# University of Asia Pacific <br> Department of Civil Engineering <br> Final Examination Fall 2019 <br> Program: B. Sc. Engineering (Civil) 

Credit Hours: $2.0 \quad$ Course Code: ECN 201
Time: 2 hours
Full Marks: $60(=3 \times 20)$
There are Four Questions. Answer three questions including Q-1 and Q-2.

1. a. Provide an overview of Size Distribution of Income.

What is Kuznets Ratio? What are its implications?
b. Explain why the level of new business startups is considered as an important indicator of economic well-being of a nation.
2. Explain why the total income and the total expenditure are the same in an economy.

Prepare a Circular Flow Diagram. Explain how GDP is linked to this diagram?
3. a. Define Labor Force.

What are the differences between labor Force and Economically Active Population?
b. Prepare and explain a Lorenz Curve.

## OR

4. a. Define Poverty and Inequality and their types.
b. Explain how Inequality is measured.

# University of Asia Pacific <br> Department of Civil Engineering <br> Final Examination Fall 2019 <br> Program: B. Sc. Engineering (Civil) 

Course \# : CE-203
Full Marks: 150

Course Title: Engineering Geology \& Geomorphology Time: 3 hours

## Answer to all questions

1. (a) Discuss Igneous rock. Giving examples distinguish between sediments and sedimentary rocks.
(b) Mention (names only) the principal zones of earth. With the aid of a schematic diagram show the thicknesses of different parts of lithosphere/geosphere.
2. (a) Mention (names only) different geomorphic processes based on origin.
(b) Distinguish between physical and chemical weathering processes.
(c) Compare weathering and erosion processes.
3. (a) What is diastraphism? Draw neat sketch of a typical fold geometry showing its major features.
(b) Differentiate faults and joints.
(c) Draw neat sketches of Graben and oblique fault.
4. (a) What is mineral? Classify mineral (mention names only) with examples.
(b) Discuss, in short, the basic mechanism of liquefaction phenomenon.
(c) Mention Modified Mercali Intensity (MMI) Scale of earthquake from X to XII.
(d) Discuss, in brief (no sketch required), any one type of wave generated due to earthquake.
5. (a) In the following basin, for what value of $x$, the flow rate $(Q)$ or runoff will be the
maximum?

t
x

+ 

(b) Write down three assumptions of Rational Formula.
(c) For the drainage area as shown below, calculate co-efficient of runoff $\left(C_{2}\right)$ for $Q_{p}=0.361$
$\mathrm{ft}^{3} / \mathrm{s}$ and $\mathrm{I}=0.25 \mathrm{inch} /$ hour.

6. (a) Cross-sectional profile of a channel is shown below. The gradient of the channel bed is

(b) For a stream having triangular $x$-section and $D \lll \lll T$, prove that $\tau \alpha D$, where symbols carry their usual meanings.
7. (a) Prove that $\mathrm{H}=\mathrm{ae}^{-\mathrm{bx}}$; where symbols carry their usual meanings.
(b) From the figure shown below, calculate the horizontal distance between the locations B and C .

8. (a) Classify and discuss, in brief with sketches, any one type of drainage pattern.
(b) Summarize your understanding regarding the ways valleys are widened.
(c) Mention the laws of stream order/rank with diagram.
(d) Rank the streams of the following drainage basin having a total catchment area of 10,000 square kilometer. The results of the survey are summarized in the table below.


| Steam Rank | Average <br> Length $(\mathrm{km})$ |
| :---: | :---: |
| 1 | 7.0 |
| 2 | 18.9 |
| 3 | 44.8 |
| 4 | 99.9 |

Calculate the following parameters:
(i) Average Bifurcation Ratio (ABR)
(ii) Average Length Ratio (ALR)
(iii) Stream Frequency

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## Answer the following questions.

## Assume reasonable number for the missing values <br> Marks Distribution [ $30+30+60+15+15]$

1. Describe the following fluid phenomenon.
(Answer any Six (6) from the following questions.)
(i) What is flow net? Write down the uses and limitations of flow net.
(ii) Write short note on Hydraulics radius
(iii) What do you know about Demarcation Point? Show it graphically.
(iv) Discuss the relationship between viscosity and temperature in case of fluid.
(v) Which one is the most elementary device for measuring the pressure? Discuss the reasons behind its limited use.
(vi) Briefly explain the application of fluid mechanics in Civil Engineering field.
(vii) Explain Capillarity in Fluid with net sketch.
2. Demonstrate the following equations related to Fluid Flow.
(Answer any Three (3) from the following questions.)
(i) Derive the general equation of continuity for flow through pipes. Reduce the equation for steady incompressible flow.
(ii) In steady uniform flow there is no acceleration. Prove mathematically.
(iii) State and prove Bernoulli's Theorem.
(iv) Prove mathematically that center of pressure and center of gravity is not same for a submerged plane surface. In which cases it becomes identical?
3. Solve the following numerical problems using basic Fluid Mechanics Equation. (Answer any Six (6) from the following questions.)
(i) A homogenous $4-\mathrm{ft}$ wide, 8 - ft long rectangular gate, weighing 800 lbf is held in place by a horizontal flexible cable as shown in the Figure 01. Water acts against the gate which is hinged at point A. Friction in the cable is negligible. Determine the tension in the cable.


Figure 01
(ii) A pump draws water from reservoir A and lifts it to reservoir B as shown in Figure 02. The loss of head from A to 1 is 3 times the velocity head in the 150 mm pipe and the loss of head from 2 to B is 20 times the velocity head in the 100 mm pipe. Compute the horsepower output of the pump and the pressure heads at 1 and 2 when the discharge is (a) $12 \mathrm{~L} / \mathrm{s}$; (b) $36 \mathrm{~L} / \mathrm{s}$


Figure 02
(iii) A pipeline 25 m long as shown in Figure 03 below is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 10 m of its length from the tank, the pipe is I 5 cm in diameter and its diameter suddenly enlarges to 30 cm . Considering major and minor losses determine the rate of flow.


Figure 03
(iv) The pressure of water flowing through a pipe is measured by the arrangement shown in Figure 04. For the values given, calculate the pressure in the pipe.


Figure 04
(v) Water is flowing in a conduit shown in the following Figure 05 with a vapor pressure of $30 \mathrm{KN} / \mathrm{m}^{2}$. Atmospheric pressure is 65 cm Hg . Find the maximum flow rate (Q) that will cause cavitation. (Assume, total head loss of the conduit to be 2.5 m .)


Figure 05
(vi) Determine the magnitude of the resultant force exerted on this double nozzle shown in Figure 06. Both nozzle jets have a velocity of $20 \mathrm{~m} / \mathrm{s}$. The axis of the pipe and both nozzles lies in a horizontal plane, $\gamma=9.81 \mathrm{KN} / \mathrm{m}^{3}$. Neglect friction.


Figure 06
(vii) In a flow the velocity vector is given by $V=2.5 x i+3 y j-9 z k$. Determine the equation of the streamline passing through a point $M(2,3,4)$.
4. Solve the following problems related to fluid phenomenon and analyze the impact on fluid system.
(Answer any two (2) from the following questions.)
(i) A piston of weight 21 lb slides in a lubricated pipe as shown in Figure 07. The clearance between piston and pipe is 0.001 inch. If the piston decelerates at $2.1 \mathrm{ft} / \mathrm{s}^{2}$ when the speed is $21 \mathrm{ft} / \mathrm{s}$, what is the viscosity of the oil?


Figure 07
(ii) A rectangular tank is shown in Figure 08. The tank contains Kerosene which has a unit weight of $12 \mathrm{KN} / \mathrm{m}^{3}$. (i) Determine hydrostatic pressure at point $A$ and $P$. (ii). Total hydrostatic force on $A B C D$ and $A B E F$.


Figure 08
(iii) The following information are given for the parallel pipe flow connection with three pipes shown in Figure 09.
$\mathrm{L}_{1}=0.45 \mathrm{~km}, \mathrm{~d}_{1}=600 \mathrm{~mm}, \mathrm{f}_{1}=0.021 ; \mathrm{L}_{2}=0.3 \mathrm{~km}, \mathrm{~d}_{2}=400 \mathrm{~mm}, \mathrm{f}_{2}=0.018 ; \mathrm{L}_{3}=0.6$ $\mathrm{km} \mathrm{m}, \mathrm{d}_{3}=800 \mathrm{~mm}, \mathrm{f}_{3}=0.019$. Rate of flow is given as $1 \mathrm{~m}^{3} / \mathrm{s}$.
Determine the head loss between A and B .


Figure 09

## 5. Explain the following problems with justification.

(i) According to Bernoullis law the total head remains constant. The total head is different for the sytem given below shown in Figure 10. How do you justify the changes in total head? Mention every issues.


Figure 10
(ii) An oil ( $\mathrm{S}=1.5$ ) having a kinematic viscosity of 60 stokes is flowing through a pipe of 30 cm radius through an operating system in a hydraulic laboratory. Suppose you are a hydraulic engineer and you have to consider the frictional losses in your operating system. You have found that the discharge rate is $100 \mathrm{l} / \mathrm{s}$. How can you relate the giving parameters to decide what type of flow the system is running?


Figure 7.13 Moody diagram. (From L. F. Moody, Trans. ASME, Vol. 66, 1944.)

# University of Asia Pacific <br> Department of Civil Engineering <br> Final Examination Fall 2019 <br> Program: B.Sc. Engineering (Civil) 

Credit Hour: 3.00

## Answer all the Questions

1. Identify the root of $f(x)=\cos x-x e^{x}$ using Newton Raphson method.
2. The deflection ( $\delta$ ) of a cantilever beam at different distances $(x)$ from one end is shown
in Table-1. Calculate $\delta$ for $\mathrm{x}=10 \mathrm{ft}$ using Interpolation Method.
Table-1

| $\mathrm{x}(\mathrm{ft})$ | 3 | 6 | 9 | 12 |
| :---: | :---: | :---: | :---: | :---: |
| $\delta(\mathrm{~mm})$ | 3 | 11 | 22 | 35 |

3. Apply the least-square method to fit an equation of the form $\delta=a x^{n}$ for the data shown in Table-1. Using the best-fit equation, calculate $\delta$ for $\mathrm{x}=10 \mathrm{ft}$.
Compare the two results (Question 2 and Question 3) of $\delta$ for $x=10 \mathrm{ft}$.
4. a. Derive Simpsons rule as a numerical method for integration.

For the cantilever beam shown in Figure 01, the equation of the variation of the distributed force on the beam is: $y=7 e^{x}+3 x^{2}+1$ where, $x$ is the distance from left. Determine the upward support reaction force using the equation $R=\int y d x$. Use the following methods and compare the result.


## Figure 01

b) Simpson's rule with 10 panels or $\mathrm{n}=10$.
c) Gauss Quadrature with 3 points or $\mathrm{n}=3$

| $x i$ | $w i$ |
| :---: | :---: |
| +0.8611363116 | 0.3178548451 |
| +0.3399810436 | 0.6521451549 |
| -0.3399810436 | 0.6521451549 |

5. Solve the following boundary value problem with step length 0.5 .

$$
\begin{equation*}
\frac{d^{2} y}{d x^{2}}+\frac{4 x}{1+x^{2}} \frac{d y}{d x}+\frac{2 y}{1+x^{2}}=0, \quad y(0)=0, y(2)=0.2 \tag{14}
\end{equation*}
$$

6. Apply Gauss-Seidel method and Jacobi method to solve the following system of linear $(8+7+1)$ equations.

$$
\begin{aligned}
5 x_{1}+x_{2}-2 x_{3} & =7.74 \\
2 x_{1}+12 x_{2}+3 x_{3} & =39.66 \\
3 x_{1}-3 x_{2}+15 x_{3} & =54.8
\end{aligned}
$$

Provide your comments on the performance of two methods.
7. Determine $y(0.8)$ by solving the following differential equation using Predictor-

Corrector method (Milne's method). Assume the accuracy. Given that,

$$
\begin{aligned}
& \frac{d y}{d x}=x-y^{2} \\
& y(0)=0 \\
& y(0.2)=0.02 \\
& \mathrm{y}(0.4)=0.0795 \\
& \mathrm{y}(0.6)=0.1762
\end{aligned}
$$

8. Write a program to calculate vertical reaction of fixed support as shown in Figure 01 using trapezoidal rule.
9. a) Write a program that reads the elements of two matrices, $\mathrm{A}[4][2]$ and $\mathrm{B}[2][4]$, and displays the product matrix of A and B i.e. $\mathrm{C}[4][4]=\mathrm{A}[4][2] \times \mathrm{B}[2][4]$.
b) Write a program using User defined function that reads a number and displays its multiplication table up to $n$ - $t h$ term.

# Department of Civil Engineering Final Examination Fall 2019 (Set 1) Program: B. Sc. Engineering (Civil) 

Course Title: Mechanics of Solids II
Credit Hours: 3.0
Course Code: CE 213
Time: 3 hours
Full Marks: $100(=10 \times 10)$

1. Calculate the equivalent polar moment of inertia $\left(J_{\text {eq }}\right)$ for the cross-section shown in Fig. 1(a) by centerline dimensions
[Given: Wall thickness $=0.10^{\prime}$ ].


Fig. 1(a)

OR
Fig. 1(b) shows a person wearing a fruit-peel as protection from corona-virus.
While covering, the peel is subjected to uniform pressure resulting in tensile and shear stresses.
Fig. 1(c) represents the tensile stresses

$$
\sigma_{x}=3 \mathrm{kPa}, \sigma_{y}=3 \mathrm{kPa}
$$

and shear stress $\tau=2 \mathrm{kPa}$ in a small element of the fruit-peel.



Fig. 1(c)
(i) Draw the Mohr's Circle of stresses for the element shown in Fig. 1(c), specifying the Principal Stresses and Principal Planes
(ii) Use Rankine criterion to determine yield strength $Y$ required to avoid yielding of the fruit-peel.
2. Fig. 2(a) shows a person wearing a giraffe-costume as protection from corona-virus.

In addition to self-weights $W_{1-4}\left(W_{1}=30 \mathrm{lb}, W_{2}=40 \mathrm{lb}, W_{3}=20 \mathrm{lb}, W_{4}=10 \mathrm{lb}\right)$, he is subjected to horizontal forces $H_{1-4}$ ( $=30 \%$ of self-weights) as shown in Fig. 2(b).
Calculate the
(i) Maximum normal stresses on his shoes ( $1.5^{\prime}$-dia circle), shown in Fig. 2(c)
(ii) Maximum shear stress and deflection of the two helical springs [Fig. 2(d)] (Mean diameter $=1.5^{\prime \prime}$, Shear Modulus $=10 \times 10^{6} \mathrm{psi}$, No. of Coils $=3$, Coil Diameter $=0.1^{\prime \prime}$ ) supporting each shoe.


Fig. 2(a)


Fig. 2(b)


Fig. 2(c)

Fig. 2(d)
3. Fig. 3 shows a beam $a b c$ defgh carrying a distributed load ( $1 \mathrm{lb} / \mathrm{ft}$ ) from a snake over the length $b c$ and concentrated loads $(1 \mathrm{lb}, 1 \mathrm{lb}, 1 \mathrm{lb})$ at $(e, f, g)$ from three bats.


Use Moment-Area Theorems to calculate
(i) The value of $E I$ to cause $d$ to deflect 2-inch vertically
(ii) Rotation at $a$, for the value of $E I$ calculated in (i).
4. Answer Question 3 using the Conjugate Beam Method.
5. If the roller support $a$ (of beam abcdefgh in Fig. 3) is replaced by a fixed support (shown in Fig. 4), use Moment-Area Theorems OR Conjugate Beam Method to calculate
(i) The value of $E I$ to cause $d$ to deflect 2-inch vertically
(ii) Rotation at $a$, for the value of $E I$ calculated in (i).


Fig. 4
6. Answer Question 3 using the Singularity Functions.

OR
(i) For Fig. 5 showing a snake on a branch (with variable self-weight)

- Write equation for load $w(x)$ using singularity functions
- Write down the boundary conditions
- Draw the qualitative deflected shape
- Determine if it is statically determinate or indeterminate.

(ii) Explain the
(a) Physical inconsistency of Euler's initial formulation for the buckling behavior of slender columns
(b) Combined effect of load eccentricity and Plastic Moment ( $M_{p}$ ) on buckling behavior of slender columns.


7. Fig. 6(a) shows an emergency corona-virus hospital built in China in two days, while Fig. 6(b) shows its simplified schematic diagram. Since the soil condition is uncertain, various foundation models are assumed at $a_{0}, b_{0}, c_{0}, d_{0}$ and $e_{0}$.
Under the circumstances, calculate the critical buckling loads ( $P_{c r}$ ) for Columns $a_{0} a_{1}, b_{0} b_{1}, c_{0} c_{1}, d_{0} d_{1}$ and $e_{0} e_{1}$, assuming $E I=$ Constant $=10,000 \mathrm{k}-\mathrm{ft}^{2}$.

8. The hospital building shown in Figs. 6(a), 6(b) is constructed of a nonlinear material with stress-strain relationship $\sigma=10[1-\operatorname{Cos}(100 \varepsilon)]$, where $\sigma$ is compressive stress (ksi) and $\varepsilon$ is strain.
Calculate the critical force for the member $b_{1} b_{2}$, which has a $\left(1^{\prime} \times 1^{\prime}\right)$ square section.
9. For the emergency hospital building shown in Figs. 6(a), 6(b), use AISC-ASD method to calculate the allowable compressive force for member $c_{1} c_{2}$, using Salama (2014) to determine $k$
[Given: $c_{1} c_{2}$ is a $\left(1^{\prime} \times 1^{\prime}\right)$ square section,
Yield strength $f_{y}=10 \mathrm{ksi}$, Modulus of elasticity $\left.E=1000 \mathrm{ksi}\right]$.

## OR

For the emergency hospital building shown in Figs. 6(a), 6(b), calculate distributed load $w(\mathrm{k} / \mathrm{ft})$ required to cause Moment Magnification Factor $M M F=2.0$ (using AISC formula) for member $c_{1} c_{2}$.
10. Fig. 7(a) shows a soil-excavator used for land development to build the emergency corona-virus hospital described in Question 7, while Fig. 7(b) represents its simplified schematic diagram.
(i) Calculate the Buckling force $P_{c r}$ of the member $m n\left(E I=10,000 \mathrm{k}-\mathrm{ft}^{2}\right.$, initial deflection $\left.v_{0 i}=7^{\prime}\right)$ and the compressive force $\left(P_{m n}\right)$ required for it to deflect $10^{\prime}$ at midspan.


Fig. 7(a)


Fig. 7(b)
(ii) Use the free-body of member rns to calculate the upward force $R$ required to cause compressive force $\left(P_{m n}\right)$ in member $m n$, as calculated in (i) for it to deflect $10^{\prime}$ at midspan.

## List of Useful Formulae for CE 213

* Torsional Rotation $\phi_{\mathrm{B}}-\phi_{\mathrm{A}}=\int\left(\mathrm{T} / \mathrm{J}_{\mathrm{eq}} \mathrm{G}\right) \mathrm{dx}$, and $=\left(\mathrm{TL} / \mathrm{J}_{\text {eq }} \mathrm{G}\right)$, if $\mathrm{T}, \mathrm{J}_{\text {eq }}$ and G are constants

| Section | Torsional Shear Stress | $\mathbf{J}_{\mathbf{e q}}$ |
| :---: | :---: | :---: |
| Circular | $\tau=\mathrm{Tc} / \mathrm{J}$ | $\pi \mathrm{d}^{4} / 32$ |
| Thin-walled | $\tau=\mathrm{T} /(2(\mathrm{~A}) \mathrm{t})$ | $4 \Theta \mathrm{~A}^{2} /(\mathrm{Jds} / \mathrm{t})$ |
| Rectangular | $\tau=\mathrm{T} /\left(\alpha \mathrm{bt} \mathrm{t}^{2}\right)$ | $\beta \mathrm{bt}^{3}$ |


| $\mathrm{b} / \mathrm{t}$ | 1.0 | 1.5 | 2.0 | 3.0 | 6.0 | 10.0 | $\propto$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | 0.208 | 0.231 | 0.246 | 0.267 | 0.299 | 0.312 | 0.333 |
| $\beta$ | 0.141 | 0.196 | 0.229 | 0.263 | 0.299 | 0.312 | 0.333 |

* Biaxial Bending Stress: $\sigma_{x}(z, y)=M_{z} y / I_{z}+M_{y} z / I_{y}$
* Combined Axial Stress and Biaxial Bending Stress: $\sigma_{z}(x, y)=-P / A-M_{x} y / I_{x}-M_{y} x / I_{y}$
* Corner points of the kern of a Rectangular Area are (b/6, 0), (0, h/6), (-b/6, 0), ( $0,-\mathrm{h} / 6$ )
* Maximum shear stress on a Helical spring: $\tau_{\max }=\tau_{\text {direct }}+\tau_{\text {torsion }}=\mathrm{P} / \mathrm{A}+\mathrm{Tr} / \mathrm{J}=\mathrm{P} / \mathrm{A}(1+2 \mathrm{R} / \mathrm{r})$
* Stiffness of a Helical spring is $\mathrm{k}=\mathrm{Gd}^{4} /\left(64 \mathrm{R}^{3} \mathrm{~N}\right)$
* $\sigma_{\mathrm{xx}}{ }^{\prime}=\left(\sigma_{\mathrm{xx}}+\sigma_{y y}\right) / 2+\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{y y}\right) / 2\right\} \cos 2 \theta+\left(\tau_{\mathrm{xy}}\right) \sin 2 \theta=\left(\sigma_{\mathrm{xx}}+\sigma_{\mathrm{yy}}\right) / 2+\sqrt{\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right] \cos (2 \theta-\alpha)}$ $\tau_{x y}^{\prime}=-\left\{\left(\sigma_{x x}-\sigma_{y y}\right) / 2\right\} \sin 2 \theta+\left(\tau_{x y}\right) \cos 2 \theta=\tau_{x y}^{\prime}=-\sqrt{ }\left[\left\{\left(\sigma_{x x}-\sigma_{y y}\right) / 2\right\}^{2}+\left(\tau_{x y}\right)^{2}\right] \sin (2 \theta-\alpha)$ where $\tan \alpha=2 \tau_{x y} /\left(\sigma_{x x}-\sigma_{y y}\right)$
* $\sigma_{\mathrm{xx}(\max )}=\left(\sigma_{\mathrm{xx}}+\sigma_{\mathrm{yy}}\right) / 2+\sqrt{ }\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right]$; when $\theta=\alpha / 2, \alpha / 2+180^{\circ}$
$\sigma_{\mathrm{xx}(\text { min })}=\left(\sigma_{\mathrm{xx}}+\sigma_{\mathrm{yy}}\right) / 2-\sqrt{ }\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right]$; when $\theta=\alpha / 2 \pm 90^{\circ}$
${ }^{*} \tau_{\mathrm{xy}(\max )}=\sqrt{ }\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right]$; when $\theta=\alpha / 2-45^{\circ}, \alpha / 2+135^{\circ}$
$\tau_{\mathrm{xy}(\text { min })}=-\sqrt{ }\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right]$; when $\theta=\alpha / 2+45^{\circ}, \alpha / 2-135^{\circ}$
*Mohr's Circle: Center $(\mathrm{a}, 0)=\left[\left(\sigma_{\mathrm{xx}}+\sigma_{\mathrm{yy}}\right) / 2,0\right]$ and radius $\mathrm{R}=\sqrt{ }\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right]$
* For Yielding to take place

Maximum Normal Stress Theory (Rankine): $\quad\left|\sigma_{1}\right| \geq \mathrm{Y}$, or $\left|\sigma_{2}\right| \geq \mathrm{Y}$.
$\begin{aligned} & \text { Maximum Normal Strain Theory (St. Venant): } \\ & \text { Maximum Shear Stress Theory (Tresca): }\end{aligned} \quad \left\lvert\, \begin{aligned} & \sigma_{1}-v \sigma_{2} \mid \geq \mathrm{Y}, \text { or }\left|\sigma_{2}-v \sigma_{1}\right| \geq \mathrm{Y} . \\ & \sigma_{1}-\sigma_{2}\left|\geq \mathrm{Y},\left|\sigma_{1}\right| \geq \mathrm{Y}, \text { or }\right| \sigma_{2} \mid \geq \mathrm{Y}\end{aligned}\right.$
Maximum Shear Stress Theory (Tresca): $\quad\left|\sigma_{1}-\sigma_{2} \geq Y, \sigma_{1}\right| \geq Y$,
Maximum Distortion-Energy Theory (Von Mises): $\sigma_{1}^{2}+\sigma_{2}^{2}-\sigma_{1} \sigma_{2} \geq Y^{2}$

* $\mathrm{M}(\mathrm{x})=\mathrm{EI} \kappa \cong \mathrm{EI} \mathrm{d}^{2} \mathrm{v} / \mathrm{dx}^{2}$
* $w(x) \cong E I d^{4} v / d x^{4}, \quad V(x)=\int w(x) d x \cong E I d^{3} v / d x^{3}, \quad M(x)=\int V(x) d x \cong E I d^{2} v / d x^{2}$
$S(x)=\int M(x) d x \cong E I d v / d x \cong E I \theta(x), \quad D(x)=\int S(x) d x \cong E I v(x)$
* Singularity Functions for Common Loadings


$$
\begin{aligned}
w(x)= & 10<x-0>^{-1} *-20<x-5>^{-1} *-2<x-9>^{0}+2<x-15>^{0} \\
& +100<x-20>^{-2}++C_{\theta}<x-20>^{-3} .
\end{aligned}
$$

* First Moment-Area Theorem: $\quad \theta_{B}-\theta_{A}=\int(M / E I) d x$
*Second Moment-Area Theorem: $\left(\mathrm{x}_{\mathrm{B}}-\mathrm{x}_{\mathrm{A}}\right) \theta_{\mathrm{B}}-\mathrm{v}_{\mathrm{B}}+\mathrm{v}_{\mathrm{A}}=\int \mathrm{x}(\mathrm{M} / \mathrm{EI}) \mathrm{dx}$
* Conjugate Beam Method

| Original Beam | Free End | Fixed End | Hinge/Roller End | Internal Support | Internal Hinge |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Conjugate Beam | Fixed End | Free End | Hinge/Roller End | Internal Hinge | Internal Support |

* Euler Buckling Load: $\quad \mathrm{P}_{\mathrm{cr}}=\pi^{2} \mathrm{EI}_{\min } /(\mathrm{kL})^{2}$
* Effect of Initial Imperfection: $\quad v(x)=v_{0 i} /\left[1-P / P_{c r}\right] \sin (\pi x / L) \Rightarrow v(L / 2)=v_{0 i} /\left[1-P / P_{c r}\right]$
* Effect of Load Eccentricity: $\quad \lambda^{2}=\mathrm{P} / \mathrm{EI} \Rightarrow \mathrm{v}(\mathrm{L} / 2)=\mathrm{e}[\mathrm{sec} \lambda \mathrm{L} / 2-1]=\mathrm{e}\left[\mathrm{sec}\left\{(\pi / 2) \sqrt{ }\left(\mathrm{P} / \mathrm{P}_{\mathrm{cr}}\right)\right\}-1\right]$
* Effect of Material Nonlinearity: $P_{c r}=\pi^{2} \mathrm{E}_{\mathrm{q}} / \mathrm{L}^{2} \Rightarrow \sigma_{\mathrm{cr}}=\pi^{2} \mathrm{E}_{\mathrm{f}} / \eta^{2}$
* Eccentric Loading with Elasto-plastic Material:
$\mathrm{v}(\mathrm{L} / 2)=\mathrm{e}\left[\sec \left\{(\pi / 2) \sqrt{ }\left(\mathrm{P} / \mathrm{P}_{\mathrm{cr}}\right)\right\}-1\right]$ for the elastic range; and
$\mathrm{v}(\mathrm{L} / 2)=\mathrm{M}_{\mathrm{p}} / \mathrm{P}-\mathrm{e}$, for the plastic range
$* \mathrm{k}=1.0$ for Hinge-Hinged Beam, 0.7 for Hinge-Fixed Beam, 0.5 for Fixed-Fixed Beam, 2.0 for Cantilever Beam
* In general, k can be obtained from $\psi_{\mathrm{A}}$ and $\psi_{\mathrm{B}}$ for braced and unbraced frames

Using approximate formulae (Salama, 2014)
For braced frame, $\mathrm{k} \cong\left\{3 \psi_{\mathrm{A}} \psi_{\mathrm{B}}+1.4\left(\psi_{\mathrm{A}}+\psi_{\mathrm{B}}\right)+0.64\right\} /\left\{3 \psi_{\mathrm{A}} \psi_{\mathrm{B}}+2.0\left(\psi_{\mathrm{A}}+\psi_{\mathrm{B}}\right)+1.28\right\}$
For unbraced frame, $\mathrm{k} \cong \sqrt{ }\left[\left\{1.6 \psi_{\mathrm{A}} \psi_{\mathrm{B}}+4.0\left(\psi_{\mathrm{A}}+\psi_{\mathrm{B}}\right)+7.5\right\} /\left(\psi_{\mathrm{A}}+\psi_{\mathrm{B}}+7.5\right)\right]$

* AISC-ASD Method, $\eta=L_{e} / r_{\text {min }}$, and $\eta_{c}=\pi \sqrt{ }\left(2 \mathrm{E} / \mathrm{f}_{\mathrm{y}}\right)$

If $\eta \leq \eta_{\mathrm{c}}, \sigma_{\text {all }}=\mathrm{f}_{\mathrm{y}}\left[1-0.5\left(\eta / \eta_{\mathrm{c}}\right)^{2}\right] / \mathrm{FS}$, where FS $=\left[5 / 3+3 / 8\left(\eta / \eta_{\mathrm{c}}\right)-1 / 8\left(\eta / \eta_{\mathrm{c}}\right)^{3}\right]$
If $\eta>\eta_{c}, \sigma_{\text {all }}=\left(\pi^{2} \mathrm{E} / \eta^{2}\right) / \mathrm{FS}$, where FS $=$ Factor of safety $=23 / 12=1.92$

* Moment magnification factor for a Simply Supported Beam

For concentrated load at midspan of $=[\tan (\lambda \mathrm{L} / 2) /(\lambda \mathrm{L} / 2)]$, subjected to end moments only $=[\sec (\lambda \mathrm{L} / 2)]$
Under UDL $=2[\sec (\lambda \mathrm{~L} / 2)-1] /(\lambda \mathrm{L} / 2)^{2}$, according to AISC code $=1 /\left(1-\mathrm{P} / \mathrm{P}_{\mathrm{cr}}\right)$

Department of Civil Engineering Final Examination Fall 2019 (Set 2)
Program: B. Sc. Engineering (Civil)

1. Calculate the equivalent polar moment of inertia
( $J_{\text {cq }}$ ) for the cross-section shown in Fig. 1(a) by centerline dimensions
[Given: Wall thickness $=0.10^{\prime}$ ].


Fig. 1(a)

## OR

Fig. 1(b) shows a person wearing a fruit-peel as protection from corona-virus.
While covering, the peel is subjected to uniform pressure resulting in tensile and shear stresses.
Fig. 1(c) represents the tensile stresses

$$
\sigma_{x}=2 \mathrm{kPa}, \sigma_{y}=2 \mathrm{kPa}
$$

and shear stress $\tau=1 \mathrm{kPa}$ in a small element of the fruit-peel.

(i) Draw the Mohr's Circle of stresses for the element shown in Fig. 1(c), specifying the Principal Stresses and Principal Planes
(ii) Use Rankine criterion to determine yield strength $Y$ required to avoid yielding of the fruit-peel.
2. Fig. 2(a) shows a person wearing a giraffe-costume as protection from corona-virus.

In addition to self-weights $W_{1-4}\left(W_{1}=50 \mathrm{lb}, W_{2}=70 \mathrm{lb}, W_{3}=30 \mathrm{lb}, W_{4}=10 \mathrm{lb}\right)$, he is subjected to horizontal forces $H_{1-4}$ ( $=30 \%$ of self-weights) as shown in Fig. 2(b).
Calculate the
(i) Maximum normal stresses on his shoes ( $1.5^{\prime}$-dia circle), shown in Fig. 2(c)
(ii) Maximum shear stress and deflection of the two helical springs [Fig. 2(d)] (Mean diameter $=1^{\prime \prime}$, Shear Modulus $=12 \times 10^{6} \mathrm{psi}$, No. of Coils $=4$, Coil Diameter $=0.1^{\prime \prime}$ ) supporting each shoe.


Fig. 2(a)


Fig. 2(b)


Fig. 2(c)

Fig. 2(d)
3. Fig. 3 shows a beam $a b c$ defgh carrying a distributed load $(1 \mathrm{lb} / \mathrm{ft})$ from a snake over the length $b c$ and concentrated loads ( $1 \mathrm{lb}, 1 \mathrm{lb}, 1 \mathrm{lb}$ ) at $(e, f, g)$ from three bats.


Use Moment-Area Theorems to calculate
(i) The value of $E I$ to cause $d$ to deflect 1 -inch vertically
(ii) Rotation at $a$, for the value of $E I$ calculated in (i).
4. Answer Question 3 using the Conjugate Beam Method.
5. If the roller support $a$ (of beam abcdefgh in Fig. 3) is replaced by a fixed support (shown in Fig. 4), use
Moment-Area Theorems OR Conjugate Beam Method to calculate
(i) The value of $E I$ to cause $d$ to deflect 1 -inch vertically
(ii) Rotation at $a$, for the value of $E I$ calculated in (i).

6. Answer Question 3 using the Singularity Functions.

## OR

(i) For Fig. 5 showing a snake on a branch (with variable self-weight)

- Write equation for load $w(x)$ using singularity functions
- Write down the boundary conditions
- Draw the qualitative deflected shape
- Determine if it is statically determinate or indeterminate.
(ii) Explain the
(a) Difference between the behavior of short columns and long columns under compression
(b) Combined effect of load eccentricity and Plastic Moment ( $M_{p}$ ) on buckling behavior of slender columns.
$1 \mathrm{lb} / \mathrm{ft}$

$3 \mathrm{lb} / \mathrm{ft}$

7. Fig. 6(a) shows an emergency corona-virus hospital built in China in two days, while Fig. 6(b) shows its simplified schematic diagram. Since the soil condition is uncertain, various foundation models are assumed at $a_{0}, b_{0}, c_{0}, d_{0}$ and $e_{0}$.

Under the circumstances, calculate the critical buckling loads ( $P_{c r}$ ) for Columns $a_{0} a_{1}, b_{0} b_{1}, c_{0} c_{1}, d_{0} d_{1}$ and $e_{0} e_{1}$, assuming $E I=$ Constant $=5,000 \mathrm{kN}-\mathrm{m}^{2}$.


Fig. 6(a)


Fig. 6(b)
8. The hospital building shown in Figs. 6(a), 6(b) is constructed of a material with stress-strain relationship $\sigma=100[1-\operatorname{Cos}(100 \varepsilon)]$, where $\sigma$ is compressive stress $(\mathrm{MPa})$ and $\varepsilon$ is strain.

Calculate the critical force for the member $b_{1} b_{2}$, which has a $\left(0.3^{\mathrm{m}} \times 0.3^{\mathrm{m}}\right)$ square section.
9. For the emergency hospital building shown in Figs. 6(a), 6(b), use AISC-ASD method to calculate the allowable compressive force for member $c_{1} c_{2}$, using Salama (2014) to determine $k$
[Given: $c_{1} c_{2}$ is a $\left(0.3^{\mathrm{m}} \times 0.3^{\mathrm{m}}\right)$ square section,
Yield strength $f_{y}=100 \mathrm{MPa}$, Modulus of elasticity $E=10,000 \mathrm{MPa}$ ].

## OR

For the emergency hospital building shown in Figs. 6(a), 6(b), calculate distributed load $w(\mathrm{kN} / \mathrm{m})$ required to cause Moment Magnification Factor $M M F=2.0$ (using AISC formula) for member $c_{1} c_{2}$,
10. Fig. 7(a) shows a soil-excavator used for land development to build the emergency corona-virus hospital described in Question 7, while Fig. 7(b) represents its simplified schematic diagram.
(i) Calculate the Buckling force $P_{c r}$ of the member $m n\left(E I=5,000 \mathrm{kN}-\mathrm{m}^{2}\right.$, initial deflection $\left.v_{0 i}=2^{m}\right)$ and the compressive force $\left(P_{m n}\right)$ required for it to deflect $3^{\mathrm{m}}$ at midspan.


Fig. 7(a)


Fig. 7(b)
(ii) Use the free-body of member $r n s$ to calculate the upward force $R$ required to cause compressive force $\left(P_{m n}\right)$ in member $m n$, as calculated in (i) for it to deflect $3^{\mathrm{m}}$ at midspan.

## List of Useful Formulae for CE 213

* Torsional Rotation $\phi_{\mathrm{B}}-\phi_{\mathrm{A}}=\int\left(\mathrm{T} / \mathrm{J}_{\mathrm{eq}} \mathrm{G}\right) \mathrm{dx}$, and $=\left(\mathrm{TL} / \mathrm{J}_{\mathrm{eq}} \mathrm{G}\right)$, if T , $\mathrm{J}_{\mathrm{eq}}$ and G are constants

| Section | Torsional Shear Stress | $\mathbf{J}_{\mathbf{e q}}$ |
| :---: | :---: | :---: |
| Circular | $\tau=\mathrm{Tc} / \mathbf{J}$ | $\pi \mathrm{d}^{4} / 32$ |
| Thin-walled | $\tau=\mathrm{T} /(2(\mathrm{~A}) \mathrm{t})$ | $4\left(\mathrm{~A}^{2} /(\mathrm{dds} / \mathrm{t})\right.$ |
| Rectangular | $\tau=\mathrm{T} /\left(\alpha \mathrm{bt} \mathrm{t}^{2}\right)$ | $\beta \mathrm{bt}^{3}$ |


| $\mathrm{b} / \mathrm{t}$ | 1.0 | 1.5 | 2.0 | 3.0 | 6.0 | 10.0 | $\propto$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | 0.208 | 0.231 | 0.246 | 0.267 | 0.299 | 0.312 | 0.333 |
| $\beta$ | 0.141 | 0.196 | 0.229 | 0.263 | 0.299 | 0.312 | 0.333 |

* Biaxial Bending Stress: $\sigma_{x}(z, y)=M_{z} y / I_{z}+M_{y} z / I_{y}$
* Combined Axial Stress and Biaxial Bending Stress: $\sigma_{z}(x, y)=-P / A-M_{x} y / I_{x}-M_{y} x / I_{y}$
* Corner points of the kern of a Rectangular Area are (b/6, 0), ( $0, \mathrm{~h} / 6$ ), (-b/6, 0), ( $0,-\mathrm{h} / 6$ )
* Maximum shear stress on a Helical spring: $\tau_{\max }=\tau_{\text {direct }}+\tau_{\text {torsion }}=P / A+T r / J=P / A(1+2 R / r)$
* Stiffness of a Helical spring is $\mathrm{k}=\mathrm{Gd}^{4} /\left(64 \mathrm{R}^{3} \mathrm{~N}\right)$
* $\sigma_{x x}{ }^{\prime}=\left(\sigma_{x x}+\sigma_{y y}\right) / 2+\left\{\left(\sigma_{x x}-\sigma_{y y}\right) / 2\right\} \cos 2 \theta+\left(\tau_{x y}\right) \sin 2 \theta=\left(\sigma_{x x}+\sigma_{y y}\right) / 2+\sqrt{ }\left[\left\{\left(\sigma_{x x}-\sigma_{y y}\right) / 2\right\}^{2}+\left(\tau_{x y}\right)^{2}\right] \cos (2 \theta-\alpha)$
$\tau_{\mathrm{xy}}{ }^{\prime}=-\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\} \sin 2 \theta+\left(\tau_{\mathrm{xy}}\right) \cos 2 \theta=\tau_{\mathrm{xy}}{ }^{\prime}=-\sqrt{ }\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right] \sin (2 \theta-\alpha)$
where $\tan \alpha=2 \tau_{\mathrm{xy}} /\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right)$
* $\sigma_{\mathrm{xx}(\text { max })}=\left(\sigma_{\mathrm{xx}}+\sigma_{\mathrm{yy}}\right) / 2+\sqrt{ }\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right]$; when $\theta=\alpha / 2, \alpha / 2+180^{\circ}$
$\sigma_{\mathrm{xx}(\text { min })}=\left(\sigma_{\mathrm{xx}}+\sigma_{\mathrm{yy}}\right) / 2-\sqrt{ }\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right]$; when $\theta=\alpha / 2 \pm 90^{\circ}$
${ }^{*} \tau_{\mathrm{xy}(\max )}=\sqrt{ }\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{\mathrm{yy}}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right]$; when $\theta=\alpha / 2-45^{\circ}, \alpha / 2+135^{\circ}$
$\tau_{x y(\text { min })}=-\sqrt{ }\left[\left\{\left(\sigma_{x x}-\sigma_{y y}\right) / 2\right\}^{2}+\left(\tau_{x y}\right)^{2}\right]$; when $\theta=\alpha / 2+45^{\circ}, \alpha / 2-135^{\circ}$
* Mohr's Circle: Center $(\mathrm{a}, 0)=\left[\left(\sigma_{\mathrm{xx}}+\sigma_{y y}\right) / 2,0\right]$ and radius $\mathrm{R}=\sqrt{ }\left[\left\{\left(\sigma_{\mathrm{xx}}-\sigma_{y y}\right) / 2\right\}^{2}+\left(\tau_{\mathrm{xy}}\right)^{2}\right]$
* For Yielding to take place

$$
\begin{array}{lc}
\text { Maximum Normal Stress Theory (Rankine): } & \left|\sigma_{1}\right| \geq \mathrm{Y}, \text { or }\left|\sigma_{2}\right| \geq \mathrm{Y} . \\
\text { Maximum Normal Strain Theory (St. Venant): } & \left|\sigma_{1}-v \sigma_{2}\right| \geq \mathrm{Y}, \text { or }\left|\sigma_{2}-v \sigma_{1}\right| \geq \mathrm{Y} . \\
\text { Maximum Shear Stress Theory (Tresca): } & \left|\sigma_{1}-\sigma_{2}\right| \geq \mathrm{Y},\left|\sigma_{1}\right| \geq \mathrm{Y}, \text { or }\left|\sigma_{2}\right| \geq \mathrm{Y} \\
\text { Maximum Distortion-Energy Theory (Von Mises): } \sigma_{1}^{2}+\sigma_{2}^{2}-\sigma_{1} \sigma_{2} \geq \mathrm{Y}^{2}
\end{array}
$$

* $\mathrm{M}(\mathrm{x})=\mathrm{EI} \kappa \cong \mathrm{EI} \mathrm{d}^{2} \mathrm{v} / \mathrm{dx}^{2}$
* $w(x) \cong E I d^{4} v / d x^{4}, \quad V(x)=\int w(x) d x \cong E I d^{3} v / d x^{3}, \quad M(x)=\int V(x) d x \cong E I d^{2} v / d x^{2}$

$$
S(x)=\int M(x) d x \cong E I d v / d x \cong E I \theta(x), \quad D(x)=\int S(x) d x \cong E I v(x)
$$

* Singularity Functions for Common Loadings


$$
\begin{aligned}
w(x)= & \left.10<x-0>^{-1} \cdot-20<x-5>^{-1} \cdot-2<x-9\right\rangle^{0}+2<x-15>^{0} \\
& +100<x-20>^{-2} \cdot+C_{\theta}<x-20>^{-3} .
\end{aligned}
$$

* First Moment-Area Theorem: $\quad \theta_{\mathrm{B}}-\theta_{\mathrm{A}}=\int(\mathrm{M} / \mathrm{EI}) \mathrm{dx}$
* Second Moment-Area Theorem: $\left(x_{B}-x_{A}\right) \theta_{B}-v_{B}+v_{A}=\int x(M / E I) d x$
* Conjugate Beam Method

| Original Beam | Free End | Fixed End | Hinge/Roller End | Internal Support | Internal Hinge |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Conjugate Beam | Fixed End | Free End | Hinge/Roller End | Internal Hinge | Internal Support |

* Euler Buckling Load: $\quad \mathrm{P}_{\mathrm{cr}}=\pi^{2} \mathrm{EI}_{\text {min }} /(\mathrm{kL})^{2}$
* Effect of Initial Imperfection: $\quad v(x)=v_{0 i} /\left[1-P / P_{c r}\right] \sin (\pi x / L) \Rightarrow v(L / 2)=v_{0 i} /\left[1-P / P_{c r}\right]$
* Effect of Load Eccentricity: $\quad \lambda^{2}=\mathrm{P} / \mathrm{EI} \Rightarrow \mathrm{v}(\mathrm{L} / 2)=\mathrm{e}[\sec \lambda \mathrm{L} / 2-1]=\mathrm{e}\left[\mathrm{sec}\left\{(\pi / 2) \sqrt{ }\left(\mathrm{P} / \mathrm{P}_{\mathrm{cr}}\right)\right\}-1\right]$
* Effect of Material Nonlinearity: $P_{c r}=\pi^{2} \mathrm{E}_{\mathrm{l}} \mathrm{I} / \mathrm{L}^{2} \Rightarrow \sigma_{\mathrm{cr}}=\pi^{2} \mathrm{E}_{\mathrm{f}} / \eta^{2}$
* Eccentric Loading with Elasto-plastic Material:
$\mathrm{v}(\mathrm{L} / 2)=\mathrm{e}\left[\sec \left\{(\pi / 2) \sqrt{ }\left(\mathrm{P} / \mathrm{P}_{\mathrm{ct}}\right)\right\}-1\right]$ for the elastic range; and
$\mathrm{v}(\mathrm{L} / 2)=\mathrm{M}_{\mathrm{p}} / \mathrm{P}-\mathrm{e}$, for the plastic, range
* $\mathrm{k}=1.0$ for Hinge-Hinged Beam, 0.7 for Hinge-Fixed Beam, 0.5 for Fixed-Fixed Beam, 2.0 for Cantilever Beam
* In general, $k$ can be obtained from $\psi_{\mathrm{A}}$ and $\psi_{\mathrm{B}}$ for braced and unbraced frames

Using approximate formulae (Salama, 2014)
For braced frame, $\mathrm{k} \cong\left\{3 \psi_{\mathrm{A}} \psi_{\mathrm{B}}+1.4\left(\psi_{\mathrm{A}}+\psi_{\mathrm{B}}\right)+0.64\right\} /\left\{3 \psi_{\mathrm{A}} \psi_{\mathrm{B}}+2.0\left(\psi_{\mathrm{A}}+\psi_{\mathrm{B}}\right)+1.28\right\}$
For unbraced frame, $\mathrm{k} \cong \sqrt{ }\left[\left\{1.6 \psi_{\mathrm{A}} \psi_{\mathrm{B}}+4.0\left(\psi_{\mathrm{A}}+\psi_{\mathrm{B}}\right)+7.5\right\} /\left(\psi_{\mathrm{A}}+\psi_{\mathrm{B}}+7.5\right)\right]$

* AISC-ASD Method, $\eta=L_{e} / r_{\text {min }}$, and $\eta_{c}=\pi \sqrt{ }\left(2 E / f_{y}\right)$

If $\eta \leq \eta_{c}, \sigma_{\text {all }}=f_{y}\left[1-0.5\left(\eta / \eta_{c}\right)^{2}\right] /$ FS, where FS $=\left[5 / 3+3 / 8\left(\eta / \eta_{c}\right)-1 / 8\left(\eta / \eta_{c}\right)^{3}\right]$
If $\eta>\eta_{\mathrm{c}}, \sigma_{\text {all }}=\left(\pi^{2} \mathrm{E} / \eta^{2}\right) / \mathrm{FS}$, where FS $=$ Factor of safety $=23 / 12=1.92$

* Moment magnification factor for a Simply Supported Beam

For concentrated load at midspan of $=[\tan (\lambda L / 2) /(\lambda L / 2)]$, subjected to end moments only $=[\sec (\lambda L / 2)]$
Under UDL $=2[\sec (\lambda L / 2)-1] /(\lambda L / 2)^{2}$, according to AISC code $=1 /\left(1-\mathrm{P} / \mathrm{P}_{\text {cr }}\right)$

## Alignment Charts for Effective Length Factors k


$\psi=$ Ratio of $\sum E I / L$ of compression members to $\sum E I / L$ of flexural members in a plane at one end a compression member $\mathrm{k}=$ Effective length factor

# University of Asia Pacific <br> Department of Basic Sciences and Humanities <br> Final Examination, Fall-2019 <br> Program: B. Sc. in Civil Engineering 

Course Title: Mathematics IV
Course Code: MTH 203
Time: 3.00 Hour
Full Marks: 150
There are Eight questions. Answer Six questions including questions 1, 2, 3 and 4. All questions are of equal value. Figures in the right margin indicate marks.

1. (a) Define Bernoulli's equation. Solve the following differential equation

$$
\frac{d y}{d x}+x \sin 2 y=x^{3} \cos ^{2} y
$$

(b) When a cake is removed from an oven, its temperature is measured at $300^{\circ}$ Fahrenheit. Three minutes later, its temperature is $200^{\circ}$ Fahrenheit. How long will it take for the cake to cool off to a temperature of $80^{\circ}$ Fahrenheit? The room temperature is $70^{\circ}$ Fahrenheit.
2. (a) Define Cauchy-Euler equation and solve the following Cauchy-Euler equation

$$
x^{2} \frac{d^{2} y}{d x^{2}}-2 x \frac{d y}{d x}+2 y=x^{3}
$$

(b) Solve the following higher order differential equations:
(i) $\quad\left(D^{3}-D^{2}-6 D\right) y=x^{2}+1$
(ii) $\left(D^{2}+3 D+2\right) y=e^{2 x} \sin x$.
3. (a) Define Laplace Transform. Find Laplace Transform of the following functions
i. $\quad \mathcal{L}\left\{\left(5 t+e^{-t}\right)^{2}\right\}$
ii. $\mathcal{L}\left\{t^{2} \cos a t\right\}$
iii. $\quad \mathcal{L}\{G(t)\}$ where $G(t)=\left\{\begin{array}{r}\sin \left(t-\frac{\pi}{2}\right), t>\frac{\pi}{2} \\ 0, t<\frac{\pi}{2}\end{array}\right.$
iv. $\mathcal{L}\left\{e^{4 t} \cosh 5 t\right\}$
v. $\mathcal{L}\left\{e^{-2 t}(\cos 6 t-\sin 6 t)\right\}$
(b) Prove that
i. $\quad \mathcal{L}\{t\}=\frac{1!}{s^{2}}$
ii. $\quad \mathcal{L}\{\cosh a t\}=\frac{s}{s^{2}-a^{2}}$
4. (a) Solve the equation using Laplace Transform $F^{\prime \prime}+F=t, F(0)=1, F^{\prime}(0)=-2$.
(b) A particle of mass 2 grams is attached to a spring moves on the $X$-axis and is attracted toward the origin with a force numerically equal to 8 . If it is initially at rest at $X=10$, find its position at any subsequent time assuming
i. No other forces act.
ii. A damping force numerically equal to 8 times the velocity acts.
5. (a) $F(t)=\left\{\begin{array}{r}2 t^{2}, 0<t<1 \\ t, 1<t<2\end{array}\right.$ is a periodic function with period 2. Then,
i. $\quad$ Sketch $F(t)$
ii. Evaluate $\mathcal{L}\{F(t)\}$
(b) State HEAVISIDE'S Expansion Formula. Then apply HEAVISIDE'S Expansion

Formula to evaluate $\mathcal{L}^{-1}\left\{\frac{2 s^{2}-4}{(s+1)(s-3)(s-2)}\right\}$

## OR

6. (a) Define Inverse Laplace Transform. Apply Partial Fraction method to evaluate $\mathcal{L}^{-1}\left\{\frac{4 s+5}{(s-1)^{2}(s+2)}\right\}$
(b) Write down the formula of the Laplace Transform of Periodic Function.
$F(t)=\left\{\begin{array}{r}\sin t, 0<t<\pi \\ 0, \pi<t<2 \pi\end{array}\right.$ is a periodic function with period $2 \pi$. Then,
i. Sketch $F(t)$
ii. Evaluate $\mathcal{L}\{F(t)\}$
7. (a) Define Full Range Fourier Series with Fourier Coefficients. Find the Full Range Fourier Series of the function $f(x)=\left\{\begin{array}{cc}-1 & ,-1<x<0 \\ 1 & , 0<x<1\end{array}\right.$ having period 2.
(b) Find the Finite Fourier Cosine transform of the function

$$
F(x)=2 x, \quad 0<x<4
$$

8. (a) Define Half Range Fourier Cosine and Sine Series. Find the Half Range Fourier Cosine and Sine Series of the function $f(x)=x^{2}, 0<x<3$
(b) Find the Infinite Fourier Cosine transform of the function $f(x)=e^{-x}, x \geq 0$
