2-2

University of Asia Pacific
Department of Civil Engineering
Final Examination Spring 2016
Program: B.Sc Engineering(Civil)

Course Title: Principles of Economics | **Course Code: ECN201**
Time: 2 Hours | **Full Marks: 50**

Part-A:

Answer the following Question:

1) Define 'Elasticity of Demand'. How would it be measured? Distinguish between price elasticity and income elasticity of demand with example. 10

Part-B:

Answer any four from the following Questions:
(10*4=40)

2) Find the cross elasticity of demand between hot dogs and hamburgers and between hotdogs and mustard, for the data in given table and also state the elastic relations among these products.

Commodity	Before		After	
	Price(per Unit) Tk.	Quantity(Units per month)	Price (per Unit) Tk.	Quantity(Units per month)
Hamburgers	80	30	70	40
Hotdogs	40	15	40	10
Mustard(Jar)	100	10	120	9
Hotdogs	40	15	40	12

3) What is supply function? With the help of diagrams explain the elasticity of supply.

4) A. Distinguish among Total revenue, Average revenue & Marginal revenue.
B. From the following table calculate Average revenue, Total revenue & Marginal revenue.

Quantity sold	1	2	3	4	5	6	7	8	9	10
Price per unit(Tk)	40	35	33	28	25	22	17	15	12	10

5) A. Write short notes:(Any Two)
Nominal Cost, Economic Cost, Implicit & Explicit Cost.

B. Calculate Total cost, Average Variable cost, Average cost & Marginal Cost from the following table:

Unit	0	1	2	3	4	5	6
Total Fixed cost	70	70	70	70	70	70	70
Total Variable cost	20	24	30	36	39	45	50

6) Explain the main features of following markets:
   i. Monopoly
   ii. Monopolistic Competition
   iii. Oligopoly

University of Asia Pacific
Department of Basic Sciences and Humanities
Final Examination, Spring- 2016
Program: B. Sc Engineering (Civil Engineering)
(2nd Year/ 2nd Semester)

Course Title: Mathematics-IV | Course No. MTH 203

Time: 3.00 Hours | Full Mark: 150

N.B: There are **Eight** questions. Answer any **Six (6)** of the following:

1. (a) Define Fourier Sine and Cosine transforms. Find the (i) finite Fourier Sine transform, (ii) finite Fourier Cosine transform of the function **5+10**

$$F(x) = x \;, \quad 0 < x < 2$$

(b) Find the Fourier transform of $f(x) = e^{-|x|}$ where x belongs to $(-\infty, \infty)$. **10**

2. (a) Define Fourier Series. Find the Fourier Series of the function **3+12**

$$f(x) = \begin{cases} 0 & , -5 < x < 0 \\ 3 & , \; 0 < x < 5 \end{cases} \text{ having period 10.}$$

(b) Show that $\int_0^\infty \frac{cosux}{u^2+1}\,du = \frac{\pi}{2}e^{-x}$, when $f(x) = e^{-x}$ is an even function. **10**

3. (a) Write down the Convolution theorem. Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$ by using Convolution theorem. **3+12**

(b) Prove that, $\mathcal{L}^{-1}\left\{\frac{2s+1}{(s+2)^2(s-1)^2}\right\} = \frac{1}{3}t(e^t - e^{-2t})$ **10**

4. (a) Define inverse Laplace transformation. Evaluate $\mathcal{L}^{-1}\left\{\frac{8s+20}{s^2-12s+32}\right\}$ as a function of t. **2+10**

(b) Prove that (i) $\int_0^\infty te^{-st}\,cosat\,dt = \frac{s^2-a^2}{(s^2+a^2)^2}$ (ii) $\int_0^\infty te^{-st}\,sinat\,dt = \frac{2as}{(s^2+a^2)^2}$ using Derivatives of Laplace transformation. **13**

5. (a) Graph the function **12**

$$F(t) = \begin{cases} Cost & , \; 0 \le t < \pi \\ 0 & , \; \pi \le t \le 2\pi \end{cases}$$

Extended periodically with period 2π and then find $\mathcal{L}\{F(t)\}$.

	(b)	Show that (i) If $F(t) = t$, then $\mathcal{L}\{F(t)\} = \frac{1}{s^2}$, if $s > 0$. (ii) If $F(t) = t^n$, then $\mathcal{L}\{F(t)\} = \frac{n!}{s^{n+1}}$, if $s > 0$	**5+8**
6.	**(a)**	Define Bernoulli's equation and Solve $\frac{dy}{dx} + xsin2y = x^3cos^2y$	**3+12**
	(b)	Solve: $p^2 + 2pycotx - y^2 = 0$	**10**
7.	**(a)**	Define Cauchy-Euler equation and Solve the differential equation $(x^2D^2 - 3xD + 4)y = 0$	**3+12**
	(b)	Find the general solution of $(D^2 - 2D)y = e^{2x}sinx$.	**10**
8.	**(a)**	Solve: (i) $(D^4 - 81)y = 0$ (ii) $((D^2 - 3D + 2)y = 0$ (iii) $(D^4 - 2D^3 + 5D^2)y = 0$	**15**
	(b)	Solve the differential equation $(D^3 - D^2 - 6D)y = 1 + x^2$	**10**

University of Asia Pacific
Department of Basic Sciences and Humanities
Final Examination, Spring- 2016
Program: B. Sc Engineering (Civil Engineering)
(2^{nd} Year/ 2^{nd} Semester)

Course Title: Mathematics-IV — Course No. MTH 203

Time: 3.00 Hours — Full Mark:150

N.B: There are **Eight** questions. Answer any **Six (6)** of the following:

1. (a) Define Fourier Sine and Cosine transforms. Find the (i) finite Fourier Sine transform, (ii) finite Fourier Cosine transform of the function **5+10**

$$F(x) = x\ , \quad 0 < x < 2$$

(b) Find the Fourier transform of $f(x) = e^{-|x|}$ where x belongs to $(-\infty, \infty)$. **10**

2. (a) Define Fourier Series. Find the Fourier Series of the function **3+12**

$$f(x) = \begin{cases} 0 & , -5 < x < 0 \\ 3 & ,\ \ 0 < x < 5 \end{cases} \text{ having period 10.}$$

(b) Show that $\int_0^\infty \frac{cosux}{u^2+1}\, du = \frac{\pi}{2} e^{-x}$, when $f(x) = e^{-x}$ is an even function. **10**

3. (a) Write down the Convolution theorem. Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$ by using Convolution theorem. **3+12**

(b) Prove that, $\mathcal{L}^{-1}\left\{\frac{2s+1}{(s+2)^2(s-1)^2}\right\} = \frac{1}{3}t(e^t - e^{-2t})$ **10**

4. (a) Define inverse Laplace transformation. Evaluate $\mathcal{L}^{-1}\left\{\frac{8s+20}{s^2-12s+32}\right\}$ as a function of t. **2+10**

(b) Prove that (i) $\int_0^\infty te^{-st} cosat\, dt = \frac{s^2-a^2}{(s^2+a^2)^2}$ (ii) $\int_0^\infty te^{-st} sinat\, dt = \frac{2as}{(s^2+a^2)^2}$ using Derivatives of Laplace transformation. **13**

5. (a) Graph the function **12**

$$F(t) = \begin{cases} Cost & ,\ 0 \le t < \pi \\ 0 & ,\ \pi \le t \le 2\pi \end{cases}$$

Extended periodically with period 2π and then find $\mathcal{L}\{F(t)\}$.

(b) Show that (i) If $F(t) = t$, then $\mathcal{L}\{F(t)\} = \frac{1}{s^2}$, if $s > 0$. 5+8

(ii) If $F(t) = t^n$, then $\mathcal{L}\{F(t)\} = \frac{n!}{s^{n+1}}$, if $s > 0$

6. (a) Define Bernoulli's equation and Solve $\frac{dy}{dx} + xsin2y = x^3cos^2y$ 3+12

(b) Solve: $p^2 + 2pycotx - y^2 = 0$ 10

7. (a) Define Cauchy-Euler equation and Solve the differential equation 3+12

$$(x^2D^2 - 3xD + 4)y = 0$$

(b) Find the general solution of $(D^2 - 2D)y = e^{2x}sinx$. 10

8. (a) Solve: (i) $(D^4 - 81)y = 0$ 15

(ii) $((D^2 - 3D + 2)y = 0$

(iii) $(D^4 - 2D^3 + 5D^2)y = 0$

(b) Solve the differential equation $(D^3 - D^2 - 6D)y = 1 + x^2$ 10

University of Asia Pacific
Department of Civil Engineering
Final Examination Spring 2016
Program: B.Sc. Engineering (Civil)

Course Title: Numerical Analysis & Computer Programming — Course Code: CE 205
Time- 3 hours — Full marks: 150

SECTION-A

There are twelve (12) questions in this section. Answer any ten (10).
Assume any missing data reasonably.

1. Find the root of the equation $xe^x = 1$ by using the Iterative method with the accuracy of 0.0001. [12]

2. Find the root of the equation $x + lnx = 2$ by the Newton-Raphson method using the initial approximation of x_o=1. Use the accuracy of 0.0001. [12]

3. Solve the following system of equations using the Gauss-Jordan method. [12]

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

4. Fit a function of the form $y = cx^d$ to the following data. [12]

x	2	4	8	10	20	40
y	45	25	20	15	10	8

5. Evaluate numerically the following equation using the Simpson's rule with 10 panels or n=10

$$\mathrm{I} = \sum_{0}^{5} \frac{e^x\,dx}{x^2 + 1}$$ [12]

6. Solve the following differential equation to get y(2) by the Euler's method which has initial value y(0)=1.Use the step length 0.4

$$\frac{dy}{dx} = x^2 + \frac{5x}{2y + 4}$$

[12]

7. Find y(1) by solving the following differential equation using the fourth-order Runge-Kutta method which has an initial value y(0)=2.Use the step length h=0.5 [12]

$$\left(1+x^2\right)\frac{dy}{dx}+y=0$$

8. Solve the following boundary value problem to estimate y(0.5) by the Finite Difference method with step length, h=0.5.Given that, y(0)=0; y(1)=4 [12]

$$\frac{d^2y}{dx^2}-7\frac{dy}{dx}+y+4=0$$

9. (i) Derive the formulas for fitting a straight line equation. [05]

   (ii) Find the cubic polynomial which takes the following values:

x	1	3	5
y	24	120	336

[07]

   Hence obtain the value of y(6)

10. Solve the boundary value problem for y (0.25), y (0.5), y (0.75) by finite difference method with the step length h=0.25.Given y(0)=y(1)=0. [12]

$$\frac{d^2y}{dx^2}-64y+10=0$$

11. Evaluate numerically the following equations using the Trapezoidal rule.[with 4 segments] [12]

   (i) $\mathrm{I}=\sum_0^{\pi} t\ sint\ dt$

   (ii) $\mathrm{I}=\sum_{-2}^{2}\frac{t\,dt}{5+2t}$

12. Fit a Lagrange polynomial to the following data. Also find y when $x = 3.5$ [12]

x	1	2	3	5
y	0	1	24	126

SECTION-B

There are four (04) questions in this section. Answer any three (03).
Assume any missing data reasonably.

13. Write down the code which can calculate the factorial of any positive integer using While loop. [10]

14. Write a program that prompts the user to input three integer values and find the maximum among those. [10]

15. Write the output of the following program. [10]

```
#include <iostream>
#include <math.h>
using namespace std;

int main()

{
    int i,j;

    for (i=1;i<=10;i++)
    {
        for (j=1;j<=i;j++)
        {
            cout << j;
        }
        cout << endl;
    }
    return 0;
}
```

16. Write the output of the following loop. [10]

```
#include <iostream>
using namespace std ;
int main ()

{
    int i=0,sum=0;
    do

    {
        i++;
        sum+=i;
    }
    while (i<10);
        cout<<sum<<endl;
    return 0;
}
```

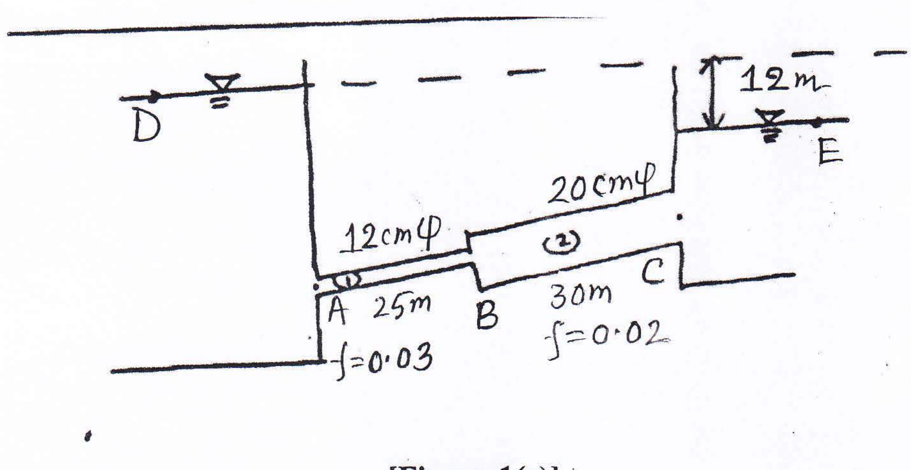
University of Asia Pacific
Department of Civil Engineering
Final Examination Spring 2016
Program: B.Sc. Engineering (Civil)

Course Title: Fluid Mechanics — Course Code: CE 221
Time- 3 hours — Full marks: 150

SECTION-A

There are four (04) questions in this section. Answer any three (03).
Assume any missing data reasonably.

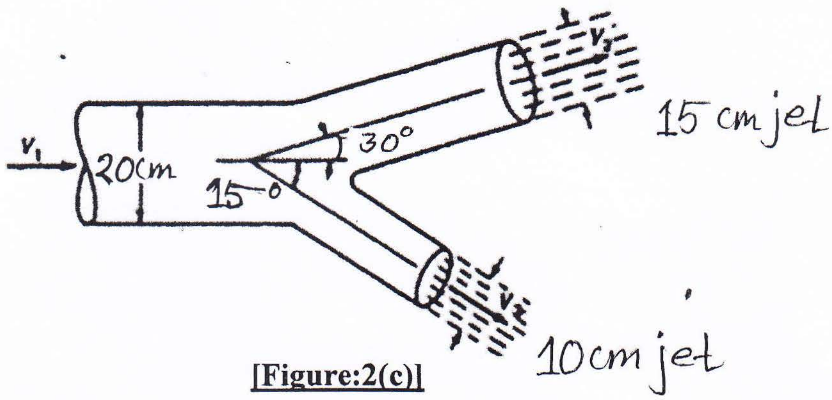
1. **(a)** Describe the relationship between velocity and head loss in a uniform pipe. **[08]**
   (b) Derive the equation of shear stress for a circular pipe. [use Darcy Weisbach equation for derivation] **[05]**
   (c) Two reservoirs with a difference in water surface elevation of 12 m are connected by a pipeline ABC which consists of two pipes AB and BC, joined in series pipe. AB is 12 cm in diameter, 25m long and has a f = 0.03. Pipe BC is of 20 cm diameter, 30 m long and has a f=0.02.The junction with the reservoirs and between the pipes are abrupt. Calculate the discharge.[Figure 1(c)] **[12]**



[Figure:1(c)]

2. **(a)** Derive an expression for the force when a jet of water strikes a stationary flat plate normally and leaves tangentially. **[06]**
   (b) Derive an expression for the force when a jet strikes the vane tangentially and is deflected through an angel 45°. **[06]**

(c) Determine the magnitude of the resultant force exerted on this double nozzle. Both nozzle jets have a velocity of 8 m/s. The axis of the pipe & both nozzles lies in a horizontal plane, $\gamma = 9.81$ KN/m^3. Neglect friction. [Figure 2(c)] **[13]**

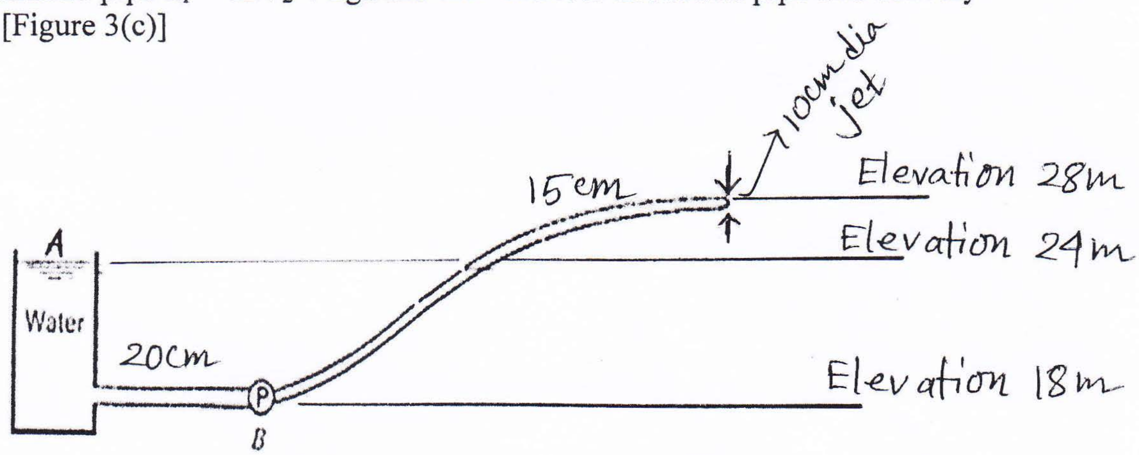


[Figure:2(c)]

3. **(a)** Derive the general equation of continuity in case of steady incompressible flow. **[10]**

(b) What is loss of head at submerged discharge? **[02]**

(c) A pipeline with a pump leads to a nozzle as shown in fig below. Find the flow rate when the pump develops a head of 30 m. Assume the head loss in the 20 cm diameter pipe may be expressed by $h_f = 5V_1^2 / 2g$, while the head loss in the 15 cm diameter pipe $h_f = 12V_2^2 / 2g$. Find the head loss in suction pipe and delivery pipe. [Figure 3(c)] **[13]**

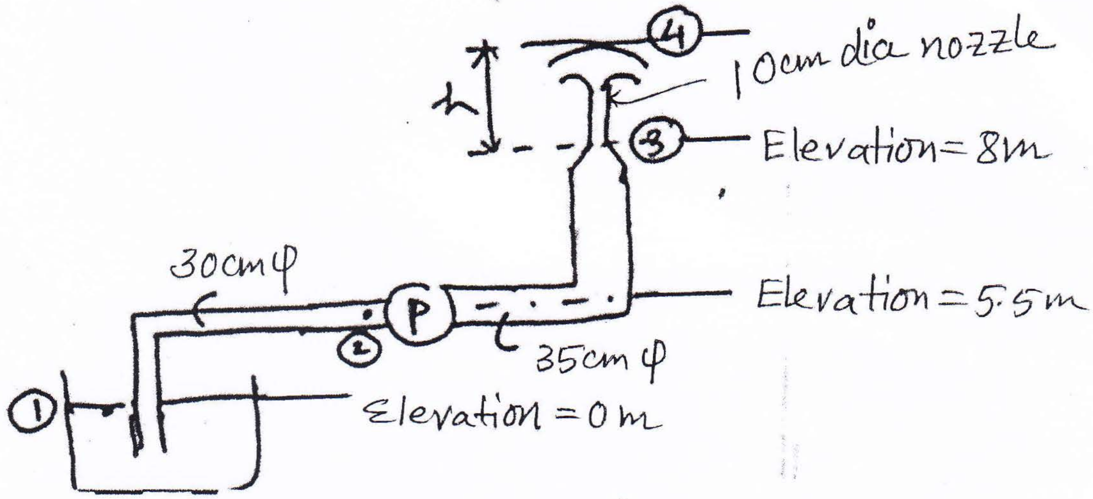


[Figure:3(c)]

4. **(a)** What do you understand by the term impulse momentum equation? **[02]**

(b) Briefly explain Reynolds Experiment. How can you determine characteristics of flow from Reynolds Experiment? **[08]**

(c) A pump is 5.5m above the water level in the sump & has a pressure of -5.00 m of the water at the suction side. The suction pipe is of 30 cm diameter & the delivery pipe is a short 35 cm diameter pipe ending in a nozzle of 10 cm dia. If the nozzle is directly vertically upwards at an elevation of 8 m above the sump water level, determine (i) the discharge, (ii) power input into the flow by the pump. [Figure 4(c)] **[15]**



[Figure:4(c)]

SECTION-B

There are four (04) questions in this section. Answer any three (03).
Assume any missing data reasonably.

5. **(a)** What is flow net? Write down the uses and limitations of flow net. **[05]**

(b) Differentiate between **[06]**

(i)Compressible and Incompressible flow.

(ii)Steady and Uniform flow.

(c) A velocity field is given by: **[14]**

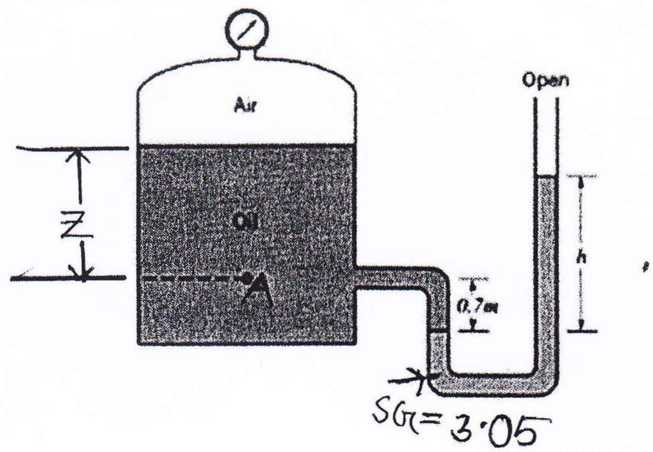
$$\bar{U} = (1 + At + Bt^2)\,\hat{\imath} + x\,\hat{\jmath}$$

Find the equation of the streamline at $t = t_0$ passing through the point (x_0, y_0)

6. **(a)** Discuss different types of manometer. **[06]**

(b) Define absolute and gage pressure. Differentiate between them. **[05]**

(c) A U-Tube manometer is connected to a closed tank. The air pressure in the tank is 120 Pa & the liquid in the tank is oil ($\gamma = 12000\ N/m^3$)· The pressure at point A is 20 Kpa. Determine: (a) the depth of oil, *z* and (b) the differential reading, *h*, on the manometer. [Figure 6(c)] **[14]**

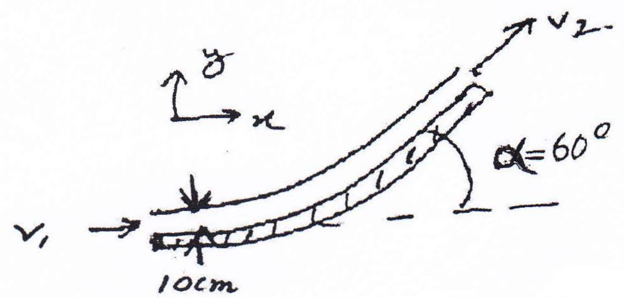


[Figure:6(c)]

7. (a) What is equivalent length method? Establish the equivalent length equation in case of pipes in series connection. **[06]**

(b) In which case center of pressure and center of gravity coincides in submerged plane surface. Prove mathematically. **[11]**

(c) Assume $\alpha=60°$,$v_1=30$ m/s ,the stream is a jet of water with an initial diameter of 10 cm as shown in figure .If friction was to be neglected ,find the resultant force on the blade. Assume flow occurs in a horizontal plane. [Figure 7(c)] **[08]**



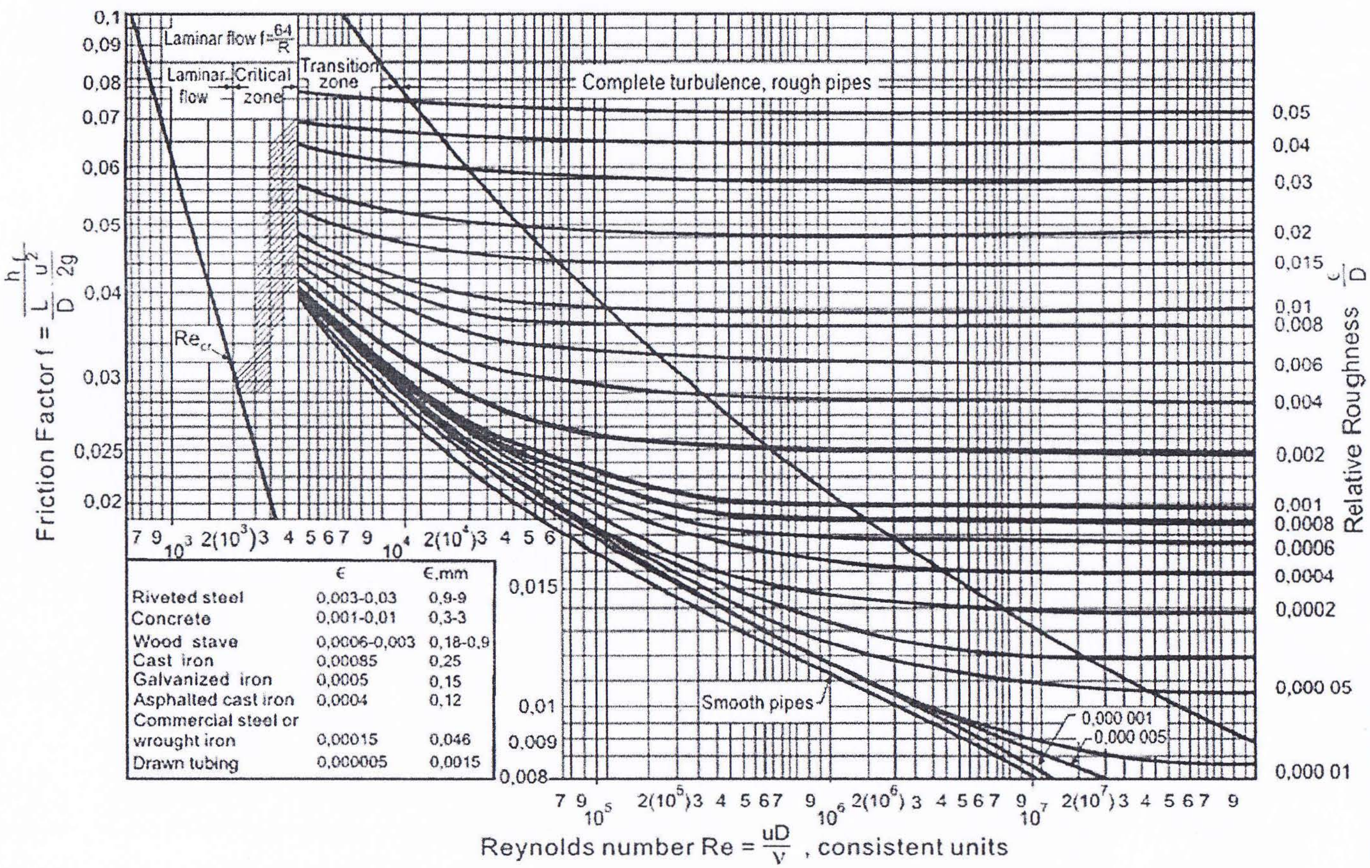
[Figure:7(c)]

8. (a) Prove Bernoulli's equation. **[11]**

(b) Derive the expression for the forces on the fluid in a reducer. **[07]**

(c) A compound pipe system consists of 1800m of 50 cm, 1200 m of 40 cm and 600m of 30 m pipes of the same material connected in a series. **[07]**

(i)What is the equivalent length of a 40 cm pipe of the same material?



	ϵ	ϵ,mm
Riveted steel	0,003-0,03	0,9-9
Concrete	0,001-0,01	0,3-3
Wood stave	0,0006-0,003	0,18-0,9
Cast iron	0,00085	0,25
Galvanized iron	0,0005	0,15
Asphalted cast iron	0,0004	0,12
Commercial steel or wrought iron	0,00015	0,046
Drawn tubing	0,000005	0,0015

[Figure:8 (Moody Diagram)]

University of Asia Pacific
Department of Civil Engineering
Final Examination Spring 2016
Program: B. Sc. Engineering (Civil)

Course # : CE-203 — Course Title: Engineering Geology & Geomorphology
Full Marks: 120 (6 X 20 = 120) — Time: 3 hours

Section A

There are **four (4)** questions in this section. Answer **any three (3) [3x20=60]**

1. (a) Discuss Igneous rock. Giving examples distinguish between sediments and sedimentary rocks. 8

(b) Classify (mention names only) geomorphic processes based on origin. Write down the names of major geomorphic agents. 5

(c) What are physical and chemical weathering processes? Discuss, in brief, the physical weathering processes. 7

2. (a) What is diastraphism? Draw neat sketch of a typical fold geometry showing its major features. ~~5~~ 7

(b) Write short notes on folds, faults and joints. ~~6~~ 7

(c) Draw neat sketches of Graben and oblique fault. ~~5~~ 6

~~(d) With the aid of a neat sketch show different elements of a typical fold geometry.~~ ~~4~~

3. (a) Classify (mention names only) folds and discuss any two types showing neat sketches. 6

(b) Discuss liquefaction phenomenon in the light of its basic mechanism and aftermaths. 8

(c) Classify and discuss briefly (no sketch required) any two types of waves generated due to earthquake. 6

4. Briefly discuss, mention or draw sketches, as asked for, on **any four** of the following topics:- 5 X 4 = 20

(i) Schematic diagram of rock cycle

(ii) Principal zones of earth (names only) with a schematic diagram showing the thicknesses of different parts of lithosphere/geosphere.

(iii) Classification of mineral with examples

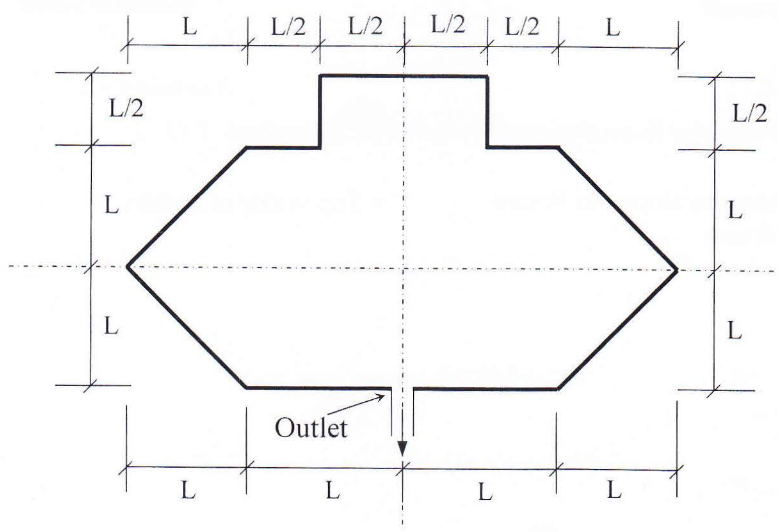
(iv) A few physical properties of mineral and distinction between Ferro-Magnesian and Non-Ferro-Magnesian Silicates

(v) Major earthquake parameters (geometric) with neat sketches

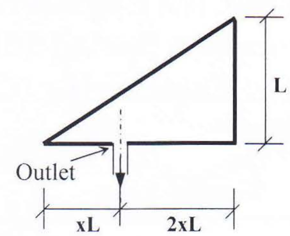
Section B

There are **four (4)** questions in this section, answer **any three (3) [3x20=60]**

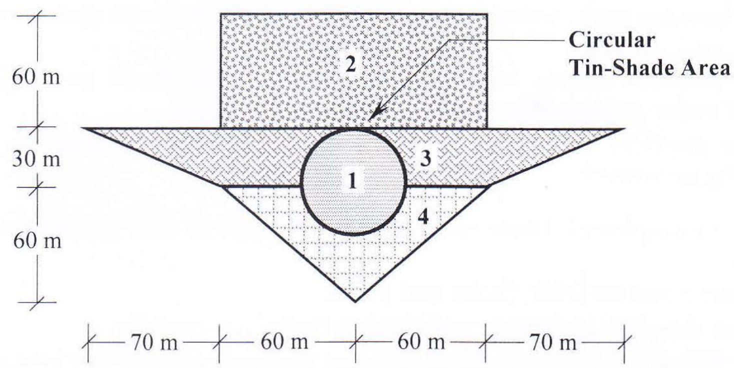
5. (a) Calculate (FF) and (CC) of the following basin. 5



(b) In the following basin, for what value of x, the flow rate (Q_p) will be the maximum? Also calculate the CC of the basin. 8



(c) For the drainage area as shown below, calculate peak runoff in ft^3/s. Use $C_2 = (C_3 + C_4 - 0.3)$, $C_3 = (C_4 - 0.2)$ and $C_4 = 0.6$ and I = 0.05 in/min. 7

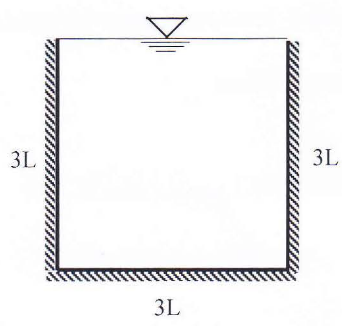


6. (a) What are the major causes of river erosion? Mention three hydraulic actions responsible for river erosion 3

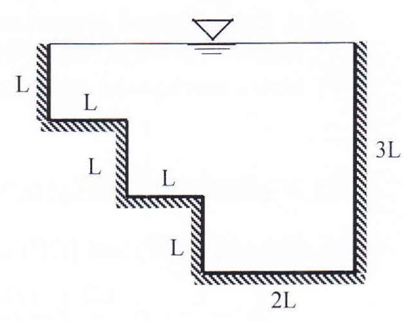
(b) Prove that $d \propto v^2$; where symbols carry their usual meanings. 7

(c) Prove that $\tau = \gamma_\omega R_H s$; where symbols carry their usual meanings. 5

(d) The cross-sectional profiles at two locations (location-1 and location-2) of a river are shown in the figures below. The gradient (s), unit weight of water and x-sectional area ($A = 9L^2$) of these two locations are same. Mention (if all other factors affecting erosion remain constant) which location will exhibit more erosion? Justify your answer. 5



Location - 1



Location - 2

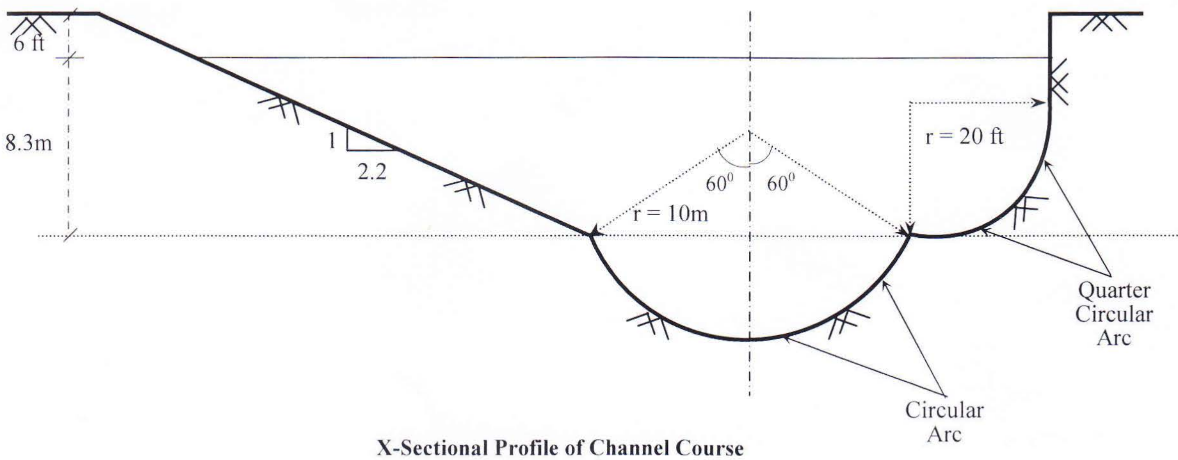
7. (a) For a stream having triangular X-section and T <<<<< D, prove that $\tau \propto T$ 4

where

τ = tractive pressure along the stream | T = Top width of stream

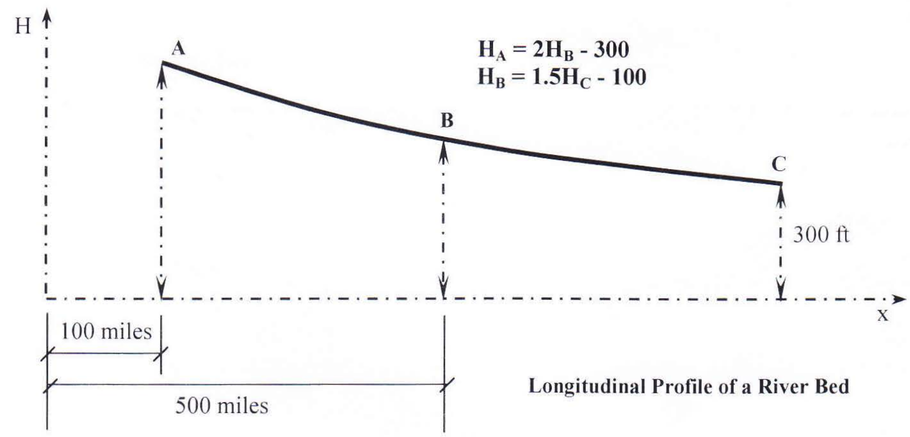
D = depth of stream

(b) Cross-sectional profile of a channel is shown below. The gradient of the channel bed is 5.67×10^{-3}. Calculate the tractive pressure along the channel. 11



X-Sectional Profile of Channel Course

(c) Using the figure shown below, calculate the horizontal distance between B and C. 5



Longitudinal Profile of a River Bed

8. (a) Mention the laws of stream order/rank with diagram. 3

(b) Calculate Stream Frequency (SF) of a catchment area (having DD = 0.0340744 Km/Km^2) from the information provided in the table below. 7

Stream Rank	No. of Streams (Ns_i)	BR	ABR	Mean Length (Lm_i, Km)	LR	ALR
1	---	2.143	2.492	---	3.0	2.5
2	---	---		---	---	
3	3	---		---	2.5	
4	---			200		

(c) Mention the factors affecting drainage pattern. Classify and discuss, in brief with sketches, any one type of drainage pattern. 5

(d) Discuss, in brief, the ways valleys are deepened. 5

6. For the beams shown in Fig. 5(a), and Fig. 5(b)
   (i) Write down the equations for load $w(x)$ using singularity functions.
   (ii) Write down the boundary conditions.
   (iii) Determine whether the beams are statically determinate or indeterminate.
   (iv) Draw qualitative deflected shapes of the beams under the given loads.

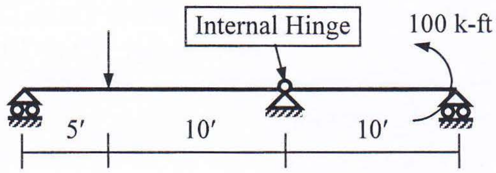


Fig. 5(a)

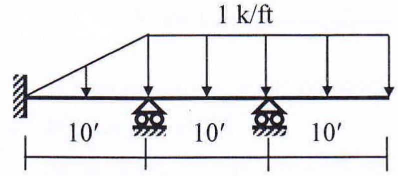


Fig. 5(b)

7. Fig. 6(a) shows floor area of a building that includes a beam $B_1B_2B_3$ supported on two stiff columns at B_1, B_3 and a flexible column at B_2. Fig. 6(b) shows an idealized view of the beam, carrying a uniformly distributed load of 50 kN/m with support B_2 modeled as a spring.

   Calculate the vertical deflection and corresponding reaction at spring B_2

   [Given: Stiffness of spring $B_2 = k_{B2} = 1000$ kN/m, EI of Beam $B_1B_2B_3$ = constant = 100×10^3 kN-m^2]

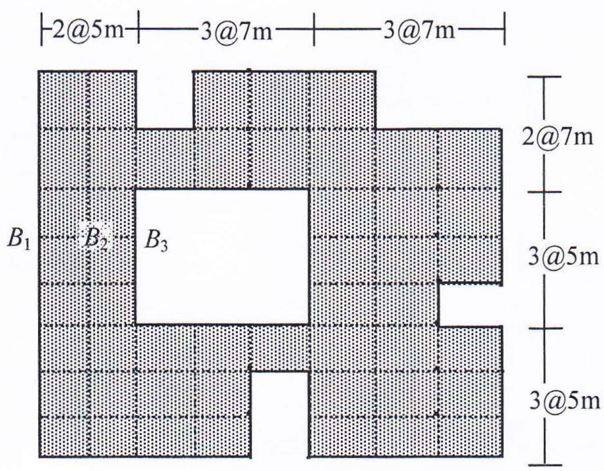


Fig. 6(a)

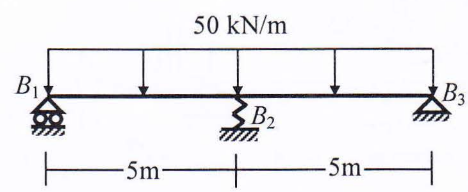


Fig. 6(b)

8. A 21-m long beam *aob* and 15-m long beam *cod* are used perpendicular to each other to connect across a ($21^m \times 15^m$) void.

   Fig. 7(a) shows the two beams intersecting at their common midspan *o*, where a concentrated force P = 100 kN acts vertically, while Fig. 7(b) shows the beams are separated and acted upon by separate forces P_1 and P_2 at midspan *o*, so that $P = P_1 + P_2$.

   Use *Singularity Functions* to calculate the
   (i) Forces P_1 and P_2 to ensure the same vertical deflection at *o* for both beams
   (ii) Corresponding vertical deflection at *o* and rotations at *a* and *d*

   [Given: $EI_{ab} = 100 \times 10^3$ kN-m^2, $EI_{cd} = 50 \times 10^3$ kN-m^2].

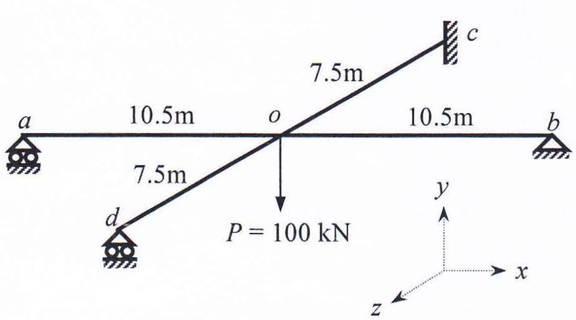


Fig. 7(a)

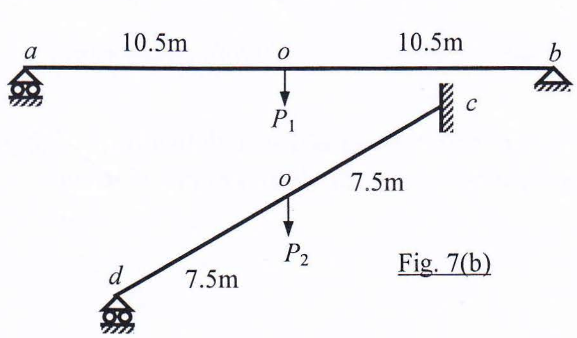


Fig. 7(b)

9. Answer Question 8 using the *Moment-Area Theorems*.

11. Fig. 8(a) shows a simply supported arch *aob* spanning as rooftop canopy over a void span of 16-m, with a concentrated force of F_0 (= 10% of its buckling load) acting upward at its midspan.

Consider *aob* as a column with initial imperfection and a cross-sectional area as shown in Fig. 8(b).

If the column is also acted upon by a horizontal force P (= Half its buckling load), calculate the

(i) Deflection

(ii) Bending Moment

at midspan (o) of the column.

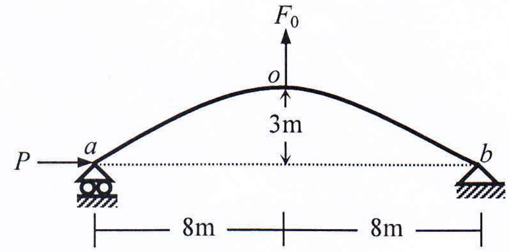


Fig. 8(a)

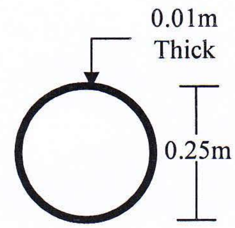


Fig. 8(b)

12. Fig. 9 shows a simply supported truss *aob* spanning as rooftop canopy over a void span of 16-m, with a concentrated force F_0 acting upward at its midspan.

Use AISC-ASD method to determine the allowable value of F_0, considering only the members across the section *x-x*, if its member cross-section is as shown in Fig. 8(b)

[Given: $E = 200$ GPa, $f_y = 500$ MPa].

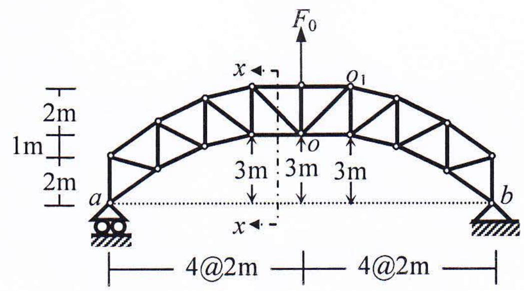


Fig. 9

13. Member oo_1 of the truss shown in Fig. 9 has a cross-section shown in Fig. 8(b) and is made of a material whose stress-strain relationship is $\sigma = 1000(\varepsilon)^{0.5}$, where σ is the stress (MPa), and ε is the strain.

Calculate the critical load for the member oo_1.

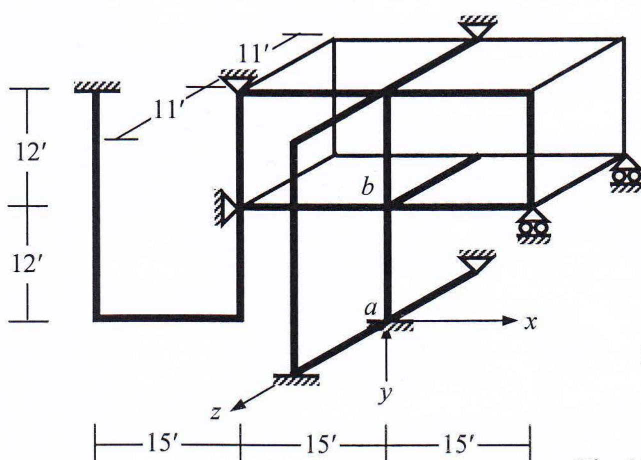
14. In the 3D frame shown in Fig. 10 (along with cross-section of all beams and columns), calculate the

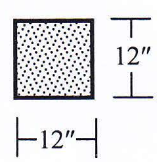
(i) Effective length factor

(ii) Critical buckling load

of the column *ab* about both the *x*- and *z*-axis

[Given: E = 3000 ksi for all members].





Beam and Column section

Fig. 10

List of Useful Formulae for CE 213

* Torsional Rotation $\phi_B - \phi_A = \int (T/J_{eq}G)\,dx$, and $= (TL/J_{eq}G)$, if T, J_{eq} and G are constants

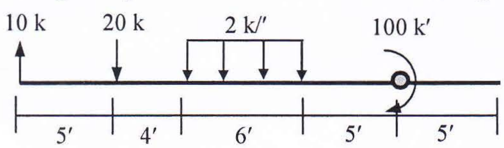
Section	Torsional Shear Stress	J_{eq}
Circular	$\tau = Tc/J$	$\pi d^4/32$
Thin-walled	$\tau = T/(2A\ t)$	$4A^2/(\oint ds/t)$
Rectangular	$\tau = T/(\alpha b t^2)$	$\beta b t^3$

b/t	1.0	1.5	2.0	3.0	6.0	10.0	∝
α	0.208	0.231	0.246	0.267	0.299	0.312	0.333
β	0.141	0.196	0.229	0.263	0.299	0.312	0.333

* Biaxial Bending Stress: $\sigma_x(z, y) = M_z\, y/I_z + M_y\, z/I_y$
* Combined Axial Stress and Biaxial Bending Stress: $\sigma_z(x,y) = -P/A - M_x\, y/I_x - M_y\, x/I_y$
* Corner points of the kern of a Rectangular Area are (b/6, 0), (0, h/6), (−b/6, 0), (0, −h/6)
* Maximum shear stress on a Helical spring: $\tau_{max} = \tau_{direct} + \tau_{torsion} = P/A + Tr/J = P/A\,(1 + 2R/r)$
* Stiffness of a Helical spring is $k = Gd^4/(64R^3N)$

* $\sigma_{xx}' = (\sigma_{xx} + \sigma_{yy})/2 + \{(\sigma_{xx} - \sigma_{yy})/2\} \cos 2\theta + (\tau_{xy}) \sin 2\theta = (\sigma_{xx} + \sigma_{yy})/2 + \sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]} \cos (2\theta - \alpha)$
  $\tau_{xy}' = -\{(\sigma_{xx} - \sigma_{yy})/2\} \sin 2\theta + (\tau_{xy}) \cos 2\theta = \tau_{xy}' = -\sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]} \sin (2\theta - \alpha)$
  where $\tan \alpha = 2\,\tau_{xy}/(\sigma_{xx} - \sigma_{yy})$
* $\sigma_{xx(max)} = (\sigma_{xx} + \sigma_{yy})/2 + \sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}$; when $\theta = \alpha/2,\ \alpha/2 + 180°$
  $\sigma_{xx(min)} = (\sigma_{xx} + \sigma_{yy})/2 - \sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}$; when $\theta = \alpha/2 \pm 90°$
* $\tau_{xy(max)} = \sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}$; when $\theta = \alpha/2 - 45°,\ \alpha/2 + 135°$
  $\tau_{xy(min)} = -\sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}$; when $\theta = \alpha/2 + 45°,\ \alpha/2 - 135°$
* Mohr's Circle: Center $(a, 0) = [(\sigma_{xx} + \sigma_{yy})/2, 0]$ and radius $R = \sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}$
* For Yielding to take place
  - Maximum Normal Stress Theory (Rankine): $|\sigma_1| \ge Y$, or $|\sigma_2| \ge Y$.
  - Maximum Normal Strain Theory (St. Venant): $|\sigma_1 - \nu\sigma_2| \ge Y$, or $|\sigma_2 - \nu\sigma_1| \ge Y$.
  - Maximum Shear Stress Theory (Tresca): $|\sigma_1 - \sigma_2| \ge Y$, $|\sigma_1| \ge Y$, or $|\sigma_2| \ge Y$
  - Maximum Distortion-Energy Theory (Von Mises): $\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \ge Y^2$

* $M(x) = EI\,\kappa \cong EI\, d^2v/dx^2$
* $w(x) \cong EI\, d^4v/dx^4$, $V(x) = \int w(x)\,dx \cong EI\, d^3v/dx^3$, $M(x) = \int V(x)\,dx \cong EI\, d^2v/dx^2$
  $S(x) = \int M(x)\,dx \cong EI\, dv/dx \cong EI\,\theta(x)$, $D(x) = \int S(x)\,dx \cong EI\, v(x)$
* Singularity Functions for Common Loadings



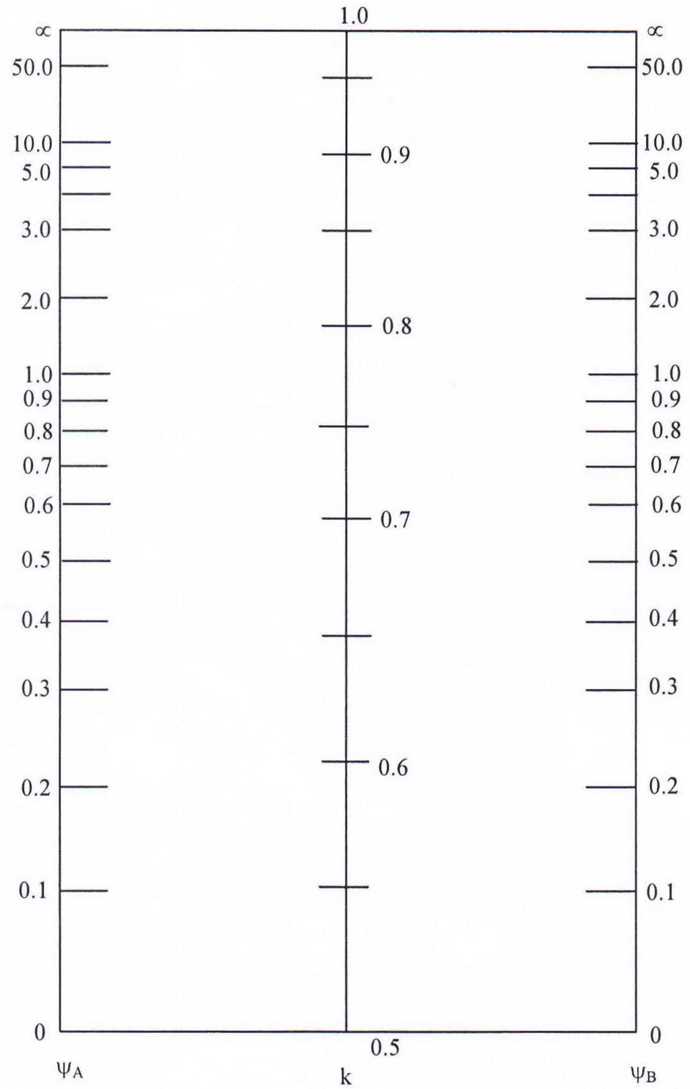
$w(x) = 10\langle x-0\rangle^{-1}{}_* - 20\langle x-5\rangle^{-1}{}_* - 2\langle x-9\rangle^0 + 2\langle x-15\rangle^0 + 100\langle x-20\rangle^{-2}{}_* + C_\theta\langle x-20\rangle^{-3}{}_*$

* First Moment-Area Theorem: $\theta_B - \theta_A = \int (M/EI)\,dx$
* Second Moment-Area Theorem: $(x_B - x_A)\,\theta_B - v_B + v_A = \int x\,(M/EI)\,dx$
* Conjugate Beam Method

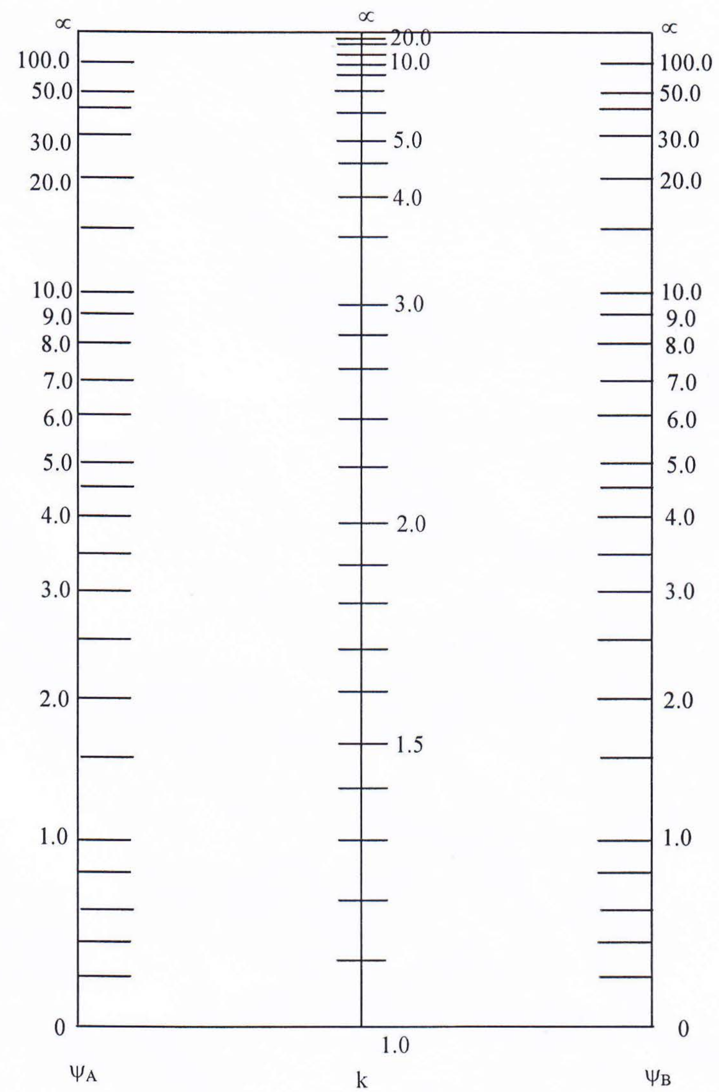
Original Beam	Free End	Fixed End	Hinge/Roller End	Internal Support	Internal Hinge
Conjugate Beam	Fixed End	Free End	Hinge/Roller End	Internal Hinge	Internal Support

* Euler Buckling Load: $P_{cr} = \pi^2 EI_{min}/(kL)^2$
* Effect of Initial Imperfection: $v(x) = v_{0i}/[1 - P/P_{cr}] \sin (\pi x/L) \Rightarrow v(L/2) = v_{0i}/[1 - P/P_{cr}]$
* Effect of Load Eccentricity: $\lambda^2 = P/EI \Rightarrow v(L/2) = e\,[\sec \lambda L/2 - 1] = e\,[\sec \{(\pi/2)\sqrt{(P/P_{cr})}\} - 1]$
* Effect of Material Nonlinearity: $P_{cr} = \pi^2 E_t I/L^2 \Rightarrow \sigma_{cr} = \pi^2 E_t/\eta^2$
* Eccentric Loading with Elasto-plastic Material:
  $v(L/2) = e\,[\sec\{(\pi/2)\sqrt{(P/P_{cr})}\} - 1]$ for the elastic range; and
  $v(L/2) = M_p/P - e$, for the plastic range
* k = 1.0 for Hinge-Hinged Beam, 0.7 for Hinge-Fixed Beam, 0.5 for Fixed-Fixed Beam, 2.0 for Cantilever Beam
* In general, k can be obtained from ψ_A and ψ_B for braced and unbraced frames
  Using approximate formulae (Salama, 2014)
  For braced frame, $k \cong \{3\,\psi_A\psi_B + 1.4\,(\psi_A + \psi_B) + 0.64\}/\{3\psi_A\,\psi_B + 2.0\,(\psi_A + \psi_B) + 1.28\}$
  For unbraced frame, $k \cong \sqrt{[\{1.6\,\psi_A\psi_B + 4.0\,(\psi_A + \psi_B) + 7.5\}/(\psi_A + \psi_B + 7.5)]}$
* AISC-ASD Method, $\eta = L_e/r_{min}$, and $\eta_c = \pi\sqrt{(2E/f_y)}$
  If $\eta \le \eta_c$, $\sigma_{all} = f_y\,[1 - 0.5\,(\eta/\eta_c)^2]/FS$, where $FS = [5/3 + 3/8\,(\eta/\eta_c) - 1/8\,(\eta/\eta_c)^3]$
  If $\eta > \eta_c$, $\sigma_{all} = (\pi^2 E/\eta^2)/FS$, where FS = Factor of safety = 23/12 = 1.92
* Moment magnification factor for a Simply Supported Beam
  For concentrated load at midspan of = $[\tan (\lambda L/2)/(\lambda L/2)]$, subjected to end moments only = $[\sec (\lambda L/2)]$
  Under UDL = $2\,[\sec (\lambda L/2) - 1]/(\lambda L/2)^2$, according to AISC code = $1/(1 - P/P_{cr})$

Alignment Charts for Effective Length Factors k



Braced Frames



Unbraced Frames

ψ = Ratio of $\sum EI/L$ of compression members to $\sum EI/L$ of flexural members in a plane at one end of a compression member
k = Effective length factor