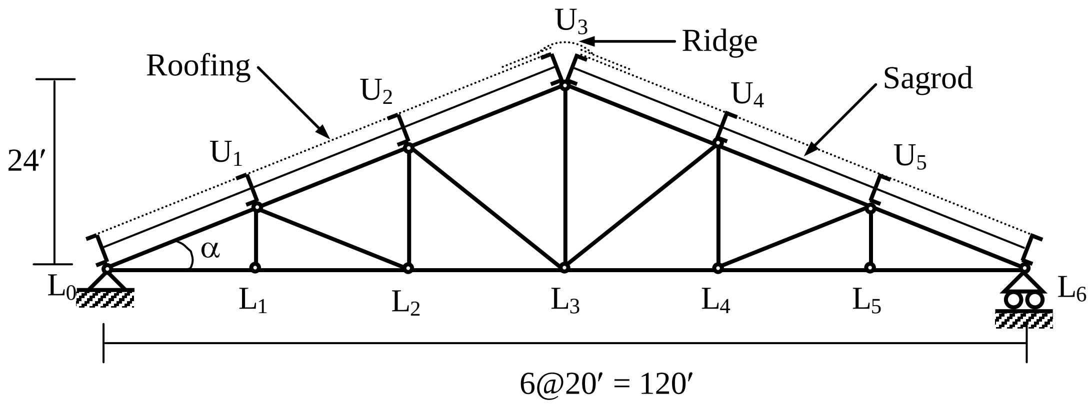
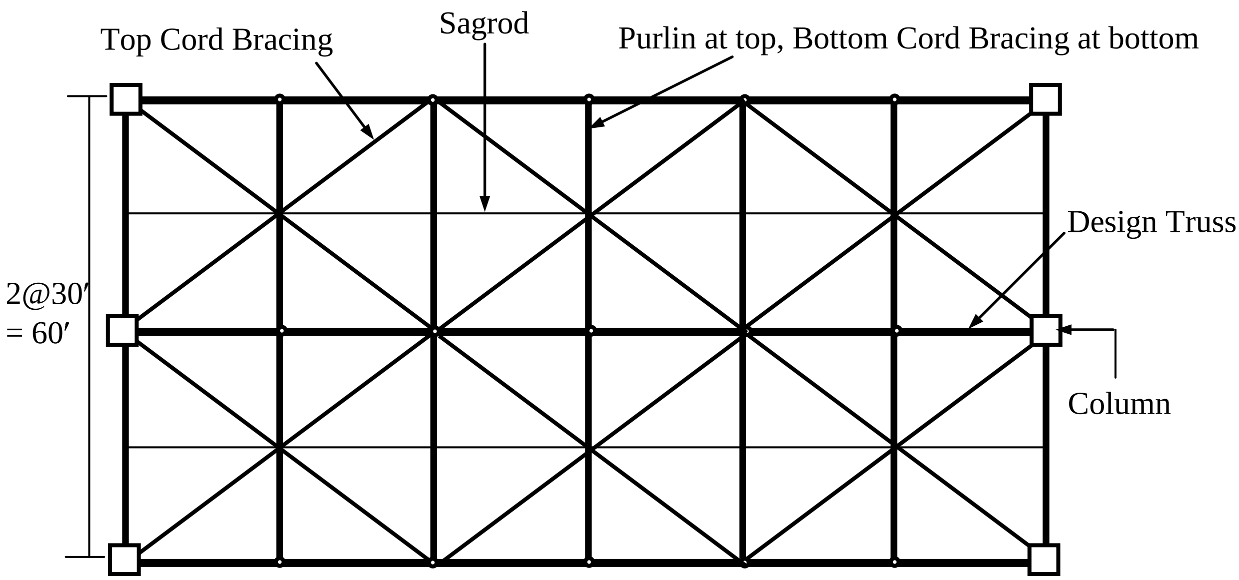
Design of an Industrial Truss



Elevation of the Truss



Building Plan

Total Span of the Truss = 6@20′ = 120′, Total Height of the Truss = 24′, Spacing = 30′

Pitch Angle $\alpha = \tan^{-1}(24/60) = 21.80°$

Dead Loads: Roofing = 2 psf, Purlins = 1.5 psf, Sagrods + Bracings = 1 psf

Basic Wind Speed V = 120 mph

Material Properties: $f'_c = 3$ ksi, $f_y = 40$ ksi

1. Design of Purlins and Sagrods

Purlin length = Spacing of truss = 30′

Dead Load:

GI Sheet roofing = 2.0 psf, Self weight of Purlins = 1.5 psf $\Rightarrow$ Total Load = 3.5 psf

Purlin Spacing S_p = Length of Top Cord $U_1U_2 = \sqrt{\{20^2 + (24/3)^2\}} = 21.54'$

$\therefore$ UDL on purlins, $w_{DL} = 3.5 \times S_p = 75.39$ lb/ft

$\therefore w_{DLx} = w_{DL} \sin\alpha = 28$ lb/ft, $w_{DLy} = w_{DL} \cos\alpha = 70$ lb/ft

Wind Load:

Basic wind speed, V = 120 mph

$\Rightarrow$ Basic wind pressure, $q = 0.00256\, V^2 = 0.00256 \times 120^2 = 36.86$ psf

$\therefore$ Wind pressure for windward surface, $p = -0.7q$, for $0 \le \alpha \le 20°$

$p = (0.07\alpha - 2.1)q$, for $20° \le \alpha \le 30°$

$p = (0.03\alpha - 0.9)q$, for $30° \le \alpha \le 60°$

$p = 0.9q$, for $60° \le \alpha \le 90°$ …………(1.1(a)~(d))

Wind pressure for leeward surface, $p = -0.7q$, for any value of α ……..………….(1.2)

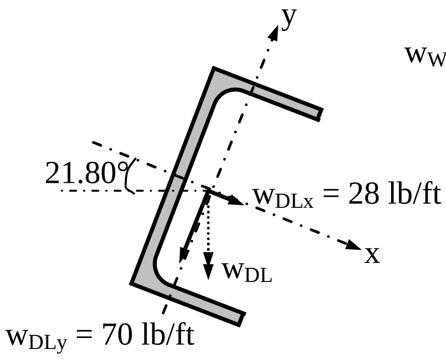
Here, pitch angle $\alpha = \tan^{-1}(24/60) = 21.80°$; i.e., $20° \le \alpha \le 30°$

$\therefore$ Wind pressure for windward surface, $p_{WW} = (0.07 \times 21.80 - 2.1) \times 36.86 = -21.16$ psf

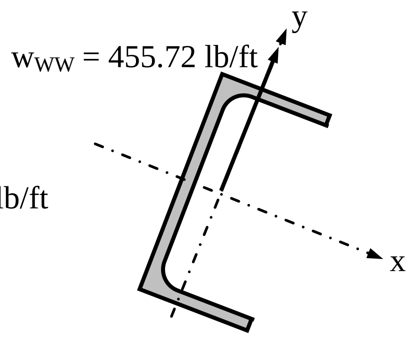
Wind pressure for leeward surface, $p_{LW} = -0.7 \times 36.86 = -25.80$ psf

$\therefore$ UDL on windward surface, $w_{WW} = 21.16 \times S_p = 455.72$ lb/ft

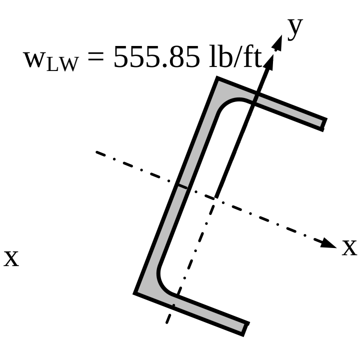
$\therefore$ UDL on leeward surface, $w_{LW} = 25.80 \times S_p = 555.85$ lb/ft



Dead Load



Windward Wind Load

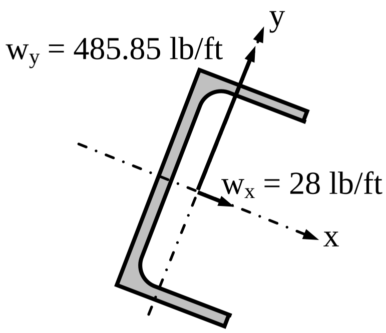


Leeward Wind Load

Load Combination and Biaxial Bending:

It is clear from the preceding analyses that leeward side is more critical for wind loading.

Therefore, the combination of dead load and leeward wind load provides the governing design condition for purlins.



$\therefore$ Combined UDL in y-direction, $w_y = 485.85$ lb/ft

Combined UDL in x-direction, $w_x = 28$ lb/ft

Span of the purlin is $S_t = 30'$

It is simply supported in y-direction

$\Rightarrow M_{xx} = w_y S^2/8 = 54.66\ k' = 655.90\ k''$, at midspan

Due to sagrod, it is 2-span continuous in x-direction

$\Rightarrow M_{yy} = w_x S^2/32 = 0.79\ k' = 9.45\ k''$, also at midspan

Design of Purlins:

The absolute maximum tensile/compressive due to biaxial bending is

$$\sigma_{zz} = M_{xx}/S_{xx} + M_{yy}/S_{yy} = 655.90/S_{xx} + 9.45/S_{yy} \quad \text{.........................(1.3)}$$

where S_{xx} and S_{yy} are the section moduli about the x- and y-axes.

Allowable bending stress $f_b = 0.66\ f_y = 26.4$ ksi

Table 1.1: Calculation for Optimum Purlin Section

Section	**S_{xx} (in^3)**	**S_{yy} (in^3)**	**σ_{zz} (ksi)**	**f_b (ksi)**	**Comments**
C3×4.1	1.10	0.202	643.06	26.4	Use much larger section
C12×30	27.0	2.06	28.88		Use slightly larger section
C15×33.9	42.0	3.11	18.66		OK

$\therefore$ The section C15×33.9 is chosen as the optimum purlin section.

Self weight of purlin = 33.9 lb/ft, i.e., 33.9/21.54 = 1.57 psf $\cong$ assumed 1.5 psf, OK.

Design of Sagrods:

The maximum axial force in the sagrods = (5/8) w_{DLx} S = (5/8)×28×30 = 525 lb = 0.525 k

The minimum size of the sagrods = 3/8″

$\therefore$ Allowing 1/16″ reduction for bolt threads, the rod area = $\pi(3/8-1/16)^2/4 = 0.077$ in^2

$\therefore$ Maximum stress $\sigma = 0.525/0.077 = 6.84$ ksi << Allowable $f_s = 20$ ksi

$\therefore$ The sagrod is chosen to have 3/8″-diameter, the weight of sagrods and bracings should be within the assumed limit of 1.0 psf.

2. Calculation of Point Dead Load, Wind Load and Truss Analysis

In this section, the uniformly distributed loads from the roof as well as from the wind are concentrated on the truss joints for subsequent analyses.

<u>Calculation of Point Dead Load</u>:

Total roof load including roofing, purlins, sagrods and bracings = 2.0 + 1.5 + 1.0 = 4.5 psf

Purlin Spacing $S_p = 21.54'$, Truss Spacing $S_t = 30'$

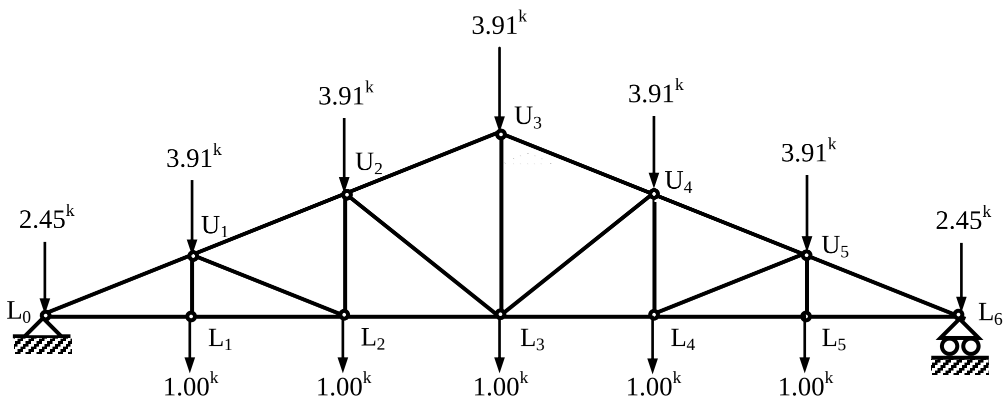
$\therefore$ Concentrated roof loads on truss joints = $4.5 \times S_p \times S_t/1000 = 2.91$ kips

Assumed self-weight of truss = 100 lb/ft = 0.10 k/ft of horizontal span, to be equally divided among the top and bottom cords.

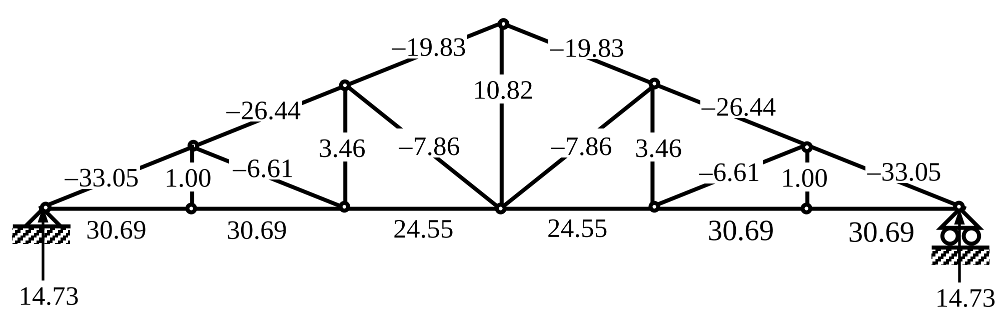
$\therefore$ Concentrated self-weight on truss joints = $0.10 \times 20/2 = 1.00$ kips

$\therefore$ Dead load on top joints, $P_{top} = 2.91 + 1.00 = 3.91$ kips, and on bottom joints, $P_{bot} = 1.00$ kips

Total dead load on end joints, $P_{end} = 2.91/2 + 1.00 = 2.45$ kips



Concentrated dead loads at truss joints



Member forces and support reactions (kips) due to dead loads

Wind Load:

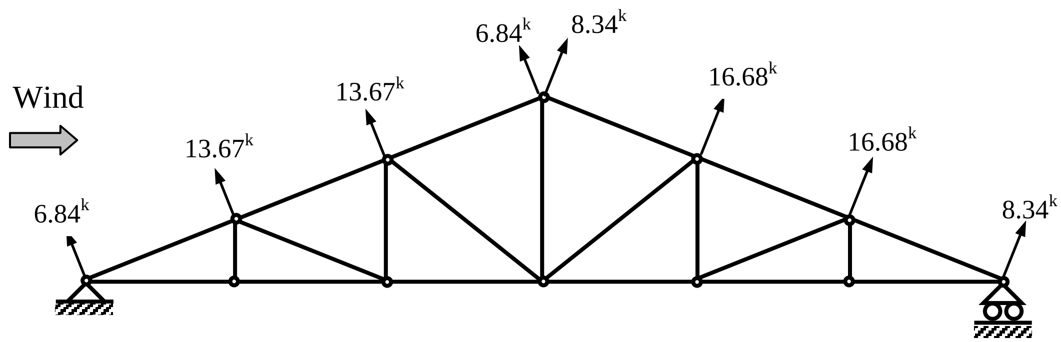
As calculated in the design of purlins and sagrods,

Wind pressure for windward surface = –21.16 psf, and for leeward surface = –25.80 psf

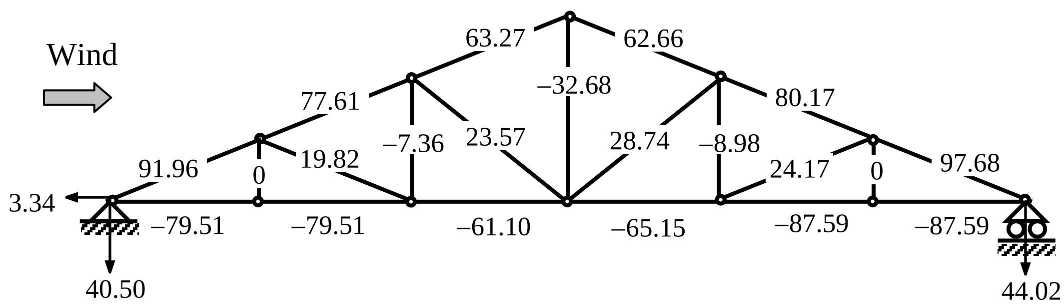
$\therefore$ Concentrated load (suction) on windward surface, $P_{WW} = 21.16 \times S_p \times S_t/1000 = 13.67$ kips

$\therefore$ Concentrated load (suction) on leeward surface, $P_{LW} = 25.80 \times S_p \times S_t/1000 = 16.68$ kips

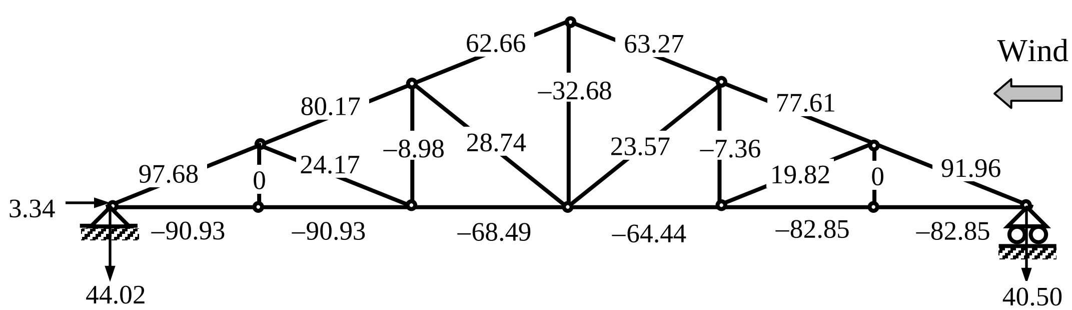
These are halved at end joints; i.e., $P_{WW(end)} = 6.84$ kips, and $P_{LW(end)} = 8.34$ kips



Concentrated wind (⟶) loads at truss joints



Member forces and support reactions (kips) due to wind (⟶) loads



Member forces and support reactions (kips) due to wind (⟵) loads

3. Load Combination and Design of Truss Members

The member forces and support reactions calculated earlier for point dead loads and wind loads are combined in this section to obtain the design member forces and support reactions.

Combination of Dead Load and Wind Load:

The calculation for the design forces is best carried out in a tabular form as shown below.

Table 3.1: Design Member Force Chart

Member Type	Member	Length (ft)	Member Forces (kips)			Design Force (kips)	
			Dead Load	Wind Load (→)	Wind Load (←)	Tension	Compression
Bottom Cord Members	L_0L_1	20.00	30.69	−79.51	−90.93	30.69	−60.24
	L_1L_2	20.00	30.69	−79.51	−90.93	30.69	−60.24
	L_2L_3	20.00	24.55	−61.10	−68.49	24.55	−43.94
	L_3L_4	20.00	24.55	−65.15	−64.44	24.55	−40.60
	L_4L_5	20.00	30.69	−87.59	−82.85	30.69	−56.90
	L_5L_6	20.00	30.69	−87.59	−82.85	30.69	−56.90
Top Cord Members	L_0U_1	21.54	−33.05	91.96	97.68	64.63	−33.05
	U_1U_2	21.54	−26.44	77.61	80.17	53.73	−26.44
	U_2U_3	21.54	−19.83	63.27	62.66	43.44	−19.83
	U_3U_4	21.54	−19.83	62.66	63.27	43.44	−19.83
	U_4U_5	21.54	−26.44	80.17	77.61	53.73	−26.44
	U_5L_6	21.54	−33.05	97.68	91.96	64.63	−33.05
Vertical Members	U_1L_1	8.00	1.00	0.00	0.00	1.00	*
	U_2L_2	16.00	3.46	−7.36	−8.98	3.46	−5.52
	U_3L_3	24.00	10.82	−32.68	−32.68	10.82	−21.86
	U_4L_4	16.00	3.46	−8.98	−7.36	3.46	−5.52
	U_5L_5	8.00	1.00	0.00	0.00	1.00	*
Diagonal Members	U_1L_2	21.54	−6.61	19.82	24.17	17.56	−6.61
	U_2L_3	25.61	−7.86	23.57	28.74	20.88	−7.86
	U_4L_3	25.61	−7.86	28.74	23.57	20.88	−7.86
	U_5L_4	21.54	−6.61	24.17	19.82	17.56	−6.61

Design Concept of Truss Members:

The design of truss members is carried out using the following equations.

Members under tension:

$\sigma_t = F_t/A$

$\sigma_{all(t)} = 0.5\ f_y$ ……………………….(3.1(a)~3.1(b))

The acceptable design condition is $\sigma_t \leq \sigma_{all(t)}$

Members under compression:

$\sigma_c = F_c/A$

Slenderness Ratio, $\lambda = L_e/r_{min}$, and $\lambda_c = \pi\sqrt{(2E/f_y)}$

If $\lambda \leq \lambda_c$, $\sigma_{all(c)} = f_y\ [1–0.5\ (\lambda/\lambda_c)^2]/[5/3 + 3/8\ (\lambda/\lambda_c) –1/8(\lambda/\lambda_c)^3]$

If $\lambda > \lambda_c$, $\sigma_{all(c)} = 0.52\ (\pi^2\ E/\lambda^2)$ ……………………….(3.2(a)~3.2(d))

The acceptable design condition is $\sigma_c \leq \sigma_{all(c)}$

Here F_t = Tensile force, F_c = Compressive force, E = Modulus of elasticity

A = Cross-sectional area, L_e = Effective length of member, r_{min} = Minimum radius of gyration

σ_t = Tensile stress, σ_c = Compressive stress, σ_{all} = Allowable stress

For the material properties used for design; i.e., E = 29000 ksi, f_y = 40 ksi

$\sigma_{all(t)} = 0.5\ f_y = 20$ ksi, $\lambda_c = \pi\sqrt{(2E/f_y)} = 119.63$, $\rho = \lambda/\lambda_c = \lambda/119.63$

If $\rho \leq 1$, $\sigma_{all(c)} = 40\ [1–0.5\rho^2]/[5/3 + 3\rho/8 –\rho^3/8]$

If $\rho > 1$, $\sigma_{all(c)} = 149000/\lambda^2 = 10.4/\rho^2$ ……………………….(3.3(a)~3.3(d))

Rather than designing for all the members individually, only one section will be chosen for all the bottom cord members, one section for all the top cord members and one section for all the other (vertical and diagonal; i.e., 'web') members.

Since the design truss has a long span, design sections will be chosen from double angle sections rather than single angle sections, which are chosen for smaller trusses.

Design of Bottom Cord Members:

For the bottom cord members (L_0L_1~ L_5L_6), the maximum tensile force = 30.69 kips and the maximum compressive force = 60.24 kips. Since all the bottom cord members are of equal length (i.e., if effective length factor k is taken = 1, then L_e = 20 ft = 240 inch), the maximum forces are taken as the design forces.

Although the design compressive force is much larger and is likely to be governing condition in this case, the sections are designed for both tension and compression for completeness of design.

Table 3.2: Bottom Cord Member Design Chart

Section	**A (in^2)**	**σ_t (ksi)**	**$\sigma_{all(t)}$ (ksi)**	**σ_c (ksi)**	**r_{min} (in)**	**λ**	**ρ**	**$\sigma_{all(c)}$ (ksi)**	**Comments**
2½×2½×1/2	4.50	6.82	20	13.39	0.74	324.32	2.71	1.41	Use much larger section
4×4×3/4	10.88	2.82		5.54	1.19	201.68	1.69	3.66	Use larger section
5×5×7/8	15.96	1.92		3.77	1.49	161.07	1.35	5.74	Use smaller section
5×5×5/8	11.72	2.62		5.14	1.52	157.89	1.32	5.97	OK

Design of Top Cord Members:

For the top cord members (L_0U_1~ U_5L_6), the maximum tensile force = 64.63 kips and the maximum compressive force = 33.05 kips. Since all the top cord members are of equal length (i.e., if k = 1, L_e = 21.54 ft = 258.48 inch), the maximum forces are taken as the design forces.

The sections are designed for both tension and compression.

Table 3.3: Top Cord Member Design Chart

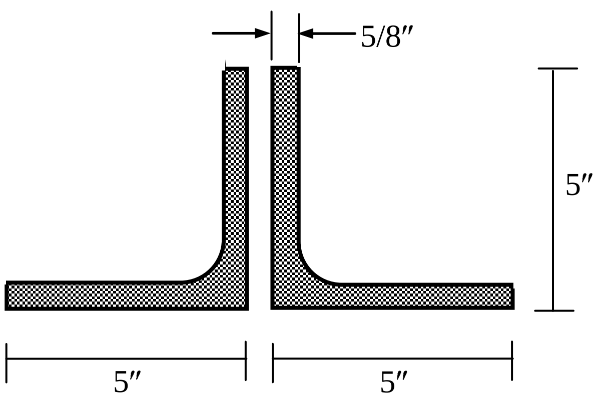
Section	**A (in^2)**	**σ_t (ksi)**	**$\sigma_{all(t)}$ (ksi)**	**σ_c (ksi)**	**r_{min} (in)**	**λ**	**ρ**	**$\sigma_{all(c)}$ (ksi)**	**Comments**
2½×2½×1/2	4.50	14.36	20	7.34	0.74	349.30	2.92	1.22	Use much larger section
4×4×5/8	9.22	7.01		3.58	1.20	215.40	1.80	3.21	Use larger section
4×4×3/4	10.88	5.94		3.04	1.19	217.21	1.82	3.15	OK

Design of Vertical and Diagonal Members:

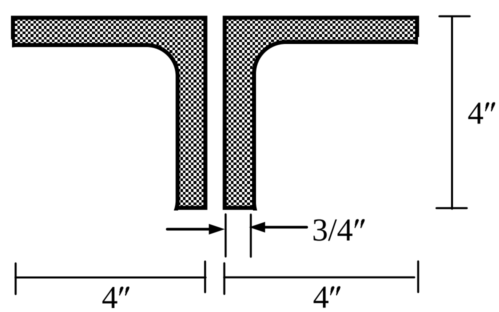
For the vertical and diagonal members (U_1L_1~ U_5L_5, U_1L_2~ U_5L_4), the maximum tensile force = 20.88 kips and the maximum compressive force = 21.86 kips. The maximum tension acts on U_2L_3 and U_4L_3 while the maximum compression acts on U_3L_3, which is 24 ft long. Although U_2L_3 and U_4L_3 are slightly longer (i.e., 25.61 ft long), the maximum compressive forces on them are much smaller (i.e., 7.86 kips). Therefore the effective length of the design members is taken as L_e = 24 ft = 288 inch. The sections are designed for both tension and compression although it is likely to be governed by compression.

Table 3.4: Web Member Design Chart

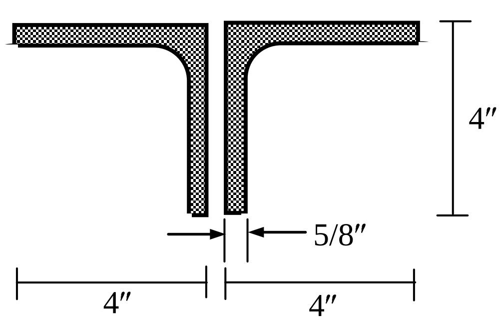
Section	**A (in^2)**	**σ_t (ksi)**	**$\sigma_{all(t)}$ (ksi)**	**σ_c (ksi)**	**r_{min} (in)**	**λ**	**ρ**	**$\sigma_{all(c)}$ (ksi)**	**Comments**
2½×2½×1/2	4.50	4.64		4.86	0.74	389.19	3.25	0.98	Use much larger section
4×4×1/2	7.50	2.78	20	2.91	1.22	236.07	1.97	2.67	Use larger section
4×4×5/8	9.22	2.26		2.37	1.20	240.00	2.01	2.58	OK



Bottom Chord Members



Top Chord Members



Web Members

4. Design of Bracings and Connections

The truss members designed in the previous section are supported against out-of-plane loads by several bracings, joined to each other by welded plate connections and connected to column or wall supports. This section discusses the design of these so-called 'non-structural members'.

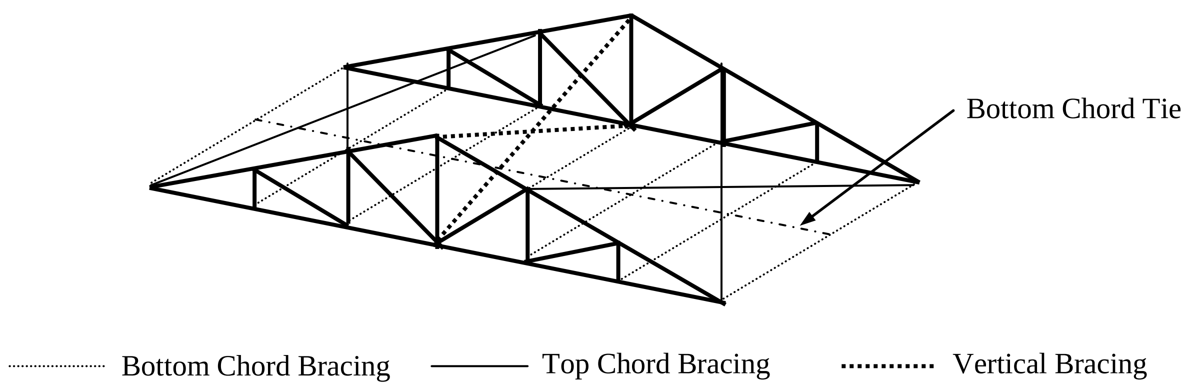
Design of Bracings:

The bracings connect joints of two successive trusses in order to provide structural support against out-of-plane loadings. The bracing system used for the truss illustrated here is shown below, consisting of three types of bracings; i.e.,

(i) Bottom cord bracings connect the corresponding bottom joints (e.g., L_0 with L_0)

(ii) Top cord bracings connect the bottom joints (e.g., L_0) with top joints (e.g., U_2) diagonally

(iii) Vertical bracings connect the bottom joints (e.g., L_3) with top joints (e.g., U_3) vertically



Since the structural analysis of the bracing system is complicated, the design follows simplified guidelines, according to which the slenderness ratio (L_e/r_{min}) ≤ 400 for bracings under tension and ≤ 300 for bracings under compression. In the absence of accurate calculations, the more conservative second criterion (i.e., $r_{min} \geq L_e/300$) is chosen for design here.

The design (using single equal angles) is best carried out in a tabular form as shown below.

Table 4.1: Design Chart for Bracings

	Length, L (ft)	**Effective Length, L_e (in)**	**r_{min} (in)**	**Chosen Section**
BC Bracing	30	$(0.7 \times 30/2 \times 12 =)$ 126	0.42	L2½×2½×3/16
TC Bracing	$\{\sqrt{(30^2 + 43.08^2)} =\}$ 52.50	$(0.7 \times 52.50/2 \times 12 =)$ 220.5	0.73	L4×4×1/4
V Bracing	$\{\sqrt{(30^2 + 24^2)} =\}$ 38.42	$(0.7 \times 38.42/2 \times 12 =)$ 161.4	0.54	L3×3×3/16

Design of Connections:

The joints provided here are actually gusset plates joining two or more members with welded connections. The design is based on the following material and structural properties

Allowable shear stress $f_v = 0.3\ f_y = 0.3 \times 40 = 12$ ksi

The gusset plate should be designed to resist the maximum possible design load condition. In this case, however, the thickness of the plate is approximately estimated based on the maximum axial force 64.63 kips. If allowable tensile stress $= 0.5\ f_y = 0.5 \times 40 = 20$ ksi, and plate thickness = 0.25″, then the maximum required width of the plate = $64.63/(20\times0.25) \cong 13''$. This should not be too large, based on the weld-lengths calculated subsequently.

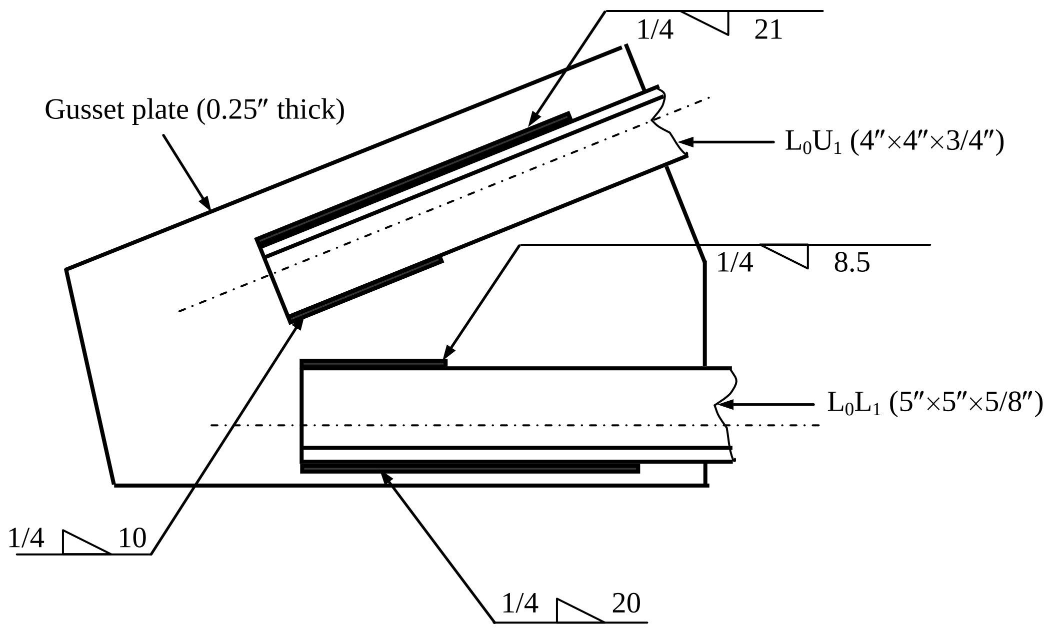
$\therefore$ Thickness of gusset plate is chosen tentatively as = 0.25″ and thickness of weld t = 0.25″

$\therefore$ The length of weld, $L_w = P/(f_v \times 0.707t) = P/(12 \times 0.707\times 0.25) = P/2.12$

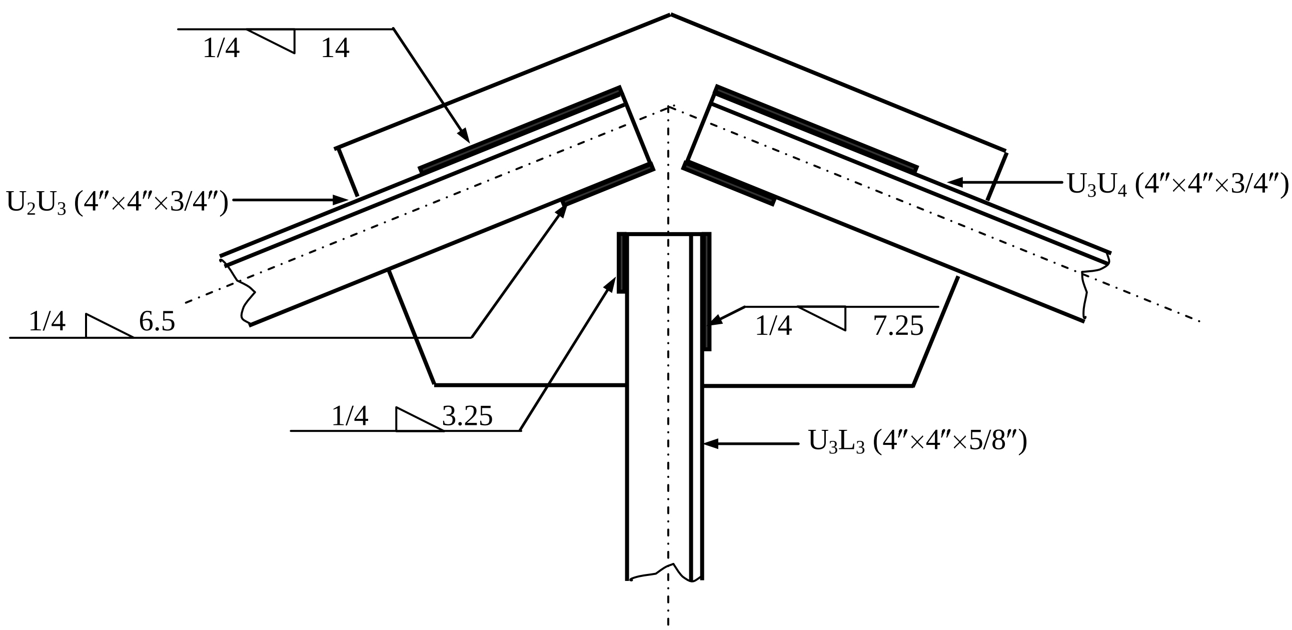
Table 4.2: Weld Design Chart

Member Type	**Member**	**Member Size**	**y : L–y (in)**	**Design Force (kips)**	**Weld Length (in)**	**Weld Length Ratio (in)**
Bottom Cord Member	L_0L_1, L_5L_6*	5×5×5/8	1.48: 3.52	60.24	28.40	≅ 20.00, ≅ 8.50
	L_1L_2, L_4L_5*			60.24	28.40	20.00, 8.50
	L_2L_3, L_3L_4*			43.94	20.71	15.00, 6.50
Top Cord Member	L_0U_1, U_5L_6	4×4×3/4	1.27: 2.73	64.63	30.47	21.00, 10.00
	U_1U_2, U_4U_5			53.73	25.33	17.50, 8.50
	U_2U_3, U_3U_4			43.44	20.48	14.00, 6.50
Vertical Member	U_1L_1, U_5L_5	4×4×5/8	1.23: 2.77	1.00	0.47	0.5, 0.25
	U_2L_2, U_4L_4			5.52	2.60	2.00, 1.00
	U_3L_3			21.86	10.30	7.25, 3.25
Diagonal Member	U_1L_2, U_5L_4			6.61	3.12	2.25, 1.00
	U_2L_3, U_4L_3			7.86	3.71	2.75, 1.25

[* Design forces assumed equal]



Joint L_0



Joint U_3

5. Design of Anchorage and Support

The truss is supported by reinforced concrete columns and footings, their reactions having been calculated earlier for point dead load and wind loads. The connections between the truss and support are designed in this section for the combined design loads.

Combination of Support Reactions from Dead Load and Wind Load:

The calculation for the design support reactions is carried out in the following tabular form.

Table 5.1: Design Support Reaction Chart

Support	Support Reactions (kips)			Design Forces (kips)		
	Dead Load	**Wind Load (→)**	**Wind Load (←)**	**Case1**	**Case2**	**Case3**
L_0	14.73 ↑	40.50 ↓ 3.34 ←	44.02 ↓ 3.34 →	14.73 (C)	25.77 (T) 3.34 (S)	29.29 (T) 3.34 (S)
L_6	14.73 ↑	44.02 ↓	40.50 ↓	14.73 (C)	29.29 (T)	25.77 (T)

Therefore the design conditions are summarized as follows

1. Compressive force = 14.73 kips
2. Tensile Force = 29.29 kips, Shear Force = 3.34 kips

Design of Base plate and Anchorage:

Since the truss is supported on base plates on concrete pedestals supported by masonry columns, the design in this study deals mainly with the connections between the truss and the columns.

The column forces are nominal, therefore a 10″×20″ masonry column is chosen.

∴ The maximum tensile stress on the column = 29.29/(10×20) = 0.146 ksi, which is within the allowable limit (Tensile strength ≅ 300 psi).

Assuming the base plate area = A_p and bearing pressure = 0.35 f_c' = 1.05 ksi

$$1.05A_p = 14.73 \Rightarrow A_p = 14.73/1.05 \Rightarrow A_p = 14.03 \text{ in}^2$$

∴ Provide 7″×14″ base plate (since the bottom cord members are 5″ + 5″ wide)

Since the free portion of the base plate is nominal, a thickness of 0.5″ is more than adequate.

The base plate is supported on a 10″×20″ concrete pedestal and connected to the column by four reinforcements to resist the entire tensile and shear force.

Allowable tensile stress = 0.5 f_y = 20 ksi and allowable shear stress = 0.3 f_y = 12 ksi

$\therefore$ Required area (based on tensile force) = 29.29/(4× 20) = 0.366 in^2

Required area (based on shear force) = 3.34/(4 × 12) = 0.07 in^2

$\therefore$ Provide 4 #6 (i.e., 3/4″ diameter) anchor bolts (Area = 0.44 in^2 each).

$\therefore$ Allowable tensile force per anchor = 0.44 × 20 = 8.8 kips

$\therefore$ Allowable bond force per unit length = $35\sqrt{f_c'}$ = $35\sqrt{3000}$ lb/in = 1.92 k/in

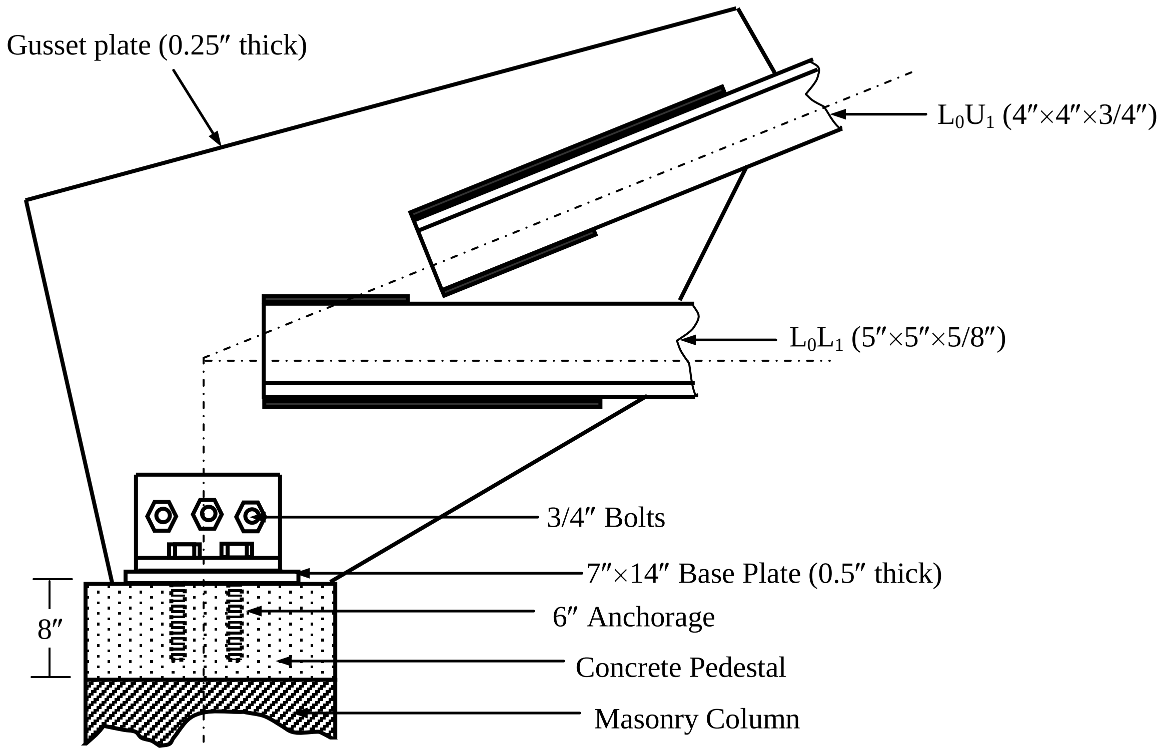
$\Rightarrow$ Development length = 8.8/1.92 = 4.59″

$\therefore$ Provide anchorage of 6″ for each bolt.

The base plate will be connected to the gusset plate by the section similar to the bottom cord (i.e., a 5″×5″×5/8″ double angle section), also with 3/4″ diameter bolts to transfer the maximum support reaction (= 29.29 kips) by shear.

$\therefore$ Required area = 29.29/12 = 2.44 in^2, i.e., provide 3-3/4″ diameter bolts in double shear.

Hinge Support L_0



Roller Support L_6