Unit Hydrograph

A unit hydrograph is defined as the hydrograph of direct runoff resulting from one unit depth (1cm) of rainfall excess occurring uniformly over the basin and at a uniform rate for a specified duration (D hours).

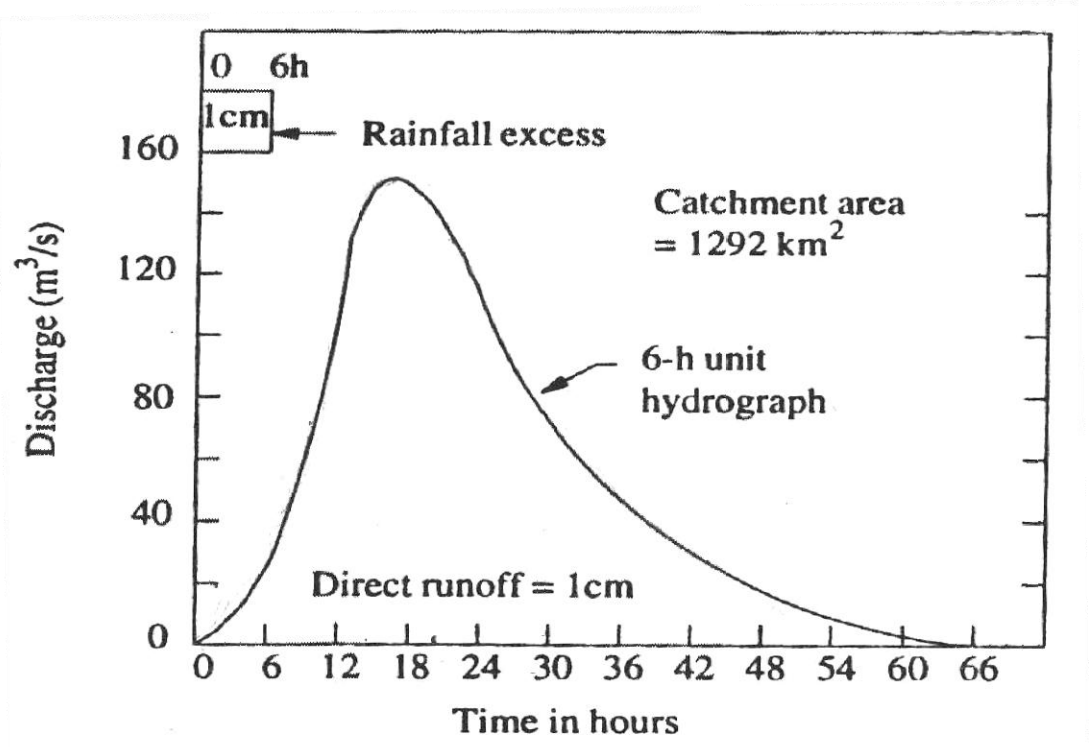


Fig. 6.9 Typical 6–h unit hydrograph

Two basic assumptions of unit hydrograph

(i) the time invariance

(ii) the linear response

EXAMPLE 6.4 *Given below are the ordinates of a 6-h unit hydrograph for a catchment. Calculate the ordinates of the DRH due to a rainfall excess of 3.5 cm occurring in 6 hr.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48	54	60	69
UH ordinate (m^3/s)	0	25	50	85	125	160	185	160	110	60	36	25	16	8	0

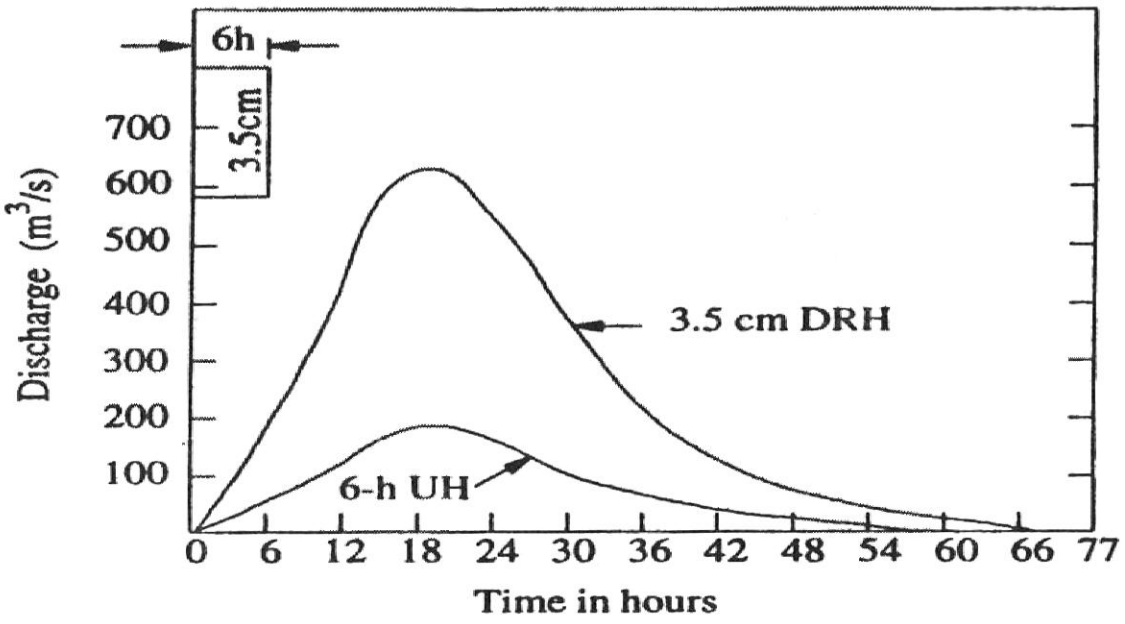


Fig. 6.10 (a) 3.5 cm DRH derived from 6–h unit hydrograph—Example 6.4

TABLE 6.3 CALCULATION OF DRH DUE TO 3.5 cm ER—*EXAMPLE 6.4*

Time (h)	Ordinate of 6-h unit hydrograph (m^3/s)	Ordinate of 3.5 cm DRH (m^3/s)
1	2	3
0	0	0
3	25	87.5
6	50	175.0
9	85	297.5
12	125	437.5
15	160	560.0
18	185	647.5
24	160	560.0
30	110	385.0
36	60	210.0
42	36	126.0
48	25	87.5
54	16	56.0
60	8	28.0
69	0	0

EXAMPLE 6.5 *Two storms each of 6-h duration and having rainfall excess values of 3.0 and 2.0 cm respectively occur successively. The 2-cm ER rain follows the 3-cm rain. The 6-h unit hydrograph for the catchment is the same as given in Example 6.4. Calculate the resulting DRH.*

TABLE 6.4 CALCULATION OF DRH BY METHOD OF SUPERPOSITION — *EXAMPLE 6.5*

Time (h)	Ordinate of 6-h UH (m^3/s)	Ordinate of 3-cm DRH (col.2) × 3	Ordinate of 2-cm DRH (col.2 lagged by 6 h) × 2	Ordinate of 5-cm DRH (col. 3 + col.4) (m^3/s)	Remarks
1	2	3	4	5	6
0	0	0	0	0	
3	25	75	0	75	
6	50	150	0	150	
9	85	255	50	305	
12	125	375	100	475	
15	160	480	170	650	
18	185	555	250	805	
(21)	(172.5)	(517.5)	(320)	(837.5)	Interpolated value
24	160	480	370	850	
30	110	330	320	650	
36	60	180	220	400	
42	36	108	120	228	
48	25	75	72	147	
54	16	48	50	98	
60	8	24	32	56	
(66)	(2.7)	(8.1)	(16)	(24.1)	Interpolated value
69	0	0	(10.6)	(10.6)	Interpolated value
75	0	0	0	0	

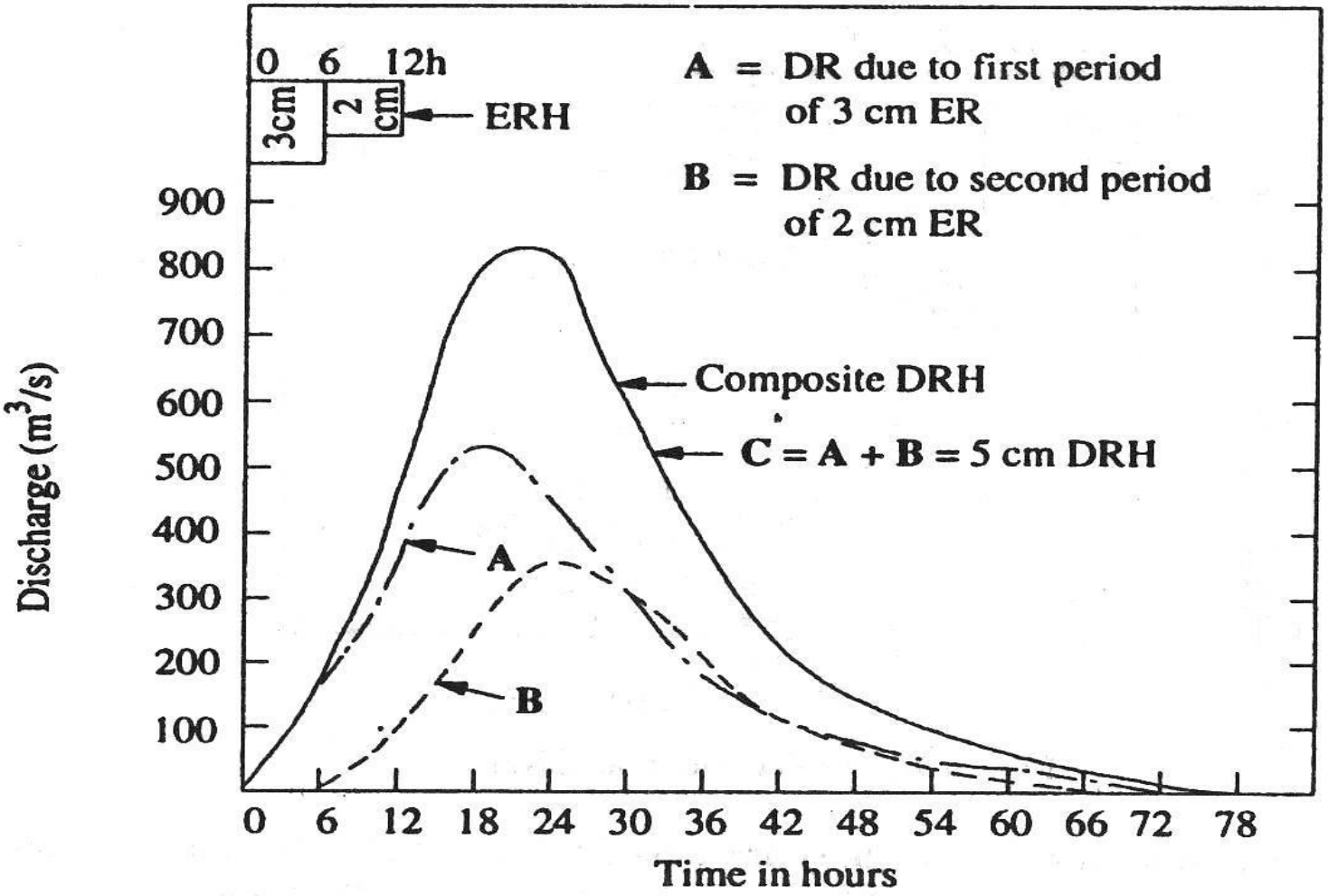


Fig. 6.10 (b) Principle of superposition—Example 6.5

Application of unit hydrograph

using the basic principles of the unit hydrograph, one can easily calculate the DRH in a catchment due to a given storm if an appropriate unit hydrograph was available

EXAMPLE 6.6 *The ordinates of a 6- hour unit hydrograph of a catchment is given below.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48
Ordinate of UH	0	25	50	85	125	160	185	160	110	60	36	25

Time	54	60	69
Ordinate of 6-h UH	16	8	0

Derive the flood hydrograph in the catchment due to the storm given below:

Time from start of storm (h)	0	6	12	18
Accumulated rainfall (cm)	0	3.5	11.0	16.5

The storm loss rate (ϕ – index) for the catchment is estimated as 0.25 cm/h. The base flow can be assumed to be 15 m^3/s at the beginning and increasing by 2.0 m^3/s for every 12 hours till the end of the direct–runoff hydrograph.

at a given time interval added. The base flow is then added to obtain the flood hydrograph shown in Col. 8, Table 6.6.

Interval	1st 6 hours	2nd 6 hours	3rd 6 hours
Rainfall depth (cm)	3.5	(11.0-3.5) = 7.5	(16.5-11.0) = 5.5
Loss @ 0.25 cm/h for 6 h	1.5	1.5	1.5
Effective rainfall (cm)	2.0	6.0	4.0

TABLE 6.5 CALCULATION OF FLOOD HYDROGRAPH DUE TO A KNOWN ERH—*EXAMPLE 6.6*

Time	Ordinaters of U.H.	DRH due to 2 cm ER Col. 2 × 2.0	DRH due to 2 cm ER × 6.0 (Advanced by 6 h)	DRH due to 4 cm ER Col.2 × 4.0 (Advanced by 12 h)	Ordinates of final DRH (Col 3+ 4+5)	Base flow (m^3/s)	Ordinates of flood hydrograph (m^3/s) (Col. 6 + 7)
1	2	3	4	5	6	7	8
0	0	0	0	0	0	15	15
3	25	50	0	0	50	15	65
6	50	100	0	0	100	15	115
9	85	170	150	0	320	15	335
12	125	250	300	0	550	17	567
15	160	320	510	100	930	17	947
18	185	370	750	200	1320	17	1337
(21)	(172.5)	(345)	960	340	1645	(17)	1662
24	160	320	1110	500	1930	19	1949
(27)	(135)	(270)	(1035)	640	1945	19	1964
30	110	220	960	740	1920	19	1939
36	60	120	660	640	1420	21	1441
42	36	72	360	440	872	21	893
48	25	50	216	240	506	23	529
54	16	32	150	144	326	23	349
60	8	16	96	100	212	25	237
66	(2.7)	(5.4)	48	64	117	25	142
69	0	0	—	—	—	—	—
72		0	16	32	48	27	75
75		0	0	—	—	—	—
78		0	0	(10.8)	(11)	27	49
81				0	0	27	27
84						27	27

Derivation of Unit Hydrograph

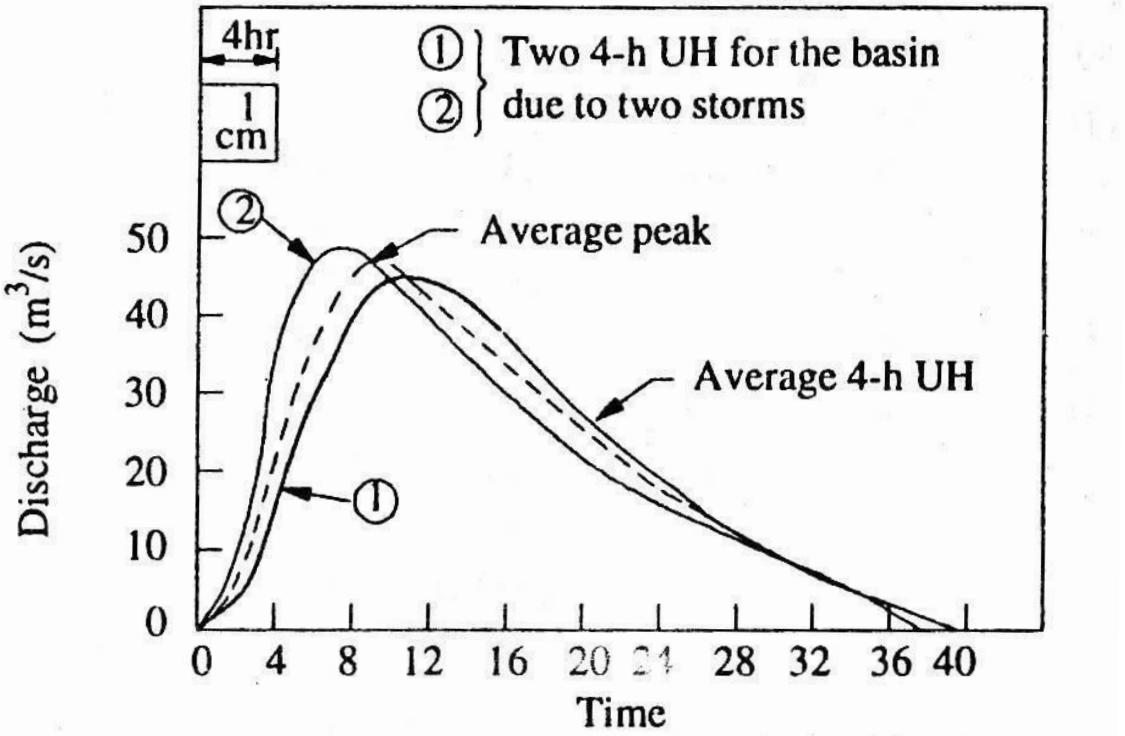


Fig. 6.12 Derivation of an average unit hydrograph

EXAMPLE 6.7 *Following are the ordinates of a storm hydrograph of a river draining a catchment area of 423 km^2 due to a 6-h isolated storm. Derive the ordinates of a 6-h unit hydrograph for the catchment.*

Time from start of storm (h)	—6	0	6	12	18	24	30	36	42	48
Discharge (m^3/s)	10	10	30	87.5	115.5	102.5	85.0	71.0	59.0	47.5
Time from start of storm (h)	54	60	66	72	78	84	90	96	102	
Discharge (m^3/s)	39.0	31.5	26.0	21.5	17.5	15.0	12.5	12.0	12.0	

SOLUTION: The storm hydrograph is plotted to scale (Fig. 6.13). Denoting the time from beginning of storm as t, by inspection of Fig. 6.12,

A = beginning of DRH $\quad t = 0$

B = end of DRH $\quad t = 90$ h

P_m = peak $\quad t = 20$ h

Hence

$$N = (90 - 20) = 70 \text{ h} = 2.91 \text{ days}$$

By Eq. (6.4),

$$N = 0.83\,(423)^{0.2} = 2.78 \text{ days},$$

However, $N = 2.91$ days is adopted for convenience. A straight line joining A and B is taken as the divide line for base-flow separation. The ordinates of DRH are obtained by subtracting the base flow from the ordinates of the storm hydrograph. The calculations are shown in Table 6.6

$$\text{Volume of DRH} = 60 \times 60 \times 6 \times (\text{sum of DRH ordinates})$$

$$= 60 \times 60 \times 6 \times 587 = 12.68 \text{ Mm}^3$$

$$\text{Drainage area} = 423 \text{ km}^2 = 423 \text{ Mm}^2$$

$$\text{Runoff depth} = \text{ER depth} = \frac{12.68}{423} = 0.03 \text{ m} = 3 \text{ cm}.$$

The ordinates of DRH (col. 4) are divided by 3 to obtain the ordinates of the 6-h unit hydrograph,, see Table 6.6

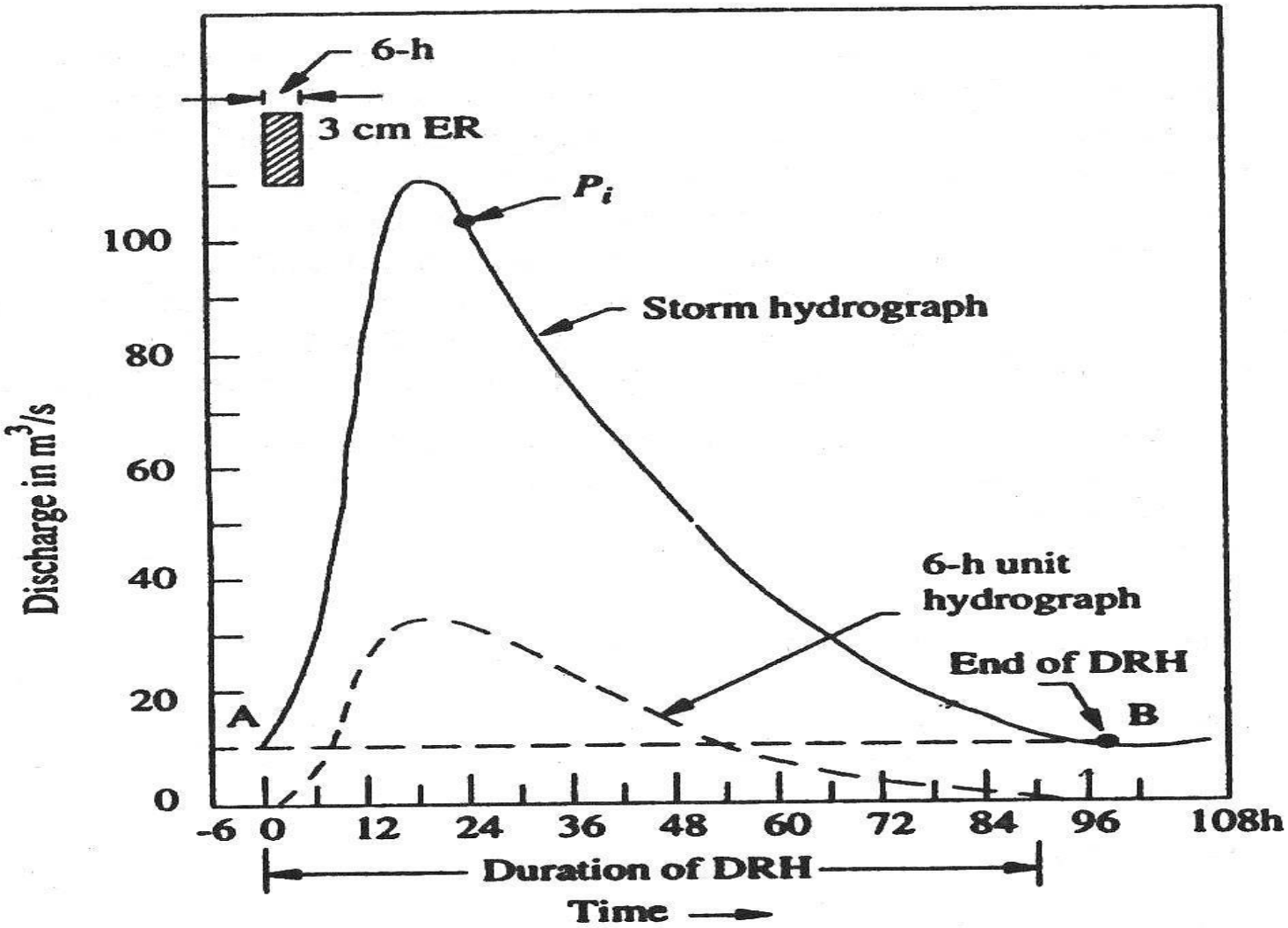


Fig. 6.13 Derivation of unit hydrograph from a storm hydrograph

TABLE 6.6 CALCULATION OF THE ORDINATES OF A 6–h UNIT HYDROGRAPH—*EXAMPLE 6.7*

Time from beginning of storm (h)	Ordinate of storm hydrograph (m^3/s)	Base flow (m^3/s)	Ordinate of DRH (m^3/s)	Ordinate of 6-h unit hydrograph (Col. 4 + 3)
1	2	3	4	5
–6	10.0	10.0	0	0
0	10.0	10.0	0	0
6	30.0	10.0	20.0	6.7
12	87.5	10.5	77.0	25.7
18	111.5	10.5	101.0	33.7
24	102.5	10.5	101.0	33.7
30	85.0	11.0	74.0	24.7
36	71.0	11.0	60.0	20.0
42	59.0	11.0	48.0	16.0
48	47.5	11.5	36.0	12.0
54	39.0	11.5	27.5	9.2

TABLE 6.6 (Continued)

1	2	3	4	5
60	31.5	11.5	20.0	6.6
66	26.0	12.0	14.0	4.6
72	21.5	12.0	9.5	3.2
78	17.5	12.0	5.5	1.8
84	15.0	12.5	2.5	0.8
90	12.5	12.5	0	0
96	12.0	12.0	0	0
102	12.0	12.0	0	0

Unit hydrograph of Different Durations

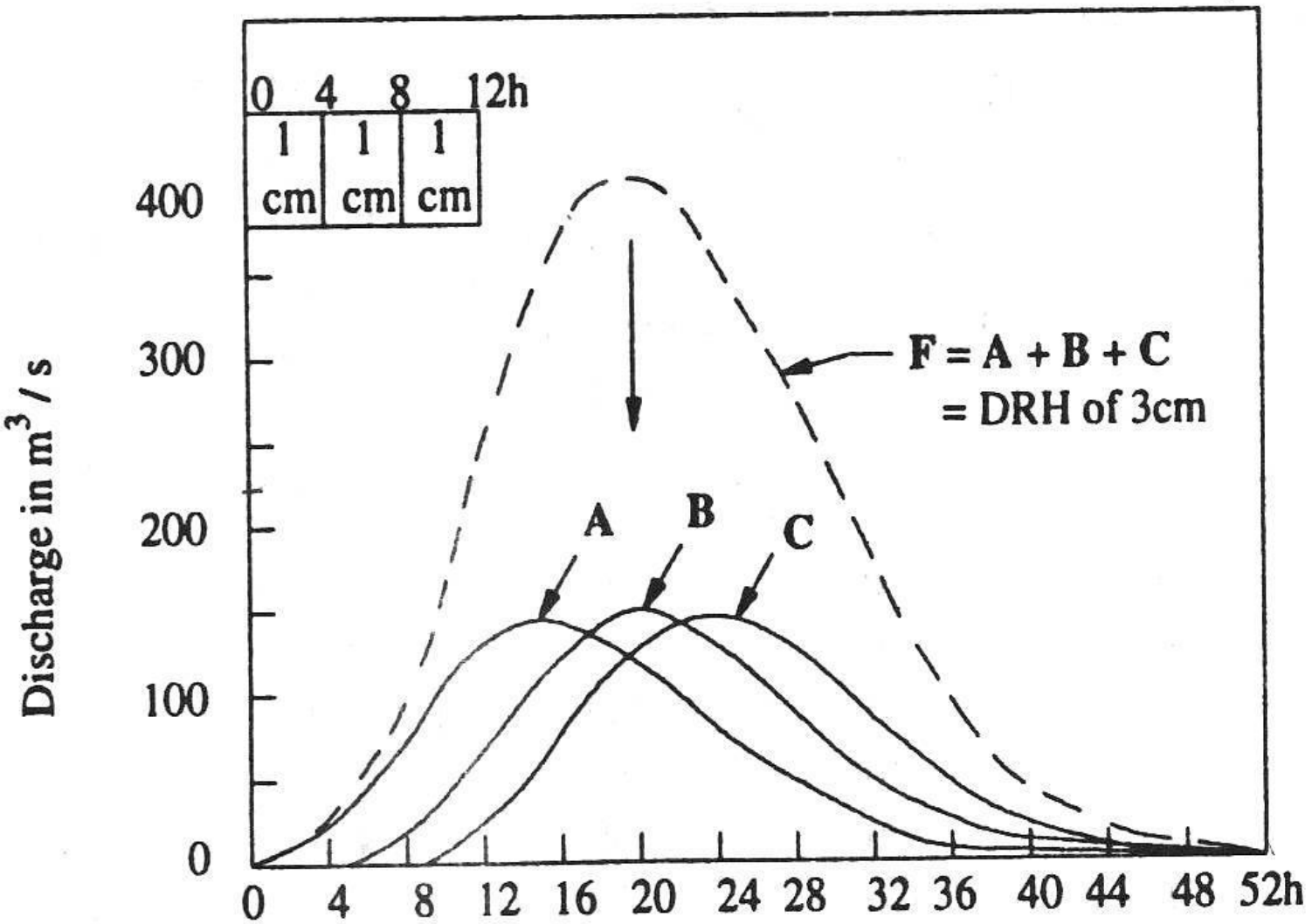
Two methods are available for this purpose

(i) Method of superposition, and

(ii) the S – curve

(i) Method of Superposition

If a D – h unit hydrograph is available, and it is desired to develop a unit hydrograph of nD h, where n is an integer.


0 4 8 12h
1 cm
1 cm
1 cm
400
300
200
100
0
Discharge in m³ / s
F = A + B + C
= DRH of 3cm
A
B
C
0 4 8 12 16 20 24 28 32 36 40 44 48 52h

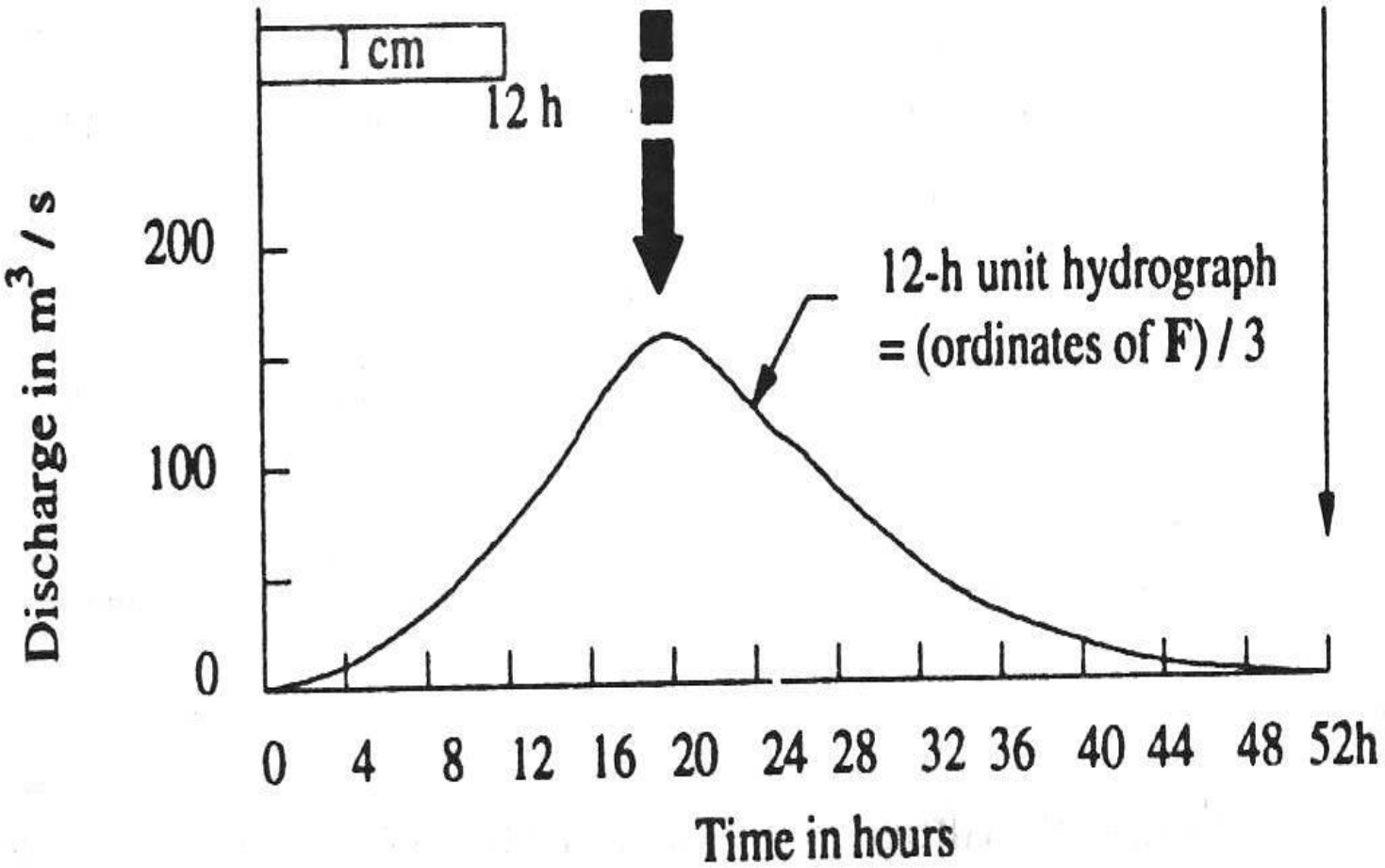


Fig. 6.15 Construction of a 12–h unit hydrograph from a 4–h unit hydrograph

EXAMPLE 6.9 *Given the ordinates of a 4-h unit hydrograph as below derive the ordinates of a 12-h unit hydrograph for the same catchment.*

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate of 4–h UH	0	20	80	130	150	130	90	52	27	15	5	0

SOLUTION: The calculations are performed in a tabular form in Table 6.7. In this

Column 3 = ordinates of 4-h UH lagged by 4 h

Column 4 = ordinates of 4-h UH lagged by 8 h

Column 5 = ordinates of DRH representing 3 cm ER in 12 h

Column 6 = ordinates of 12-h UH = (Column 5)/3

The 12-h unit hydrograph is shown in Fig. 6.15.

TABLE 6.7 CALCULATION OF A 12-h UNIT HYDROGRAPH FROM A 4-h UNIT HYDROGRAPH—*EXAMPLE 6.9*

Time (h)	Ordinates of 4-h UH (m^3/s)			DRH of 3 cm in 12-h (m^3/s) (Col.2+3+4)	Ordinate of 12-h UH (m^3/s) (Col. 5)/3
	A	B Lagged by 4 h	C Lagged by 8 h		
1	2	3	4	5	6
0	0	—	—	0	0
4	20	0	—	20	6.7
8	80	20	0	100	33.3
12	130	80	20	230	76.7
16	150	130	80	360	120.0
20	130	150	130	410	136.7
24	90	130	150	370	123.3
28	52	90	130	272	90.7
32	27	52	90	169	56.3
36	15	27	52	94	31.3
40	5	15	27	47	15.7
44	0	5	15	20	6.7
48		0	5	5	1.7
52			0	0	0

(ii) The S – Curve

If it is desired to develop a unit hydrograph of duration mD, where m is a fraction, the method of superposition cannot be used.

Ordinates (S_A – S_B) = T h DRH of (1/D x T) cm ER

UH – T h = (ordinates of DRH)/ (1/D x T)

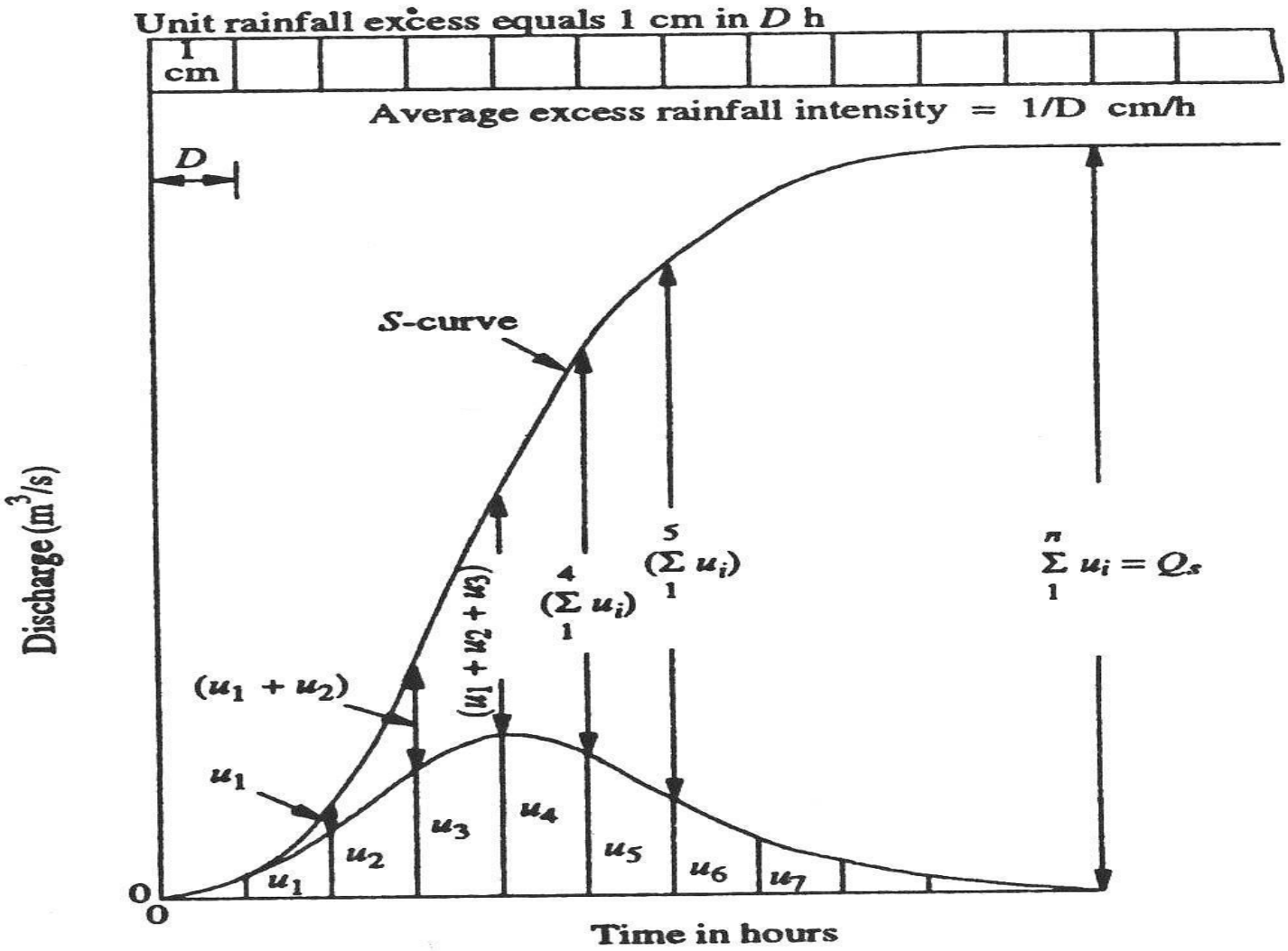


Fig. 6.16(a) S–curve

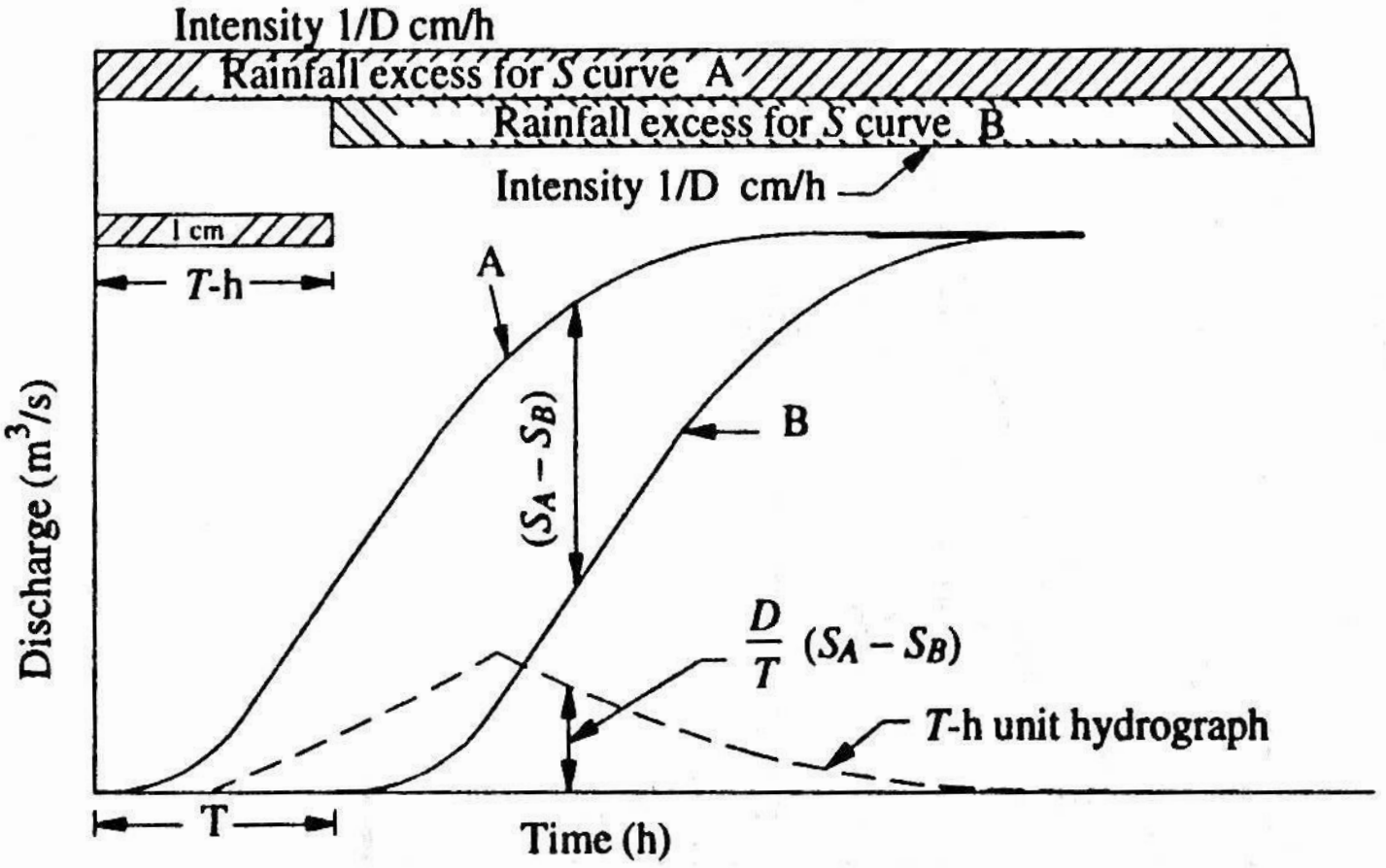


Fig. 6.16 (b) Derivation of a *T*-h unit hydrograph by *S*-curve lagging method

Example 6.10 – Solve example 6.9 by the S – curve method.

Solution:

TABLE 6.8 DETERMINATION OF A 12-h UNIT HYDROGRAPH BY *S*-CURVE METHOD—*EXAMPLE 6.10*

Time (h)	Ordinate of 4–h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (m^3/s) (Col.2 + Col. 3)	S-curve lagged by 12 h (m^3/s)	(Col. 4– Col. 5)	Col. 6 / 12/4 = 12–h UH ordinates (m^3/s)
1	2	3	4	5	6	7
0	0	—	0	—	0	0
4	20	0	20	—	20	6.7
8	80	20	100	—	100	33.3
12	130	100	230	0	230	76.7
16	150	230	380	20	360	120.0
20	130	380	510	100	410	136.7
24	90	510	600	230	370	123.3
28	52	600	652	380	272	90.7
32	27	652	679	510	169	56.3
36	15	679	694	600	94	31.3
40	5	694	699	652	47	15.7
44	0	699	699	679	20	6.7
48		699	699	694	5	1.7
52			699	699	0	0

EXAMPLE 6.11 *Ordinates of a 4-h unit hydrograph are given. Using this derive the ordinates of a 2-h unit hydrograph for the same catchment.*

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate or 4-h UH (m^3/s)	0	20	80	130	150	130	90	52	27	15	5	0

Solution:

TABLE 6.9 DETERMINATION OF 2-h UNIT HYDROGRAPH FROM A 4-h UNIT HYDROGRAPH—*EXAMPLE 6.11*

Time (h)	Ordinate of 4-h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (Col. (2) + (3)) (m^3/s)	S-curve lagged by 2 h	(Col. (4) – Col. (5)	2–h UH ordinates Col. (6) / (2/4) (m^3/s)
1	2	3	4	5	6	7
0	0	—	0	—	0	0
2	8	—	8	0	8	16
4	20	0	20	8	12	24
6	43	8	51	20	31	62
8	80	20	100	51	49	98
10	110	51	161	100	61	122
12	130	100	230	161	69	138
14	146	161	307	230	77	154
16	150	230	380	307	73	146
18	142	307	449	380	69	138

20	130	380	510	449	61	122
22	112	449	561	510	51	102
24	90	510	600	561	39	78
26	70	561	631	600	31	62
28	52	600	652	631	21	42
30	38	631	669	652	17	34
32	27	652	679	669	10	20
34	20	669	689	679	10	(20)15
36	15	679	694	689	5	(10)10
38	10	689	699	694	5	(10)6
40	5	694	699	699	(0)	(0)3
42	2	699	701	699	(2)	(4)0
44	0	699	699	701	(-2)	(-4)0