# Comparison of Sway Analysis of RC Frames using Cracked Moment of Inertia

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**Abstract**

*Sway of reinforced concrete (RC) frames subjected to lateral load is an important design criterion from serviceability point of view. In conventional structural design, a reasonably accurate and practical method of determination of sway still lags proper guideline. The recommendations for deflection calculation of the widely followed code of American Concrete Institute (ACI) as well as Bangladesh National Building Code (BNBC) are far from complete. In reality flexural cracks occur in the service life of RC structures, which may render part of the section ineffective. Thus there is a scope of investigation and improvement of the methods of sway estimation. In this paper, stiffness matrix incorporated with effective moment of inertia is used and a parametric study is carried out to investigate the effects of the flexural cracks developed under service load. A computer-aided tool is developed to determine the sway of RC frames more reasonably. Values of sway of sample frame structures calculated by the proposed method are compared with the conventional method and are also found to be closer to the results finite element analysis.*

**Keywords:** Serviceability; Frame sway; Gross moment of inertia; Effective moment of inertia; Finite Element analysis

## Introduction

Lateral deflection or sway of frame structures is an important design criterion from serviceability point of view. This sway property can turn out to be a governing factor in member proportioning of large Reinforced Concrete (RC) buildings. As a general rule the deflection of a frame structure should be a consideration in selecting the sections of its members. Generally the smaller the member the larger is the deflection or vice versa. Therefore to reduce the sway and in turn to reduce member stresses, sections are to be increased. This leads to a rise in cost of the structure and renders the structure uneconomical. Determination of sway of RC frames subjected to lateral loads by conventional methods tends to underestimate the sway values. Sometimes such estimation becomes excessively low resulting in functionally weak sections of members. Efforts are therefore required to avoid this situation and proper tools need to be designed which would help to estimate the sway of RC structures more accurately under lateral loads. With the help of such tools, reasonably accurate determinations of the lateral deflections of RC buildings would be possible in design offices, which would ultimately contribute to functional serviceability of structure in designing the various structural members.

With increasing use of high strength concrete and steel, frame members are getting thinner resulting in higher amount of deflection. In order to keep deflection or sway within permissible limit defined by various codes of practices, designers are often forced to choose larger sections for frame members nullifying the benefits of using high strength material. American Concrete Institute (ACI) is one of the leading authorities in the world in formulating design practices for RC construction. The guidelines laid out by ACI are widely practiced throughout the world. Prior to 1993, ACI code was virtually the only design code followed in the design and construction of RC structures in Bangladesh. Bangladesh National Building Code (BNBC), formulated in 1993, also gives guidelines for RC construction. However the BNBC specifications are not much different from those of ACI and ACI still remains to be more elaborate and comprehensive compared to BNBC (Amanat et al. 1999).

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Apart from providing design criteria for various concrete constructions ACI also recommends analytical procedures for the design of RC structures. Following this code one can calculate the design moments and shears of a structure under various loading conditions and hence design various members with adequate factor of safety. However the ACI code does not provide any specific recommendation regarding sway estimation of frame structures. Since most of the modern day RC building constructions consist of beam and column arrangements, the most elementary structural form of these buildings would be a simple portal frame. The ACI code proposes methods to calculate the member forces of a portal frame. But the only deflection estimation method it proposes is for simply supported and continuous beams.

Besides, an RC member is likely to crack under service load. This crack reduces the effective moment of inertia of the members. This will result in higher sway value. Therefore, in the design of RC frames, particularly those to be used in high-rise buildings, sway estimation is very important and in which the cracking effect of the member is to be considered.

## Review of Basic equations

Gross moment of inertia ($I_g$) is the second moment of area of the cross section about the neutral axis.

Cracked moment of inertia ($I_{cr}$) is the moment of inertia after tension cracking, concrete being much weaker in tension than in compression. While calculating the cracked moment of inertia, the cracked portion of the cross section is neglected and therefore is always less than the gross moment of inertia. This is most likely to happen when a concrete member is subjected to bending; i.e., for beams and frames.

According to extensive studies by Branson (1977), an effective moment of inertia ($I_e$) of the section can be evaluated using combinations of the moment of inertia of cracked section and the moment of inertia of gross section. This effective moment of inertia then can be used to calculate the deflection once the maximum actual moment of the section has exceeded the cracking moment.

Branson's equation for the effective moment of inertia for short-term deflections is as follows:

$$I_e = (M_{cr}/M_a)^3 I_g + [1 - (M_{cr}/M_a)^3]\, I_{cr} \le I_g \qquad (1)$$

where $M_{cr}$ and $M_a$ are the cracking moment and applied moments at the section. For continuous members, ACI 318-

99 stipulates that effective moment of inertia may be taken as the average value obtained from the same equation as the one for simply supported beams for the critical positive and negative moment sections. For prismatic members, effective moment of inertia may be taken as the value obtained at mid span for continuous beams.

If one chooses to average the effective moment of inertia, then according to ACI 318 (1995), the following expression should be used,

$$I_e = 0.5\, I_{e(m)} + 0.25\, (I_{e(1)} + I_{e(2)}) \qquad (2)$$

where the subscripts m, 1 and 2 refer to mid span, and two beams ends respectively.

**Software Development**

The use of computer and development of several softwares has made it more convenient to analyze a structure. Many software packages such as ANSYS, STAAD PRO, STRAN, Micro-FEAP etc. are available for this purpose. The primary objective of this paper is to present computer program for sway analysis of plane frame structure considering the effective moments of inertia of the sections rather than their gross moments of inertia. Since the effective moment of inertia depends on the bending moment of the section, it needs to be updated by iteration.

All the members of the structures to be analyzed by the developed program are assumed to be straight, prismatic members. Different material properties can be assigned for different members of the structures. Only the effects of loads are taken into account and no other influences like temperature changes are considered. The complete program consists of a commercial plane frame analysis program named as SKELETON-9 and a self-developed iteration program written in TURBO C. These two programs are combined by a batch file.

SKELETON-9 is a commercial package for solving plane frame problems developed by the British firm Techno Consultants Ltd. It is available in the Internet as a shareware in *www.techno.man.uk*.

SKELETON carries out linear elastic analysis of plane frameworks subjected to static loads. At supports, joints can have full restraints or elastic springs in any combination of the X, Y or angular direction of the support axes. Joints can therefore be fixed, free to rotate or be on rollers. SKELETON analyses any framework be it a truss, a rigid frame, or a combination of truss and rigid frame. It also analyses frameworks having joints where some member-ends are pinned and others rigidly connected. SKELETON combines its ability to analyze complex frameworks with an extremely powerful input system to describe complicated member-end connections with ease and rapidity. To this end, a new term used is HINGED BAR MEMBERS for describing members in the framework with hinges at both ends. As mentioned, to use the SKELETON-9 package the authors had to generate a program in TURBO C to enhance the updatation of the true cracked moment of inertia in each iteration.

**Parametric Study**

The analyses performed as described reveals the fact that there is significant difference in deflection pattern of a RC structure while subjected to conventional analysis and the improved analysis using the effective moment of inertia.

In this parametric study an empirical procedure is used which can be termed as a 'pseudo finite element analysis' procedure. This procedure includes the steps of dividing the members into a number of segments and calculating the sway of all the individual segments. The accumulated sway of the member parts was taken as the total sway of the member.

The pseudo finite element analysis improves the results further. There is, however, no scope of abandoning the conventional analysis altogether and adopt the proposed pseudo finite element analysis for all regular analyses. As stated previously, the proposed system requires certain level of expertise and knowledge of the subject matter. Also it could be a complicated process, which is not suitable for analyses of a large number of structures in design offices. The intricacy develops exponentially with an increase in the size of the structure. Hence some simpler methods are required that would be easier to grasp, yet have an acceptable accuracy compared to the proposed analytical process. With this goal, a parametric study is worked out to find the applicability of the proposed method in comparison to other available methods like the conventional method and the finite element analysis.

To make the comparison an sample five-storied two-bay RC building frame structure is chosen for analysis. Only lateral loads are applied in the joints and no vertical load is imposed upon the frame. A lateral load of 225 kN is applied at each story. Using 3 m bays and 3 m column height the sway was found in the conventional method as 1.58 cm and in the proposed method the sway comes as 2.03 cm, which is 28% higher than the conventional method. Fig. 1 shows a schematic diagram of the frame (Fame 1) and the subsequent analyses demonstrate the comparative results from the conventional method and the proposed new method.

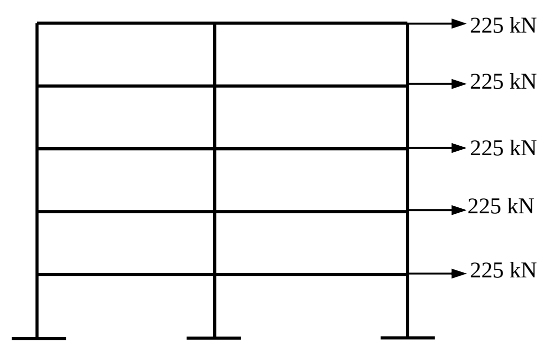


**Fig. 1** Sample Frame1

The variation of sway with the column height (shown in Fig. 2) reveals that the sway of a RC frame increases quite significantly with the column height. Also, the difference between the conventional method (using $I_g$) and proposed method (using $I_e$) increases with column height, which can be attributed to the increased bending moments in the structure that causes more cracking and therefore larger difference $I_g$ and $I_e$. Similar arguments apply for the sway profile in relation to beam span shown in Fig. 3. It also shows the increased sway of RC frame and increased difference between the methods with beam span. However the increase is not as significant as for the column height.

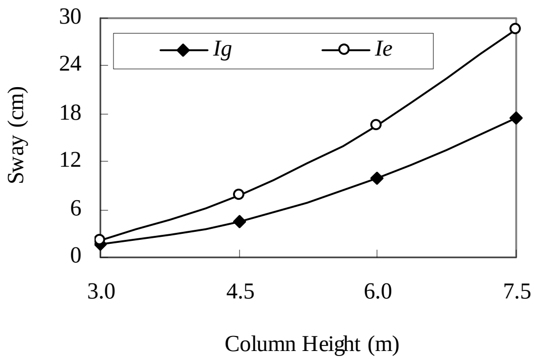


**Fig. 2** Frame1 sway vs. column height

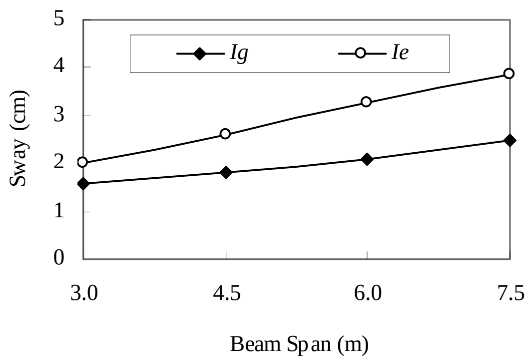


**Fig. 3** Frame1 sway vs. beam span

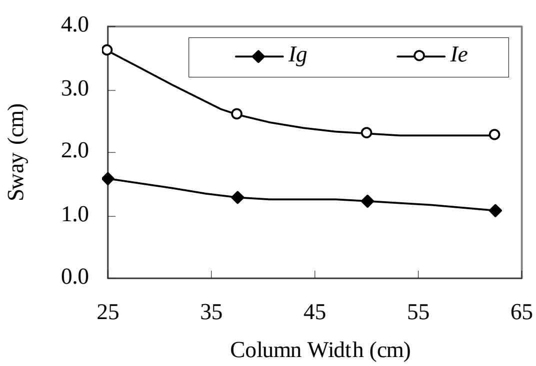


**Fig. 4** Frame1 sway vs. column width

Although increased column height and beam span increase the flexibility and therefore the sway of the frame, increased column width, beam depth and percentage of column reinforcement would decrease it. The variations of sway with column width, beam depth and percentage of column reinforcement are shown in Figs. 4, 5 and 6 respectively, which demonstrate the decreased sway of the RC frame with these parameters. The results from conventional and proposed method also converge simultaneously, as the decrease in the amount of cracking with decreased frame sway would gradually converge the values of $I_g$ and $I_e$.

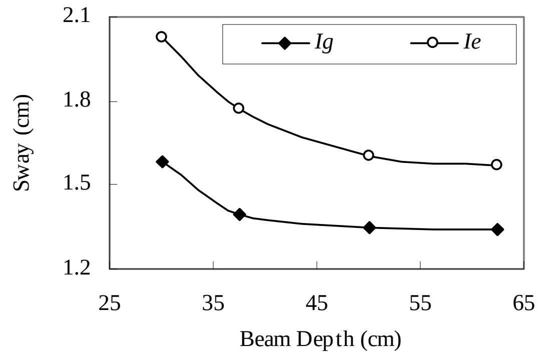


**Fig. 5** Frame1 sway vs. beam depth

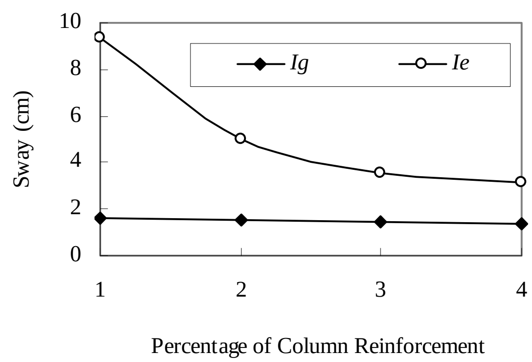


**Fig. 6** Frame1 sway vs. column reinforcement

**Comparison of Proposed Method with Finite Element Analysis**

Although the proposed method is expected to predict better the sway of cracked RC frames, its accuracy can only be judged while comparing with the 'exact' finite element method. The comparisons in this section show the limitations of the proposed method. Although the proposed method is named the pseudo finite element analysis, it is based on the analysis of one-dimensional frame elements using stiffness method of structural analysis. The discretization of the members and iterative update of their effective moments of inertia do have some effect on the improvement of the sway calculation but the method does not provide the same level of accuracy as the finite element analysis.

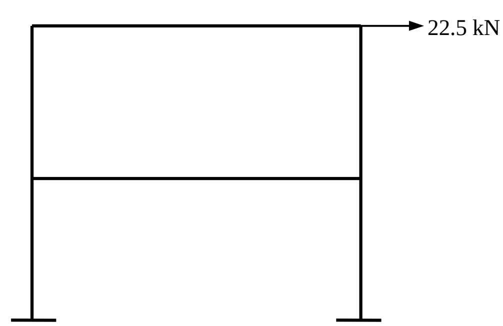


**Fig. 7** Sample Frame2

To make the comparison the authors use data available in a finite-element study of RC frames (Enam 2001), which uses a two-storied frame (named Frame2) with a 22.5 kN load at the top floor (Fig. 7).

For the frame under consideration the relation between the frame sway with beam span is shown in Fig. 8. It shows that while using the finite element (FE) method, the increase in sway with the beam span is not very significant. Results from both the conventional and the proposed method compare poorly with the FE results but predictions from the proposed method are better, particularly for larger beam spans.

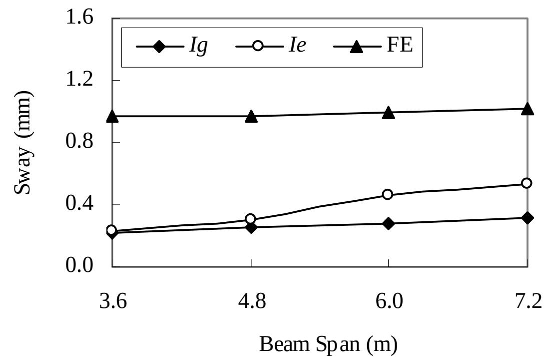


**Fig. 8** Frame2 sway vs. beam span

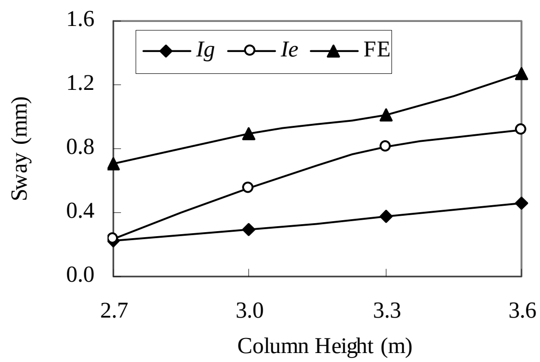


**Fig. 9** Frame2 sway vs. column height

Fig. 9 reveals that the sway of the RC frame increases appreciably with the column height, even while using FE. Here also, the proposed method is found to predict better the sway compared to the conventional method and its accuracy improves with the column height.

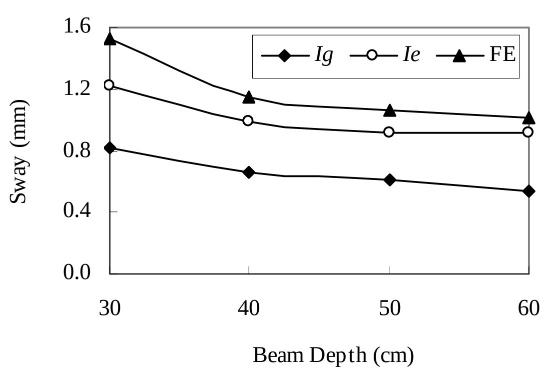


**Fig. 10** Frame2 sway vs. beam depth

Figs. 10, 11 and 12 show that the frame sway decreases with beam depth, column width and column reinforcement. Also, the results from the proposed method are closer to the FE results in all the cases.

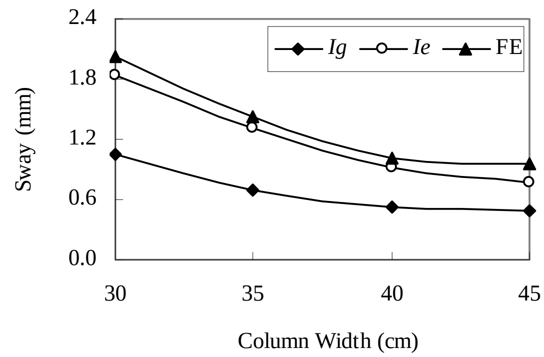


**Fig. 11** Frame2 sway vs. column width

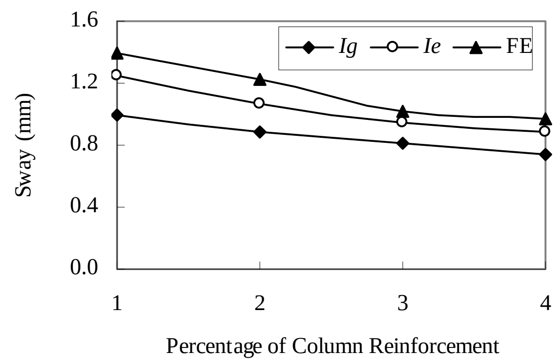


**Fig. 12** Frame2 sway vs. column reinforcement

**Conclusions**

The purpose of this work was to devise a rational guideline for the estimation of sway of RC frame structures. The guideline should be one that can easily be employed and overcomes any unrealistic estimation of sway, thereby ensuring the serviceability of the structure that may be hampered by excessive lateral deflection.

In this regard the guidelines proposed by the ACI committee report 435R-95 (1995) was adopted as the basis. The guidelines being laid down for beams, an approach dictating its adaptability to RC frame structure is discussed. To check the performance of the approach a simple five-storied two-bay frame was selected for parametric study. The frame was first analyzed using the conventional method in which gross moment of inertia is used and then the result was compared with the analysis using the ACI guideline incorporating the Branson's equation; i.e., the analysis using the effective moment of inertia after flexural cracks are developed under service loads. The method was verified by comparing the results with the finite element analysis results reported by Enam (2001) for a two-storied portal frame. Finally, a comparison of the three methods was discussed.

From the comparison it is evident that the proposed method remains in between the finite element and the conventional method using gross moment of inertia. Therefore it improves the accuracy of the analysis in a much simpler way than the finite element method does. The proposed method of the sway estimation using effective moment of inertia can be used in practical design with a certain degree of accuracy instead of the finite element analysis, which is much more rigorous particularly for RC structures.

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