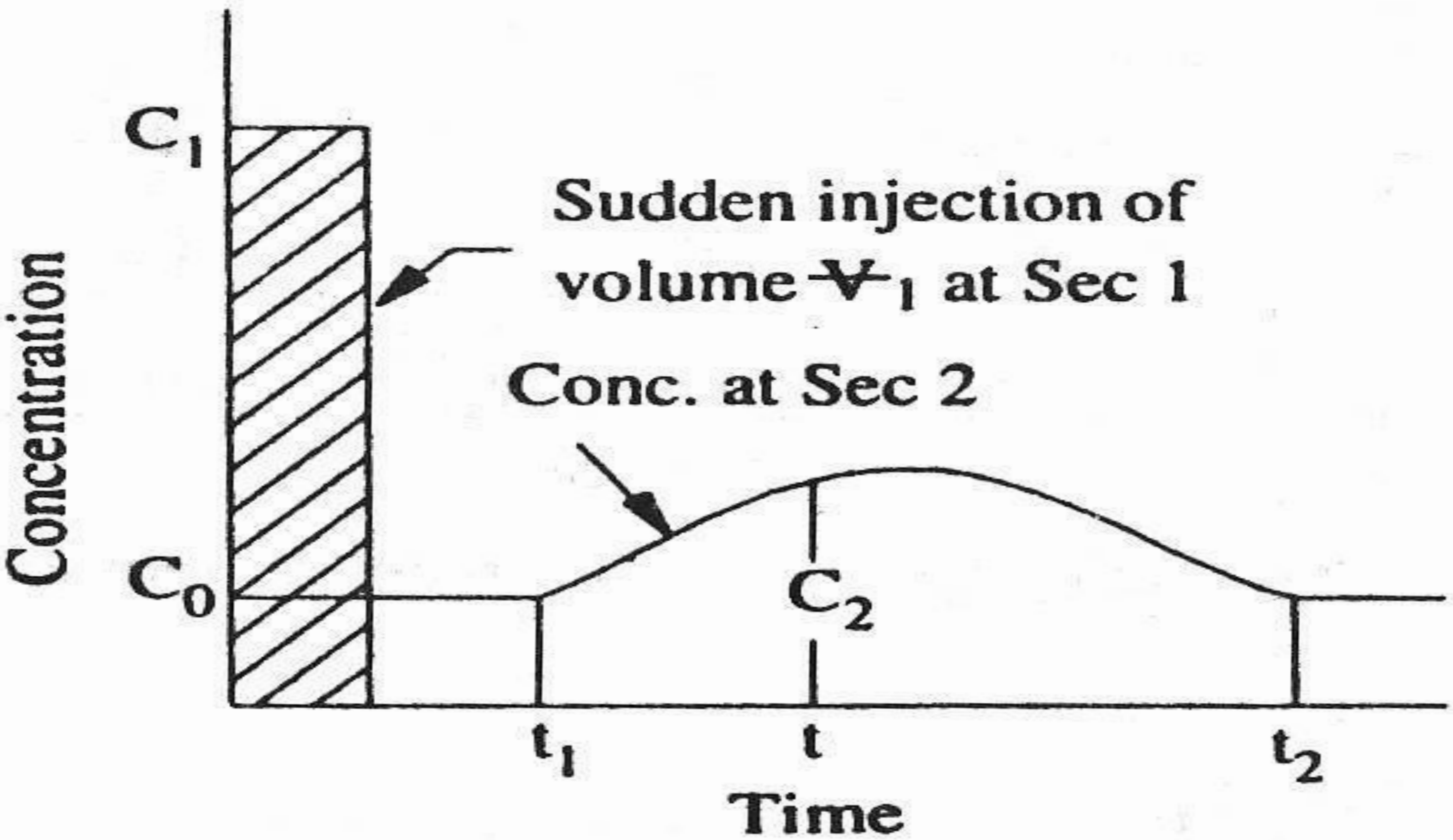


Fig. 4.16 Sudden-injection method

$$M_1 = \text{mass of tracer added at section } 1 = \forall_1 C_1$$

$$= \int_{t_1}^{t_2} Q\,(C_2 - C_0)\,dt + \frac{\forall_1}{t_2 - t_1} \int_{t_1}^{t_2} (C_2 - C_0)\,dt$$

Neglecting the second term on the right-hand side as insignificantly small,

$$Q = \frac{\forall_1 C_1}{\int_{t_1}^{t_2} (C_2 - C_0)\,dt}$$

Another way of using the dilution principle is to inject the tracer of concentration C_1 at a constant rate Q_t at section 1. At section 2, the concentration gradually rises from the background value of C_0 at time t_1 to a constant value C_2 (Fig. 4.17). At the stready state, the continuity equation for the tracer is

$$Q_t C_1 + Q C_0 = (Q + Q_t) C_2$$

i.e.
$$Q = \frac{Q_t (C_1 - C_2)}{(C_2 - C_0)}$$

This technique in which Q is estimated by knowing C_1, C_2, C_0 and Q_t is known as *constant rate injection method* or *plateau gauging.*

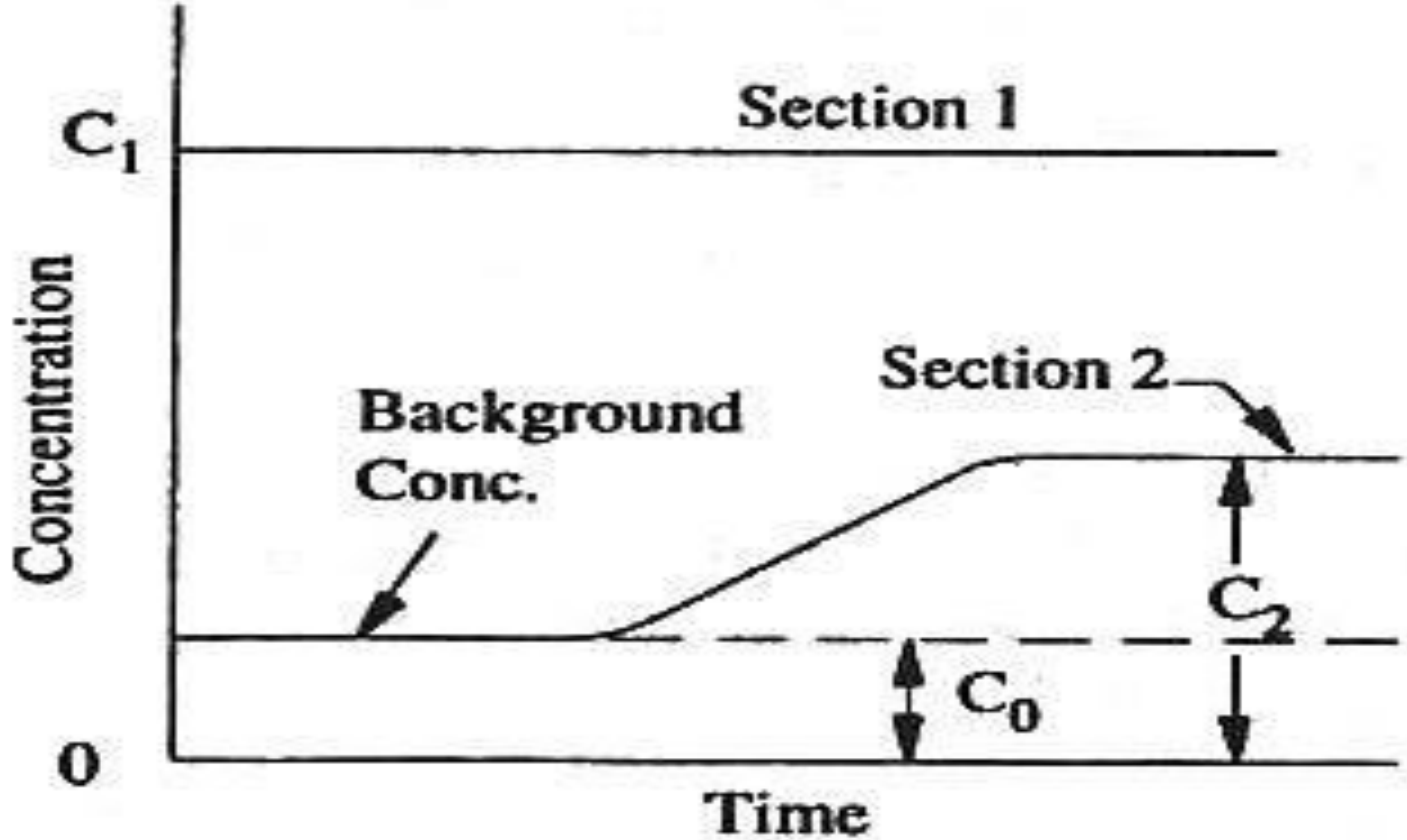


Fig. 4.17 Constant rate injection method