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An Electronic Eye: Sight in Semiconductor

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Abstract—This paper presents a model of the eye movement. Using the natural process of seeing any object by the eye, a simple block diagram was considered. From this block diagram the mathematical model of eye movement was developed. The developed model was then converted into state equations which were used to draw the signal flow diagram. This diagram was simulated using simulink of MATLAB. An electrical circuit of the model was developed and output was checked. Finally, the condition of the stability of the model and the steady state error were determined both from the simulation and practically developed circuit. The model can describe the normal operation of the eye and can also describe the eye diseases like hypermetropia and myopia.

Index Terms—state equation, signal flow diagram, simulink, MATLAB, stability, steady state error.

1 Introduction

A mathematical model uses mathematical language to describe a system in the real world. The process of developing a mathematical model is termed as mathematical modeling or modeling [1]. It is often difficult to identify the appropriate level of modeling for a particular problem [2]. A crucial part of the modeling process is the evaluation of whether a proposed mathematical model describes a system accurately or not. When we face any problems with any real world task, then we convert the task into mathematical model, apply assumption if required, solve this and interpret for real world to see whether the developed model is correct for the system or not.

Eye is one the most important organs of the body. The human eye is an organ which reacts to light for several purposes. The visual system in the brain is too slow to process information if the images are slipping across the retina at more than a few degrees per second [3]. Thus, for humans to be able to see while moving, the brain must compensate for the motion of the head by turning the eyes. Another complication for vision in frontal-eyed animals is the development of a small area of the retina with a very high visual acuity. This area is called the fovea, and covers about 2 degrees of visual angle in people. To get a clear view of the world, the brain must turn the eyes so that the image of the object of regard falls on the fovea. Eye movements are thus very important for visual perception, and any failure to make them correctly can lead to serious visual disabilities. Some works on human-machine interaction [4], [5], [6], [7], [8] and eye position [9], [10] have been published. This study represents the model of eye and eye movement.

2 Mathematical Modeling

A model for eye movement consists of the closed-loop system shown in fig. 1, where an object's position is the input and the eye position is the output. As the brain detects any object, the brain sends signals to the muscles that move the eye. These signals consist of the difference between the object's position and the position and rate information from the eye sent by the muscles spindles. In these process two types of delays should be considered: delay due the signal processing in the brain and the propagation delay of the signals through the nervous system [11].

Each eye has six muscles that control its movements: the lateral rectus, the medial rectus, the inferior rectus, the superior rectus, the inferior oblique, and the superior oblique. When the muscles exert different tensions, a torque is exerted on the globe that causes it to turn, in almost pure rotation, with only about one millimeter of translation.[12] Thus, the eye can be considered as undergoing rotations about a single point in the center of the eye.

Muscle spindles are sensory receptors within the belly of a muscle, which primarily detect changes in the length of this muscle. They convey length information to the central nervous system via sensory neurons. This information can be processed by the brain to determine the position of body parts. The responses of muscle spindles to changes in length also play an important role in regulating the contraction of muscles, by activating motoneurons via the stretch reflex to resist muscle stretch.

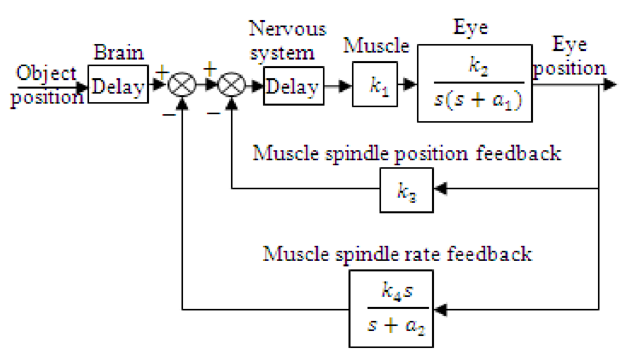

Fig. 1. Block diagram of eye movement

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Let, r(t) represents the object position, y(t) represents the eye position, k1, k2, k3 and k4 are the gain constants, a1 and a2 determine the time constant of eye and muscle spindle respectively.

Using the block reduction techniques, the above figure is converted to its simplest form which results the determination of the transfer function as follows:

$$\frac{Y(s)}{R(s)} = \frac{k_1k_2(s+a_2)}{s^3+(a_1+a_2)s^2+(a_1a_2+k_1k_2k_3)s+k_1k_2k_3a_2+k_1k_2k_4} \quad (1)$$

Taking cross multiplication

$$[s^3 + (a_1 + a_2)s^2 + (a_1a_2 + k_1k_2k_3)s + k_1k_2k_3a_2 + k_1k_2k_4]Y(s) = [k_1k_2s + k_1k_2a_2]R(s) \quad (2)$$

Let,

$A = a_1 + a_2$ (3a)

$B = a_1a_2 + k_1k_2k_3$ (3b)

$C = k_1k_2k_3a_2 + k_1k_2k_4$ (3c)

$D = k_1k_2, E = k_1k_2a_2$ (3d)

Thus,

$$[s^3 + (a_1 + a_2)s^2 + (a_1a_2 + k_1k_2k_3)s + k_1k_2k_3a_2 + k_1k_2k_4]Y(s) = [k_1k_2s + k_1k_2a_2]R(s)$$

$$Or, s^3Y(s) + As^2Y(s) + BsY(s) + CY(s) = DsR(s) + ER(s) \quad (4)$$

Applying inverse Laplace Transformation assuming zero initial condition to the above equation

$$\frac{d^3y(t)}{dt^3} + A\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + Cy(t) = D\frac{dr(t)}{dt} + Er(t) \quad (5)$$

This differential equation represents the mathematical model of the eye movement.

3 State Space Representation

Separating the transfer function of the system into two cascaded blocks, the system looks like as shown in fig. 2.

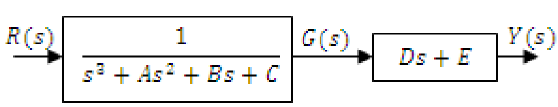

Fig. 2. Simplified transfer function

For first block, the corresponding differential equation

$\dddot{g} + A\ddot{g} + B\dot{g} + Cg = r$ (6)

Choosing the state variables as successive derivatives

$x_1 = g$ (7a)

$x_2 = \dot{g}$ (7b)

$x_3 = \ddot{g}$ (7c)

Differentiating both sides and equating equivalent values, the state equations are obtained. Since the output is g = x_1 , the combined state and output equations are:

$\dot{x_1} = x_2$ (8a)

$\dot{x_2} = x_3$ (8b)

$\dot{x_3} = -Cx_1 - Bx_2 - Ax_3 + r$ (8c)

In vector-matrix form,

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -C & -B & -A\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}r \quad (9)$$

From second block of fig. 2,

$$Y(s) = (Ds + E)G(s) \quad (10)$$

Taking inverse Laplace Transform with zero initial condition,

$y = D\dot{x_1} + Ex_1$ (11)

But $\dot{x_1} = x_2$

So, $y = Dx_2 + Ex_1$ (12)

Thus the second block of figure.2b collects the states and generates the output equations as shown below:

$$y = [E \quad D]\begin{bmatrix}x_1\\ x_2\end{bmatrix} \quad (13)$$

Using eq.9 and eq.13, the signal flow diagram [12] is constructed and is shown in fig. 3.

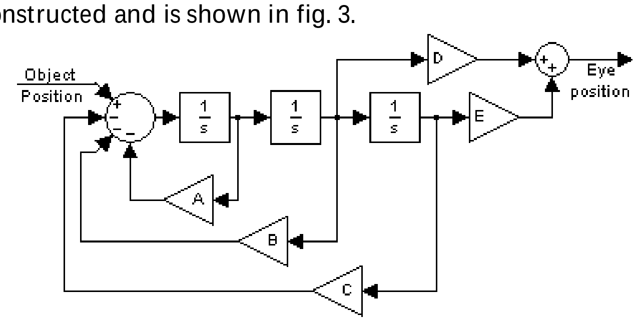

Fig. 3. Signal flow diagram

4 Electrical Equivalent Circuit

The signal flow diagram in fig. 3 can be replaced by electrical circuit components: integrators, inverting amplifiers and summing amplifiers [13]. The electrical circuit realization of the eye movement is given in fig. 4. The integrator consists of an op-amp, resistor and capacitor; an amplifier consists of an op-amp and resistors.

In the figure below:

$$\frac{R_5}{R_8} = A, \frac{R_5}{R_7} = B, \frac{R_5}{R_6} = C, \frac{R_5}{R_9} = 1$$

$$R_1C_1 = R_2C_2 = R_3C_3 = 1$$

$$\frac{R_{14}}{R_{12}} = D, \frac{R_{14}}{R_{13}} = E, \frac{R_{10}}{R_{11}} = 1$$

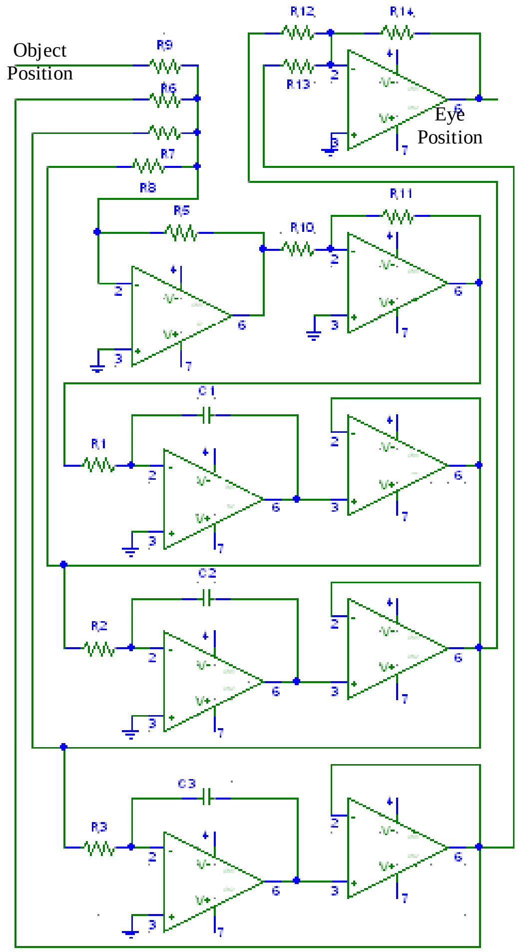

Fig. 4. Electronic equivalent circuit

5 Stability of The Model

A system is said to be stable if there is no poles in the right half plane in s-domain. Recalling the transfer function, the Routh-Hurwitz table is created using the denominator of the transfer function.

Denominator, $D(s) = s^3 + As^2 + Bs + C$

TABLE 1

STABILITY TEST OF THE MODEL

s^3	1	B
s^2	A	C
s^1	$\frac{AB - C}{A}$	0
s^0	C	0

According to Routh-Hurwitz criteria for stability, each term of the second column must have same sign to ensure all the poles in the left half side of the s-plane. The following conditions must be satisfied to ensure the stability of the system.

1. Condition 1:
   $A > 0$
   $Or, a_1 + a_2 > 0$

2. Condition 2:
   $(AB - C)/A > 0$
   $Or, AB - C > 0$
   $Or, AB > C$
   $Or, B > C/A$
   $Or, a_1a_2 + k_1k_2k_3 > \frac{k_1k_2k_3a_2 + k_1k_2k_4}{a_1+a_2}$
   $Or, a_1a_2 > \frac{k_1k_2k_3a_2 + k_1k_2k_4}{a_1+a_2} - k_1k_2k_3$
   $Or, a_1a_2 > \frac{k_1k_2k_4 - k_1k_2k_3a_1}{a_1+a_2}$
   $Or, a_1a_2 > \frac{k_1k_2(k_4 - k_3a_1)}{a_1+a_2}$
   $Or, k_1k_2 < \frac{a_1a_2(a_1+a_2)}{(k_4 - k_3a_1)}$

3. Condition 3:
   $C > 0$
   $Or, k1k2k3a2 + k1k2k4 > 0$
   $Or, a2 > -k4/k3$

To make the model stable all of the three conditions described above must be satisfied.

6 Results

Fig. 5 and fig. 6 show the eye response to an object position. The object position or the input is considered as the unit step function in fig. 5 and the sine wave in fig. 6. The figures show that the eye can detect the position of the object after a certain time. This certain time includes the rise time resulted from delay of signal processing in the brain, propagation delay in the nervous system and from time constant determining factor of the eye.

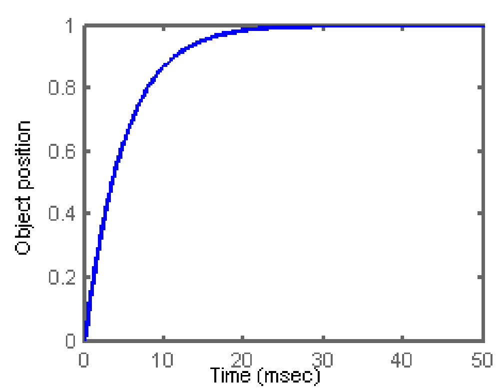

Fig. 5. Normal eye response to unit step function

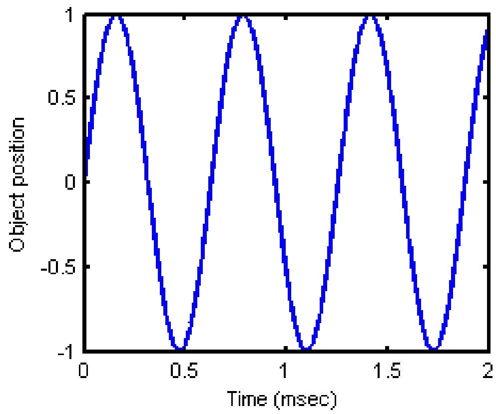

Fig. 6. Normal eye response to sine wave

If the eye suffers from diseases, the object position is not detected accurately. Eye disease means the changes of the different parameters in the model. When it occurs, the model suffers from instability and the eye cannot detect the actual position of the object which is described below.

Fig. 7 shows that when the eye suffers from myopia then there is a steady state error. This error means that the eye cannot reach to the actual object position. Myopia occurs due to increment of convex power of the eye which is determined by the decrement of gain constant of the muscle spindle. As the muscle spindle works as the negative feedback network, an increment means the decrement of power and vice versa.

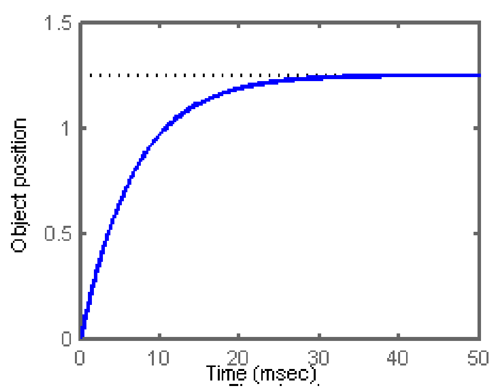

Fig. 7. Response of eye suffering from myopia

As myopia occurs due the increment of convex power of the eye, it is required to reduce the convex power of eye to get removed from this eye disease. A concave lens is used to reduce the power. In the model, the value of gain constants is increased so that the effect reduces by the negative feedback. Fig. 8 shows the response of the eye suffering from myopia with a concave lens of suitable power. This response is same as normal eye response.

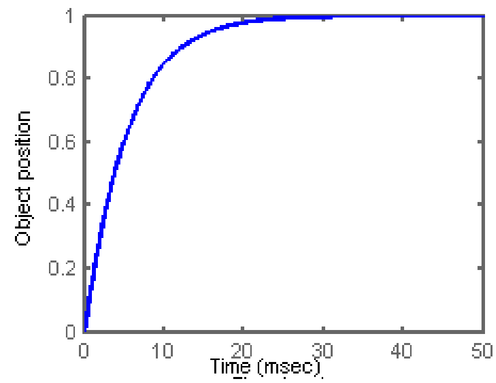

Fig. 8. Response of eye suffering from myopia with suitable concave lens

Fig. 9 shows that when the eye suffers from hypermetropia then there is a steady state error. This error means that the eye cannot reach to the actual object position. Hypermetropia occurs due to decrement of convex power of the eye which is determined by the increment of gain constant of the muscle spindle. As the muscle spindle works as the negative feedback network, an increment means the decrement of power and vice versa.

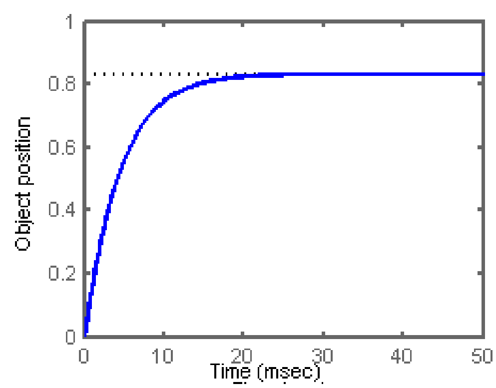

Fig. 9. Response of eye suffering from hypermetropia

As hypermetropia occurs due the decrement of convex power of the eye, it is required to increase the convex power of eye to get removed from this eye disease. A convex lens is used to increase the power. In the model, the value of gain constants is decreased so that the effect increases by the negative feedback. Fig. 10 shows the response of the eye suffering from hypermetropia with a convex lens of suitable power. This response is same as normal eye response.

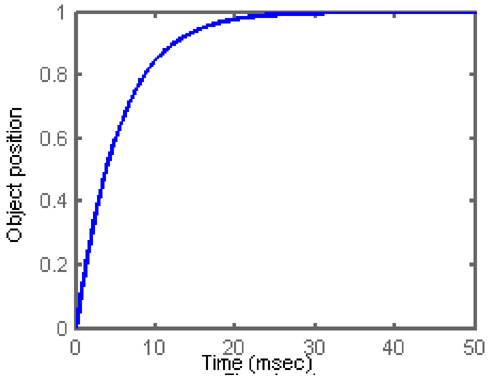

Fig. 10. Response of eye suffering from hypermetropia with suitable convex lens

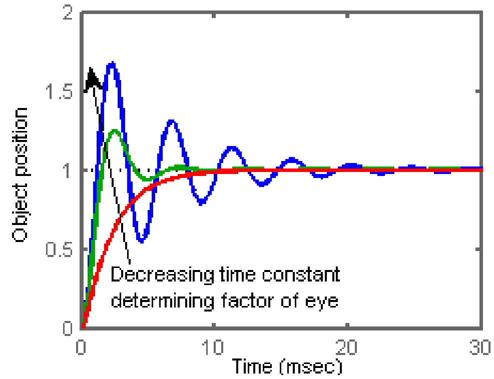

Fig.12. Eye response to various time constant determining factor of eye

Muscle and eye gain constants determine the rise time to detect the object position. The rise time increases as the gain constants decrease. It is due to aging effect, when the power of the eye muscle reduces. Rise time increases means it will take more time to detect the position of the object fully. Fig. 11 shows the variation of rise time for various gain constants.

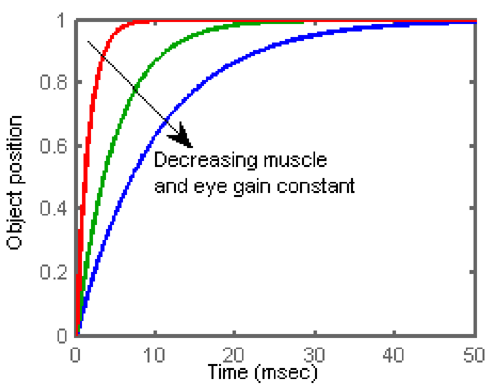

Fig. 11. Eye response to various gain constants

Time constant determining factor of eye affects both the rise time and clearness of the object detection. As the factor decreases, rise time increases but clearness of the object reduces. This phenomenon is described in fig.12.

Time constant determining factor of muscle spindle affects the initial clearness of the object detection but does not affect the rise time. A change in this factor can lead a normal eye, eye with myopia and eye with hypermetropia as described in fig.13. Hypermetropia is significant than that of the myopia.

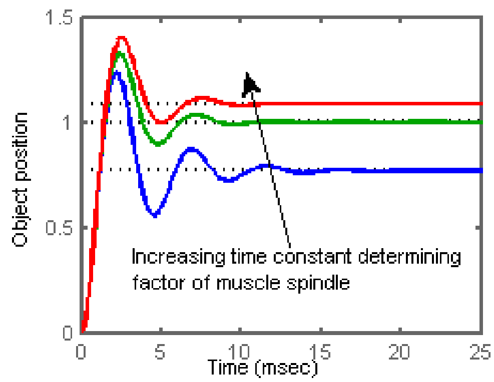

Fig. 13. Eye response to various time constant determining factor of muscle spindle

7 Conclusion

The goal of this paper is to represent a model of the eye movement. The mathematical model is presented in two forms: differential equation form and space state representation. This model is replaced by electronic circuits to develop an electrical model of eye movement.

A limited number of internal parameters are considered in developing the model. So, possible improvements of the study would include the integration of more sophisticated, more realistic model and a more complex constitutive law. Although some assumptions have been considered, the model can be treated as a valid one, because, this model is able to show the normal eye response to an object position, can describe the aging effect on eye. The model can also show the diseases of eye and the treatment of those diseases using convex or concave lens with suitable power.

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