Mathematics of Growth

Exponential Growth:

N_0 = initial amount

N_t = amount after t years

r = growth rate (fraction per year)

Then $N_{t+1} = N_t + r N_t = N_t (1+r)$

For example , $N_1 = N_0 (1+ r)$; $N_2 = N_1 (1+ r) = N_0 (1+ r)^2$;

and in general, $N_t = N_0 (1+ r)^t$

Continuous Compounding

Rate of change of quantity N is proportional to N

$$dN/dt = rN$$

$$N = N_0 e^{rt}$$

Doubling Time

The doubling time (T_d) of a quantity that grows at a fixed exponential rate r is easily derived from

$$N = N_0 e^{rt}$$

Doubling Time

Doubling time can be found by setting $N=2N_0$ at $t = T_d$

$$2N_0 = N_0 e^{rt}$$

N_0 appears on both sides of the equation and can be canceled out and taking the natural log of both sides gives $\ln 2 = r\,T_d$

$$T_d = \ln 2/r = 0.693/r$$

If the growth r is expressed as a percentage instead of as a fraction , we get the following important result

$$\mathbf{T_d = 69.3/r(\%) = 70/r(\%)}$$

Problem

It took the world about 300 years to increase in population from 0.5 billion to 4.0 billion. If we assume exponential growth at a constant rate over the period of time, what would that growth rate be? Do it using both approach.

Logistic Growth

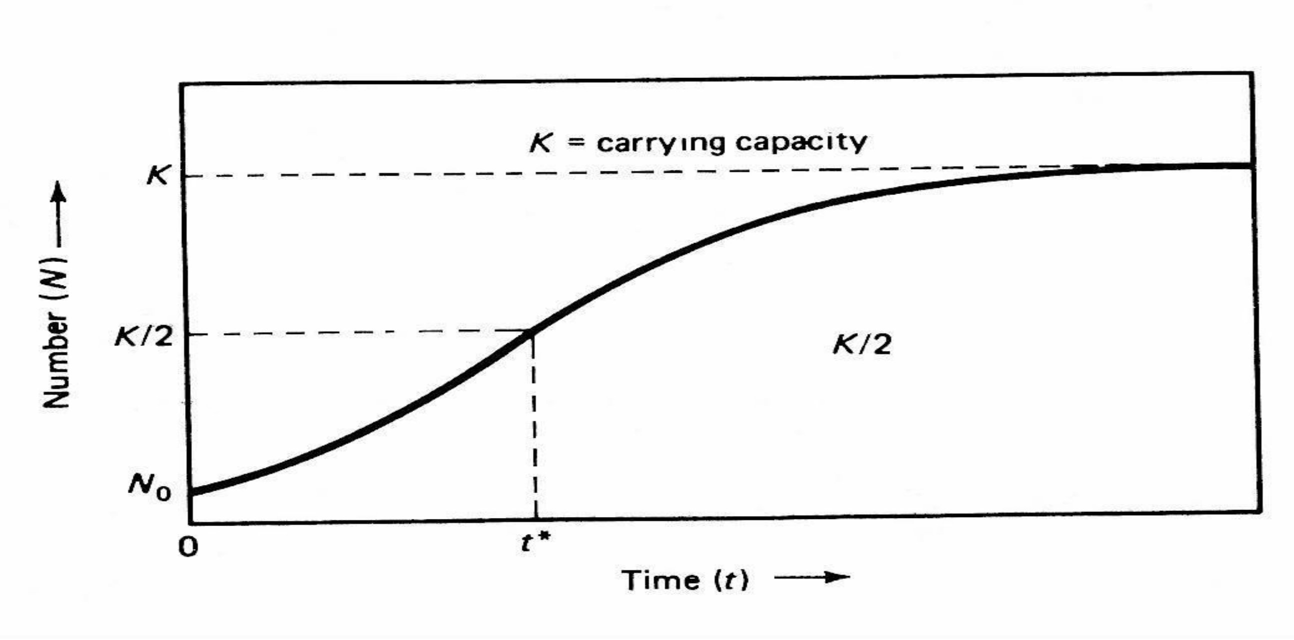

Figure : The logistic growth curve suggest a smooth transition from exponential growth to a steady – state population

- Maximum Sustainable Yield

$dN/dt = rN\,(1 - N/K)$

$dN/dt = \text{Maximum}$

Setting the derivative equal to zero gives

$d/dt\,(dN/dt) = 0$

$d/dt\,(rN\,(1 - N/K)) = 0$

$d/dt\,(rN - rN^2 / K) = 0$

$r\,dN/dt - r/K\,d/dt\,(N^2) = 0$

$r\,dN/dt - r/K\,2N\,dN/dt = 0$

$r\,dN/dt\,(1 - 2N/k) = 0$

$1 - 2N/k = 0$

$2N/k = 1$

$N = K/2$

- ## Age Structure

A graphical presentation of the data, indicating numbers of people in each age category, is called an age structure.